

**PERMANENT AND TEMPORARY PRICE VARIATIONS:
A Decomposition Analysis based on
Stochastic Forecasting Models**

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This study decomposes major price indices in Pakistan into permanent and transitory components using the Beveridge and Nelson (1981) methodology. The results show that most of the price variations are permanent with a major portion being deterministic. The transitory variations in the consumer price index are only slightly greater than those in the wholesale price index. Therefore, rent-seeking activities at the retail level are limited. Although a similar pattern holds for the open market versus official exchange rates, rent seeking can be prevalent because the market is thin and operates on a spot basis. The study concludes that a forecasting strategy based on the Beveridge-Nelson approach removes the bulk of uncertainty from prices, though risk cannot be avoided.

I. Introduction

One of the usually advocated adverse consequences, of the high inflation rate, is the uncertainty it causes among economic agents. It is argued that during periods of high inflation, the level of volatility in the general price level increases, resulting in distortions in inter-temporal allocation of resources. The resulting uncertainty also makes contractual transactions more risky. Similar consequences are usually associated with continuous erosion in the value of a currency in the foreign exchange market. Although, insurance markets can assist in risk sharing arrangements among agents in the form of risk-free contingent prices for future dates, but in presence of transaction costs and other inefficiencies in the insurance market, resource allocations remain inefficient.

The above proposition would hold if it was assumed that changes in the price level and the exchange rate include significant temporary components. The basis of

this argument is as follows: since permanent changes are predictable and cannot be the source of uncertainty, it is the temporary variation that can cause uncertainty simply because it does not follow a set trend, and is unpredictable. Furthermore, although relative prices remain unstable even during periods of steady inflation, the degree of uncertainty is likely to rise when the rate of inflation becomes unstable. This means that uncertainty about inflation is also likely to increase the degree of distortions in intra-temporal allocation of resources.

The distinction between permanent and temporary changes in economic variables is also important in predicting agents' response to economic impulses, as has been emphasised in economic models based on rational expectations.

In Pakistan, no serious attempt has been made to decompose the time series of economic variables between permanent and temporary components on a scientific basis. A crude method has been applied to measure the permanent portion of GDP by moving averages with an arbitrary weighting scheme [Mangla (1979), and Khan (1980)]. This procedure is now out-dated, as are the conventional detrending methods, like first differencing or filtering through regression on time. Time series models now recognise that assumption of a pure deterministic trend is a gross approximation. Stochastic shocks can occur with a set time pattern, thereby producing a stochastic trend. For example, in the famous random walk plus drift model, which is a pure trend model, all the stochastic variations are part of the trend [Enders (1995)]. This recognition is important in deviating from the conventional wisdom that all stochastic variations in a series are temporary and, therefore, the permanent variation cannot be more than the deterministic portion of the series.

In light of the above background, this study aims to decompose the time series of major price indices in Pakistan into permanent and temporary components using the Beveridge and Nelson (1981) approach. The price indices considered are the consumer and wholesale price indices, the official and the open market exchange rates, and the share price index. According to the Beveridge and Nelson (1981) approach, the permanent component of a series is given by deviation of long-run forecast from the benchmark forecast based on pure deterministic trend, while the temporary component is derived as deviations of the realised series from the permanent component. The long run forecasts are derived as the out-of-sample prediction of the series on basis of the best-fitted ARIMA model. Although the approach is forward looking, yet unlike the conventional method of moving averages, it does not require any future information to predict the present trend.

The paper is structured as follows. Section II reviews the conventional approaches to time series decomposition and provides details on the Beveridge and Nelson methodology of time series decomposition. Data and estimation procedures are discussed in Section III. The results are presented in Section IV. Finally, Section V concludes the paper.

