THE CHARACTERISTICS OF MARINE CURRENT POWER BASED ON STOCKES MODELS OF MARINE CURRENT SPEED

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Keywords: *Marine power, Marine current speed, Power coefficients, Stockes models*

ABSTRACT: The mathematical representation of the marine current power requires knowledge of the mathematical models of the marine current speed and of the power coefficients. A short review of the mathematical models of marine currents speed and of the power coefficients is given in this paper. For any of the listed marine current mathematical models simulation results of marine current velocity as a function of time are presented. Also, the experimental results of the marine power coefficients, found in literature, and its fitted mathematical curve are presented. Finaly, as the most common model of the marine current speed, the Stockes models of the swell are used for analyzing the impact of the turbine depth on the marine current avalilable power. All the mentioned mathematical models are implemented, and simulation results obtained, using the software package MATLAB.

1. INTRODUCTION

Nowdays it has become imperative to search for new ways to compensate the lack of energy as the rate of using of the fossil fuels becomes unsustainable. This primarily refers to the development of new renewable energy technologies such as the use of solar energy, wind energy, ocean energy and similar [1]. Ocean energy conversion represents a huge potential in term of renewable energy resource and receives more and more attention from many developers around the world although it still remains immature compared to other renewable technologies [2].

The main potential sources of ocean energy are waves, currents and ocean thermal energy [2, 3]. Currently, the most interesting form of ocean energy is the energy of the ocean currents [4-18]. The kinetic energy of the ocean current can be converted into electricity using conventional turbine technology [4-8, 12, 17-18]. Namely, as marine current turbines

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are similar in many aspects to wind turbine technologies, their theoretical and experimental studies as well as turbine technology can be essentially based on wind turbine experiences [16].

It is well known that wind turbine tower height has a high impact on the wind turbine available power [1]. However, in any of cited papers which analyze the marine current power [2-14, 17-18] the impact of the turbine depth on the marine turbine available power is not analyzed. For this reason, the main goal of this paper is an analysis of the impact of the turbine depth on the mean and maximal available marine turbine power. This investigation will be based on the usage of the Stockes models of the marine current speed. As mathematical model of the marine current speed is very important for this analysis, the mathematical model of the marine current speed will be reviewed in this paper. Also, as marine power depends on marine current power coefficients its mathematical review will be given.

The paper is organized as follows. A short review of the mathematical models of the marine current speed is given in Section II. For any of the listed marine current mathematical models, simulation results of marine current velocity as a function of time is presented. Section III discusses the mathematical models of the marine current power coefficients. In this section, the concrete experimental results of the marine power coefficient are fitted using several mathematical equations of marine power coefficients. The certain characteristics of the marine current power based on Stockes models of the marine current speed are given in Section IV. The conclusion is given in Section V.

2. SHORT REVIEW OF THE MARINE CURRENT SPEED MATHEMATICAL MODELS

Detailed review and comparison of the marine current speed mathematical models is given in [15-16]. Some of them are particularly analyzed in [9-13].

Generally, in literature one can find next mathematical models of the marine current speed:

- Model based on the JONSWAP spectrum [11], [18] – Zhou model
- Model based on the Pierson-Moskowitz spectrum [9, 10]
- Benelghali model of the marine current speed [12]
- Amin model of the marine current speed [14]
- Stockes models of the marine current speed [12, 13]

2.1 Marine current speed mathematical models based on the JONSWAP and Pierson-Moskowitz spectrum

Mathematical model of the marine current speed, based on the JONSWAP spectrum, can be represented by the following formula:

$$V(t) = V_{tide} + \sum_i \frac{2\pi a_i}{T_i} \frac{\cosh\left(2\pi \frac{z+d}{L_i}\right)}{\sinh\left(2\pi \frac{d}{L_i}\right)} \cos 2\pi \left(\frac{t}{T_i} - \frac{x}{L_i} + \phi_i\right), \tag{1}$$

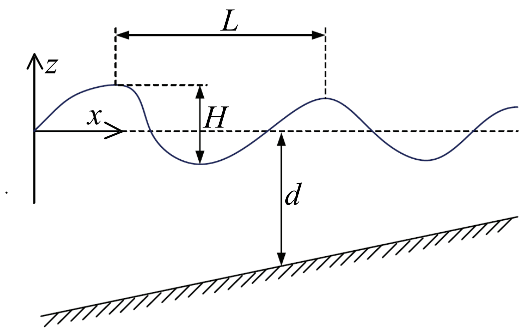

Fig. 1. Swell characteristics

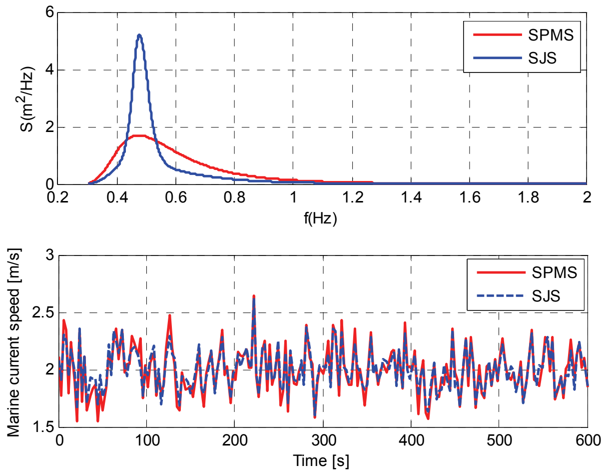

Fig. 2. JONSWAP (SJS) and Pierson-Moskowitz (SPMS) spectrum as well as the simulation waveform of total marine current speed, represented by JONSWAP (SJS) and Pierson-Moskowitz (SPMS) spectrum

where first term V_{tide} represents the predicted tidal speed, which can be regarded as a constant during a period less than an hour, while the second term represents the current speed oscillation caused by the swell. The swell characteristics d, L, T_i, y and x are given in Fig. 1, while the angle φ_i, which is given randomly, represents the initial phase angle of each frequency component. The amplitude of each frequency components a_i can be calculated by

$$a_i = \sqrt{2S(f_i)\Delta f_i}, \tag{2}$$

where $S(f_i)$ represents the value of the JONSWAP spectrum, defined by:

$$S(f) = \beta_i \frac{H_S}{T_P^4} \frac{1}{f^5} \exp\left(-\frac{4}{5} \frac{1}{T_P^4} \frac{1}{f^5}\right) \gamma^Y, \tag{3}$$

where $Y = \exp\left[-\frac{(T_P f - 1)^2}{2\sigma^2}\right]$, $\sigma = \begin{cases} 0.07, f \le 1/T_P \\ 0.09, f \ge 1/T_P \end{cases}$, and $\beta_i = -\frac{0.0624(1.094 - 0.0295\ln\gamma)}{0.22 + 0.0338\gamma - 0.185(1.9+\gamma)}$.

The parameter γ, which controls the sharpness of the spectral peak, is called peak enhancement factor, T_p is dominant period of the swell spectar and *f* represent any spectral frequency.

Mathematical model of the marine current speed based on Pierson-Moskowitz spectrum can be represented also by (1). However, the amplitude of each frequency components are calculated by using Pierson-Moskowitz spectrum:

$$S_{PM}(f) = \frac{5}{16} H_S \frac{f_P}{2\pi f^5} \exp\left(-\frac{5}{4}\left(\frac{f_P}{f}\right)^4\right), \tag{4}$$

where f_p is dominant frequency of the swell spectar.

Fig. 2 shows the simulation waveform of total marine current speed, represented by JONSWAP (SJS) i Pierson-Moskowitz (SPMS) spectrum, for the following parameters: V_{tide}=2*m/s*, H_s=3*m*, T_p=13.2*s*, γ=7, x=z=0, d=100*m*.

2.2 Benelghali model of the marine current speed

Zhou model and model based on the Pierson-Moskowitz spectrum assume that the predicted tidal speed is constant. However, in Benelghali model this component of the marine current speed is not constant, and can be presented by the following equation:

$$V_{tide} = V_0 + A\cos\omega_1 t + B\cos\omega_2 t \tag{5}$$

where A, B, V_0, ω_1 and ω_2 are constants.