II. Modelling Time Series Decomposing

Before presenting the Beveridge Nelson procedure, the conventional approaches to time series decomposition into permanent and temporary components are reviewed briefly. One of these approaches is to measure the permanent portion of the series by centered moving and temporary component by deviations of actual series from the series of moving averages. As noted in Beveridge and Nelson (1981), since moving averages are placed at the centre, some values for the permanent and temporary components are missed at the end of the series. Since in most cases, the objective of time series analysis is to determine the current position of series under consideration, the practice of extrapolation at the most recent end of the series is an awkward approximation. This difficulty arises because the detrending procedure does not rely completely on the past; it requires future data as well.

An alternative procedure is to decompose the series, say Y_t , between a deterministic function of time and a white-noise error term, that is:

$$Y_t = f(t) + \varepsilon_t$$

where $f(t)$ represents trend and ε_t the irregular portion [Beveridge and Nelson (1981)]. In practice, it is likely that the series ε_t contains cycles and seasonal variation.¹

The assumption that permanent variations in a series are purely deterministic is unsatisfactory. If a series moves in a particular direction over time, there is no reason to believe that the forces underlying this movement are purely deterministic in nature and contain no stochastic innovations. The Random walk plus drift (RWD) model:

$$Y_t = \alpha + Y_{t-1} + \varepsilon_t$$

where ε_t is a white noise, overcomes this limitation most conveniently. According to the RWD process, the series Y_t moves in each period by a fixed magnitude α known as drift, and a stochastic magnitude ε_t , known as innovation. Furthermore, since the series Y_t contains no movement other than the deterministic and stochastic trends, the RWD model is also known as the pure trend model. It also follows, that for the estimation of temporary movements, the data generating process must be more general than the simple RWD model.

Beveridge and Nelson (1981) suggest a procedure for separating pure trend from a general ARIMA model. As is well known, under the maintained assumption

¹ Another practice has been to difference the series to eliminate trend. See, Enders (1995) for more details.

of stationarity any ARIMA model has an equivalent moving average representation:

$$\Delta Y_t = \alpha_0 + \sum_{j=0}^{\infty} b_j \varepsilon_{t-j}, \quad b_0 = 1 \quad (1)$$

In order to estimate trend in the series, Beveridge and Nelson consider a forecasting profile to time $t+s$ conditional on the information at time t and shows that for a very long run forecasting horizon, the long run forecast is given by:

$$\begin{aligned} \lim_{s \rightarrow \infty} E_t(Y_{t+s}) &= Y_t + s - \lim_{s \rightarrow \infty} [E_t(\Delta Y_{t+1}) + E_t(\Delta Y_{t+2}) + \dots + E_t(\Delta Y_{t+s})] \\ &= Y_t + \alpha_0 s + \left(\sum_{i=1}^{\infty} b_i\right) \varepsilon_t + \left(\sum_{i=2}^{\infty} b_i\right) \varepsilon_{t-1} + \left(\sum_{i=3}^{\infty} b_i\right) \varepsilon_{t-2} + \dots \end{aligned} \quad (2)$$

Beveridge and Nelson (1981) define the stochastic level of the series as the excess of forecastable level over the deterministic, or mean, trend $\alpha_0 s$. Since the stochastic level is based on long-run forecasts and includes all the predictable changes in the series other than the deterministic change, it represents the permanent component of the series at time t . Thus, denoting the permanent component by P_t , we have:

$$\begin{aligned} P_t &= \lim_{s \rightarrow \infty} E_t(Y_{t+s}) - \alpha_0 s \approx Y_t + \left(\sum_{i=1}^{\infty} b_i\right) \varepsilon_t + \left(\sum_{i=2}^{\infty} b_i\right) \varepsilon_{t-1} + \left(\sum_{i=3}^{\infty} b_i\right) \varepsilon_{t-2} + \dots \\ &= Y_t + \lim_{s \rightarrow \infty} [E_t(\Delta Y_{t+1}) + E_t(\Delta Y_{t+2}) + \dots + E_t(\Delta Y_{t+s}) - \alpha_0 s] \end{aligned} \quad (3)$$