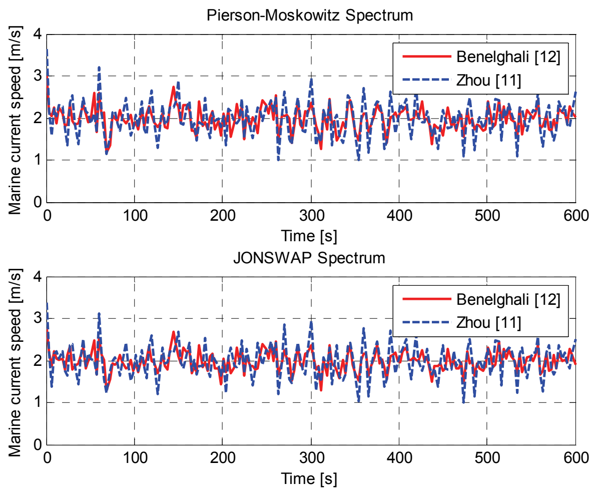

Fig. 3. The simulation waveforms of total marine current speed, represented by JONSWAP i Pierson-Moskowitz spectrum for Benelghali [12] and Zhou [11] model

Fig. 3 shows the simulation waveforms of total marine current speed, presented by JONSWAP (SJS) and Pierson-Moskowitz (SPMS) spectrum for Benelghali [12] i Zhou [11] model, for A=0.3252m/s, B=0.2749m/s, ω_1=0.4189s^{-1} i ω_2=0.6283s^{-1}.

2.3 Amin model of the marine current speed

Amin model of the marine current speed is represented by certain Fourier expression which is used to create random current velocity from several sinusoidal wave, which have different amplitudes and frequencies. Namely, the mathematical expression of Amin model is given by the following formula:

$$V(t) = V_0 + \sum_{i=1}^{n} A V_0 \sin(2\pi f_i \cdot t) \tag{6}$$

where V_0 is the initial steady current velocity, A is current amplitude, f_i is fluctuating current frequency, and t is time [14, 16]

Fig. 4 represents the simulation waveforms of Amin total marine current speed. In this case three sinusoidal wave are used. Its current amplitudes are 0.2 m, 0.1 m and 0.05 m, while its current frequencies are 100 rad/sec, 30 rad/sec and 5 rad/sec, respectively.

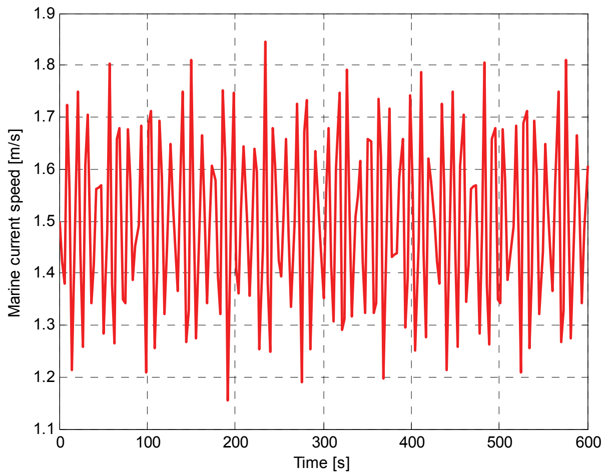

Fig. 4. The simulation waveforms of total marine current speed – Amin model

2.4 Stockes models of the marine current speed

Stockes models from the fluid theory [12, 13] can be used for marine current speed representation. In [12], for mathematical representation of the marine current speed, the first order Stockes model is used. It can be presented as follows:

$$V_{(x)} = \frac{H}{2}\frac{gk}{\omega}\frac{\cosh\left[k(z+d)\right]}{\cosh(kd)}\sin(kx-\omega t) \tag{7}$$

Beside first order Stockes model of the marine current speed the second order Stockes model is also given in [13],

$$V_{2(x)} = V_{(x)} - \left(\frac{H}{2}\right)^2 \frac{gk}{\omega} \frac{3\cosh\left[2k(z+d)\right]}{4\sinh^2(kd)\cosh(kd)} \sin\left(2(kx-\omega t)\right), \quad (8)$$

Also, in [12] the third order Stockes model can be found:

$$V_{3(x)} = V_{2(x)} - \left(\frac{H}{2}\right)^2 \frac{gk^2}{\omega} \frac{33-6\cosh(2kd)\cosh\left[3k(z+d)\right]}{64\cosh(kd)\sinh^8(kd)} \sin\left(3(kx-\omega t)\right). \quad (9)$$

The simulation waveforms of total marine current speed, represented by first, second and third order Stockes model for ocean depth d=30m are presented in Fig. 5.

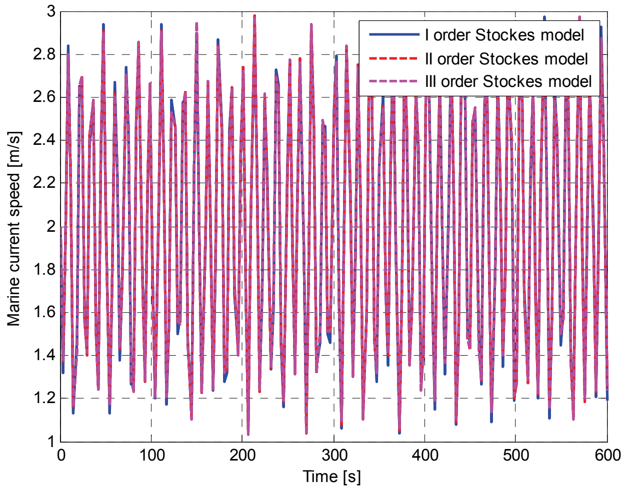

Fig. 5. The simulation waveforms of total marine current speed, represented by first, second and third order Stockes model for ocean depth d=30m

3. MATHEMATICAL MODELS OF THE MARINE CURRENT ROTOR EFFICIENCY

For analysing marine current power mathematical model of the rotor efficiency is, also, necessary. The rotor efficiency, or power coefficient, represent the percentage of mechanical power that can be extracted from the fluid stream by the turbine [1]. This coefficient, which depends on rotor blade geometry, can be represented as a nonlinear function of the water speed, turbine speed and blade pitch angle [6].

In many papers mathematical models of the marine power coefficients are not presented [4-5, 7-8, 12]. On the other hand, in [6, 17] the following equation for the marine power coefficient, obtained by fitting experimental result, is presented:

$$C_{P\_[6]} = 0.0195\lambda^2\left(1.3172e^{-0.3958\lambda+1.539} - 0.0867\cos\left(0.4019\lambda - 5.6931\right)\right), \quad (10)$$

where:

$$\lambda = \frac{R \cdot \omega_{turb}}{v_{wind}} \tag{11}$$

R is turbine radius, ω_{turb} is angular turbine speed and ν_{wind} is wind speed.
So, general form of eq.(10) is as follows:

$$C_{P\_[6]\_general} = A\lambda^2 \left(Be^{C \cdot \lambda + D} - E\cos\left(F \cdot \lambda - G\right)\right), \tag{12}$$

where A, B, C, D, E, F and G are constant. However, as marine current turbines are similar in many aspects to wind turbine technologies, based on wind turbine power coefficients, different mathematical models of the marine current power coefficients have been presented in [16]:

$$C_{P1} = \left(\frac{a_1}{\lambda + b_1} - c_1\right) e^{-\frac{d_1}{\lambda + b_1}}, \tag{13}$$

$$C_{P2} = \left(\frac{a_2}{\lambda + b_2} - c_2\right) e^{-\frac{d_2}{\lambda + b_2}} + e_2 \cdot \lambda, \tag{14}$$

$$C_{P3} = a_3 \cdot \sin\left(b_3 \cdot \lambda - c_3\right) - d_3 \cdot \lambda + e_3, \tag{15}$$

where a_k, b_k, c_k, d_k, and e_i are konstant, k=1,2,3 and i=1,2.