It can be shown that the permanent component is a RWD process [Beveridge and Nelson (1981)], and therefore, it consists of pure trend only. To complete the decomposition, the transitory or temporary component of the series is derived and denoted by T_t , and is given by the deviation of realised series from the permanent component as:

$$T_t = Y_t - P_t \quad (4)$$

III. Data and Estimation Procedure

It is obvious from the above that the application of the Beveridge-Nelson decomposition requires a long time series. Due to limited availability of temporally disaggregated data, we confine our analysis to five major price indices. These are the wholesale price index (WPI), consumer price index (CPI), official exchange rate of rupee with US dollar (OER), open market exchange rate (MER), and the

general share price index (SPI). We use the SPI compiled by the State Bank of Pakistan, which is based on the 100 major companies listed at the Karachi Stock Exchange (KSE).

Since the Pakistani rupee was let floating in February 1982, we exclude the period prior to this month from our analysis. Data for a few months after the de-linking of rupee from the US dollar are also not useful for stationarity tests, as some period must be allowed for adjustment due to the change of regime.² Thus we choose January 1983 to December 1997 as the period of analysis. This gives a sample of 180 monthly observations sufficient for the time series analysis. All the data are taken from Monthly Bulletin of the State Bank of Pakistan.

The practical approach for estimation of the model is simple and is explained well in Enders (1995). However, despite availability of efficient computing facilities, the procedure is quite laborious. It involves the following four steps.

a) Determining the Order of Integration

The first step is to determine the order of integration of the series with the help of unit root tests and apply appropriate filtering to make the series stationary. In practice, most time series in economics are non-stationary and their natural logs are integrated of order one; since the exponential growth rates are usually stationary. For the unit root test the Augmented Dickey-Fuller (ADF) tests was applied.

b) Diagnosing and Estimating ARIMA Specification

Once the order of integration is known, the next step is to use the Box-Jenkins procedure [Box and Jenkins (1976)] to diagnose an appropriate ARIMA model. This is done by choosing autoregressive (AR) and moving average (MA) terms from a careful study of correlograms for the simple and partial autocorrelation functions [Enders (1995), and Greene (1993)]. The resulting ARIMA model is estimated and correlogram for the regression residuals is examined for further improvement in the model. If some autocorrelation is still left, the ARIMA specification needs to be adjusted for re-estimation in light of the additional information. This step-wise procedure is continued until the regression residuals approximate white noise. To confirm that residuals are white noise, Q-statistic is applied on the cumulative autocorrelation for sufficiently lengthy lags [Maddala (1992)]. The diagnos-

² Apart from the unit root conditions, stationarity also requires that the specified data generating process has been followed from a distant past and the series have not been subjected to a major shock. This condition would probably not be fulfilled if we include the period of adjustment associated with the change of exchange rate regime. The condition would certainly fail if the period around February 1982 was included in the analysis.

tic-estimation procedure is continued until Q-statistics for all meaningful lag lengths turn insignificant. The performance indicators such as Akaike Information Criterion (AIC) and Schwartz Bayesian Criteria (SBC) are used in order to choose among competing ARIMA specifications [Enders (1995), and Maddala (1992)].

c) Making Conditional Forecasts

For the conditional forecasts $E_t(\Delta Y_{t+s})$, in equation (2) to be meaningful, the required information must be available sufficiently. If the maximum lag of MA terms is q , we need to abandon the first q periods for the purpose of forecasting. The same applies to the pAR terms. This means that if the ARIMA model is estimated over the period 0 to N , we can make conditional forecast $E_t(\Delta Y_{t+s})$ for each t , such that $\max(p, q) < t \leq N$. For each starting date t , the long-run forecasts require one-step to s -step conditional forecasts $E_t(\Delta Y_{t+s})$ for sufficiently large s . Since the ARIMA model is a stationary process, the forecast $E_t(\Delta Y_{t+s})$ converge to the drift parameter a_0 as the forecasting horizon increases. Furthermore, since with each one period increase in the forecasting horizon, the deterministic or mean trend also increases by the parameter a_0 , the permanent component of the series as given by equation (3) and converges to a constant value as s increases. Beveridge and Nelson (1981) recommend setting the length of forecasting horizon at 100 periods. In our own experiment, the permanent component for each price series is seen to converge upto 15 decimal points within 50 periods. Thus, we set the forecasting horizon at 50.