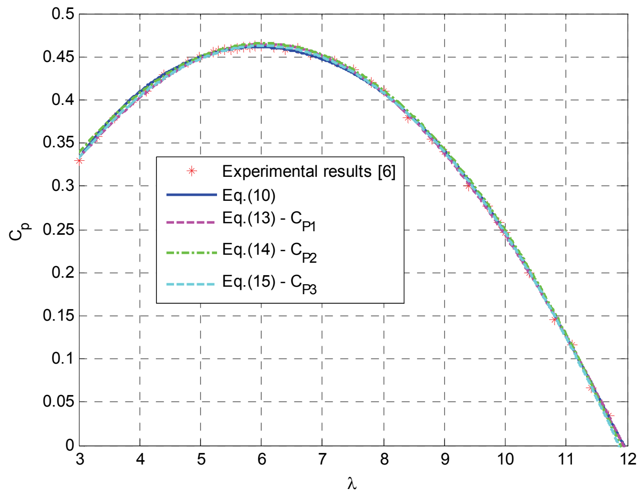

Fig. 6. Experimental, Eq.(10), Eq.(13), Eq.(14) and Eq.(15) C_p curves

Using, for example, the Least Square Method (LMS), the experimental results for power coefficient, can be fitted by using Eq. (12-15). The C_p vs. λ characteristics for experimental results taken from [6, 17], and for Eq.(10, 13-15) are shown on Fig. 6. The detailed review of the marine current rotor efficiency mathematical models can be found in [16].

4. MARINE CURRENT TURBINE POWER

The power harnessed by a marine current turbine can be calculated by the following equation:

$$P_{meh} = \frac{1}{2}\rho C_p \pi R^2 v_{water}^3, \tag{16}$$

where ρ is water density, R is turbine blade radius and v_{water} is water speed. Extractable powers of the marine current turbine, with turbine blade radius R=8m, at different marine current speeds are calculated based on (16) and illustrated by Fig. 7.

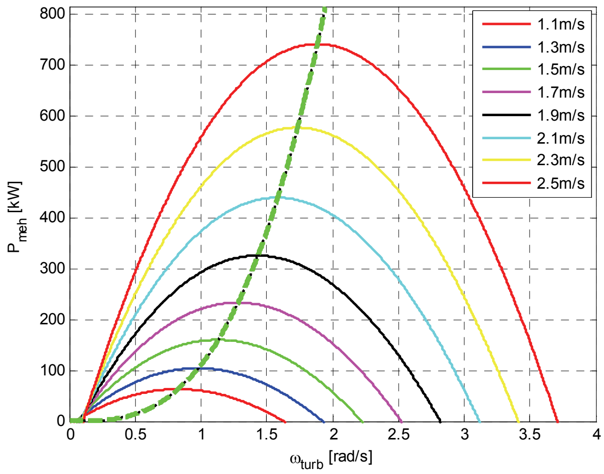

Fig. 7. Power curves for different marine current speed

As it is noted in Introduction, in any of cited paper the impact of the turbine depth on the available power is not considered. In [3] it is only noted that approximately 75 percent of the energy can be found in the upper 50 percent of the flow depth.

The impact of the ocean depth on the marine current speed value is analysed in [16]. In the above mentioned paper it is noted that Stockes model enable marine current speed representation for any depth of the turbine. For this reason, by using eq. (16), as well as eq. (7) or (8-9) the marine power can be expressed as:

$$P_{meh} = \frac{1}{2}\rho C_p \pi R^2 \left(\frac{H}{2}\frac{gk}{\omega}\frac{\cosh\left[k\left(z+d\right)\right]}{\cosh\left(kd\right)}\sin\left(kx-\omega t\right) \right)^3. \tag{17}$$

The impact of swell height H on the marine current power is presented on Fig. 8. As it can be seen, the higher oscilation of swell height causes higher value of the marine power oscilation. Similary, the impact of the turbine position (z) on the marine current power is presented in Fig. 9. If the turbine position is close to the water surface the oscilations of the marine power are higher.

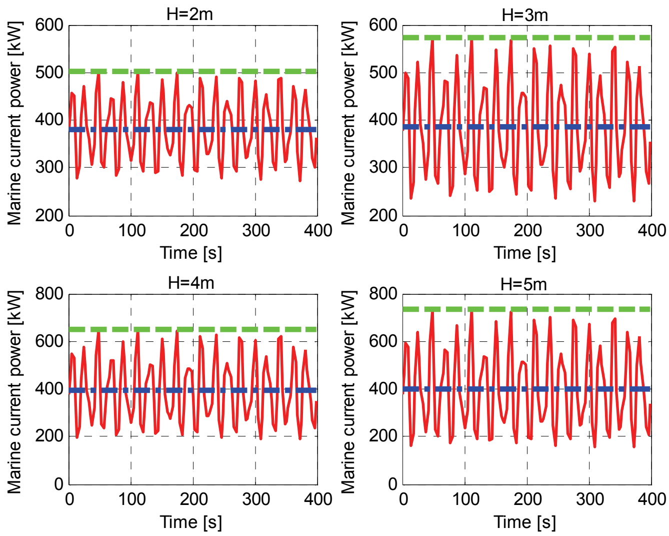

Fig. 8. Marine power – current value (solid line), mean value (dash-dot line) and maximal value (dashed line) for different value of swell height H (d=50, L=200m, x=0m, z=-10m, R=8m)

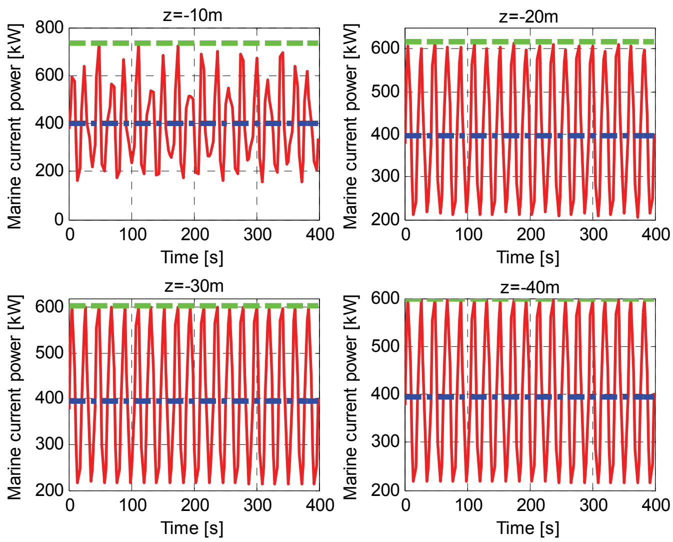

Fig. 9. Marine power – current value (solid line), mean value (dash-dot line) and maximal value (dashed line) for different value of heigth *z* (d=50, L=200m, x=0m, z=-10m, H=5m, R=8m)

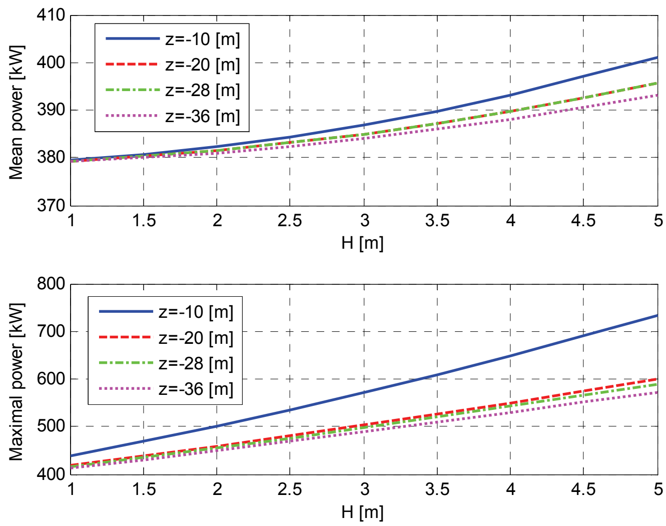

Fig. 9. The impact of swell heigth on the marine current maximal and mean power for different value of the turbine position – turbine depth (First order Stockes model)