To summarise, conditional forecasts $E_t(\Delta Y_{t+s})$ are made from the first usable observation that is for the period $\max(p, q)$ over each of the future period $s: 1 \leq s \leq 50$. Then using all 50 forecasts $E_t(\Delta Y_{t+s})$, we compute $E_t(Y_{t+s})$. This exercise is then repeated for each period: $\max(p, q) < t \leq N$.

d) Estimating Permanent and Transitory Components

Once the conditional forecasts are available, the final step is to estimate permanent and the transitory components of the series using equation (3) and (4). Our model and the estimation procedure are now complete and we are ready to apply it to the selected price indices.

IV. The Results

Following the step-wise procedure outlined above, we now present and discuss our empirical results. As a first step we test the presence of unit root in the five price variables using ADF tests using finite sample critical values at the 5 per cent level of significance. The tests are applied with and without an intercept and time

trend. In addition, alternative lag structures are allowed for the lagged first difference terms in the Dickey-Fuller equation. With all the specifications, we find that the natural logs of the price series are integrated of order one. We therefore conclude that the price series are non-stationary, while their growth rates are stationary.

On the basis of the diagnostic procedure explained in Section III, we have specified appropriate ARIMA models for the five price indices. The results presented in Table 1 show that the estimated ARIMA models satisfactorily remove serial correlation of all types, as the Q-statistics at lag lengths upto 36 are statistically insignificant. The t-statistics for the AR and MA parameters are mostly significant, suggesting that the models fit quite well to the data. As expected, the value of R^2 is low because most of the variations in prices are already filtered out through the deterministic trend while taking the first differences of the series.

TABLE 1

Estimates of ARIMA Model

Variable	ARIMA Equation	R^2	DW Statistic
Market Exchange Rate (MER)	$(1 - 0.1914L) (\Delta \log MER_t - 0.0073) = 0.2119\varepsilon_t + \varepsilon_t$ (2.71*) (4.75*) (2.84*)	0.0698	2.04
Official Exchange Rate (OER)	$(1 - 0.02811L) (\Delta \log OER_t - 0.0074) = 0.1282\varepsilon_t + \varepsilon_t$ (4.13*) (5.25*) (1.7***)	0.0965	2.05
Consumer Price Index (CPI)	$(\Delta \log CPI_t - 0.0068) = 0.2823\varepsilon_t + \varepsilon_t$ (9.12*) (3.91*)	0.0917	1.79
Wholesale Price Index (WPI)	$(1 - 0.0281L) (\Delta \log WPI_t - 0.0071) = 0.1657\varepsilon_t + \varepsilon_t$ (2.91*) (7.08*) (2.28**)	0.0594	1.99
Share Price Index (SPI)	$(1 - 0.2541L) (\Delta \log SPI_t - 0.0087) = 0.2857\varepsilon_t + \varepsilon_t$ (3.57*) (1.37) (4.00*)	0.1375	1.97

Note: The t-statistics significant of 1, 5, and 10 per cent levels are indicated by * **, and *** respectively. The notation L stands for lag operator and ε for white noise error term.