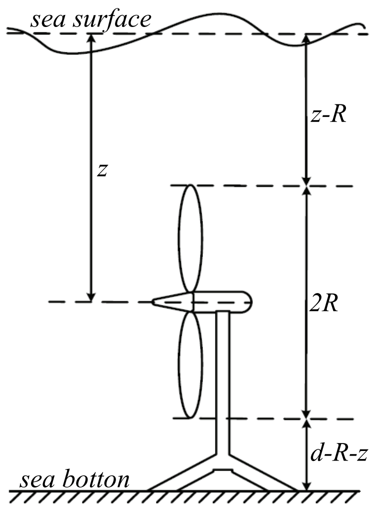

Fig. 10. Turbine position under water surface

The impact of swell heigth on the marine current maximal and mean power for different value of the turbine position (turbine depth) is presented on Fig. 9. As it can be seen, higher value of the swell height cause higher oscilation of the marine power (maximal value). Also, these oscilations are much more pronounced for turbines whose position is near the water surface. For this reason, the turbine should not be placed near to water surface as power oscilation are very expressed and value of marine power is different at upper and lower sides of the turbine propeller. The power oscilation are smaller for higher values of the turbine position (turbine depth). All simulation have been done for turbine which position under sea is presented on Fig. 10, and using first order Stockes model of the marine current speed. The usage of the second order Stockes model of the marine current speed shows the same conclusion as usage of the first order Stockes model (Fig. 11.)

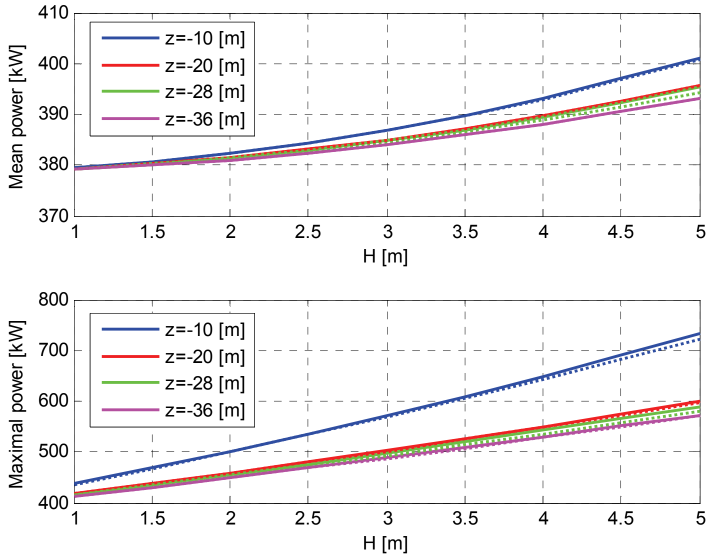

Fig. 11. The impact of swell heigth on the marine current maximal and mean power for different value of the turbine position – turbine depth (First order Stockes model – solid line; Second order Stockes model – dashed line)

5. CONCLUSION

In this paper the impact of swell heigth on the marine current maximal and mean power, for different value of the turbine depth, is presented. This study is performed using the first and second order of Stockes model of the marine current speed. It is shown that higher value of the swell oscilation, as well as if turbine position is near water surface, cause large oscilations of the marine current power.

Also, in this paper is presented a short review of the mathematical models, as well as power coefficients, of the marine current speed. The simulation results of the marine current speed, represented by different mathematical models, are also presented. Similary, mathematical expressions for marine power coefficients have been given.

In the future investigation, mathematical model of the marine generator system will be analysed.

ACKNOWLEDGMENT

The results shown in this paper represent the product activity in the actual bilateral project „*Research on Novel Switched Reluctance Ocean Current Generator System*“, between University of Montenegro, Faculty of Electrical Engineering (Head of the project team – Phd Gojko Joksimović, Full professor) and China University of Mining and

Technology, School of Information and Electrical Engineering (Head of the project team – Phd Chen Hao, Full professor).

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