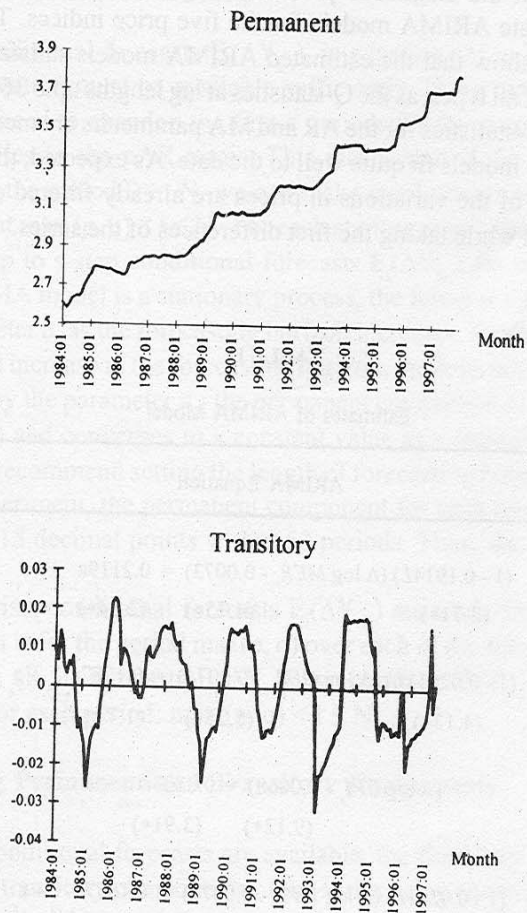


Figure 1

(a) Market Exchange Rate
 Permanent and Transitory Components of Price Indices

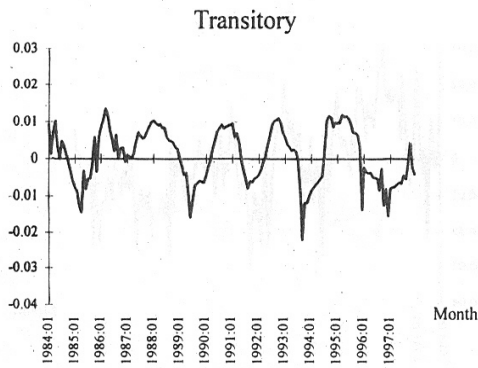
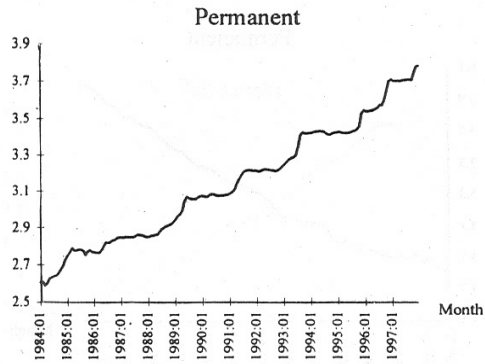


Figure 1

(b) Official Exchange Rate
Permanent and Transitory Components of Price Indices

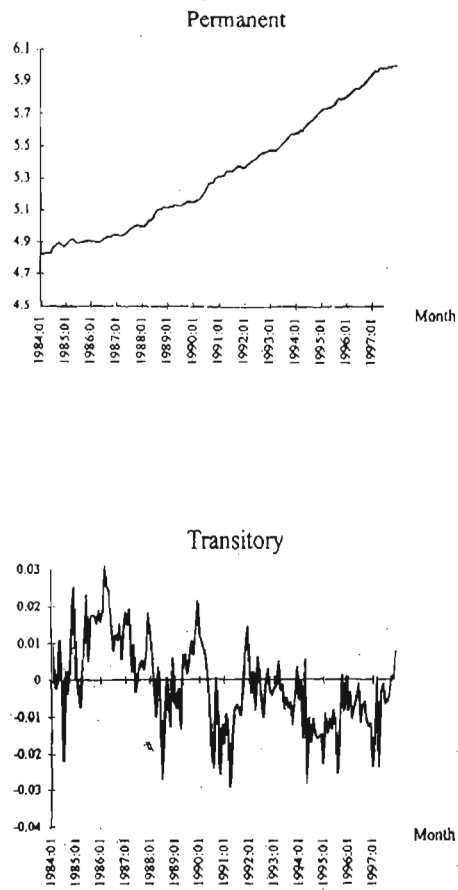


Figure 1

(c) Consumer Price Index
Permanent and Transitory Components of Price Indices

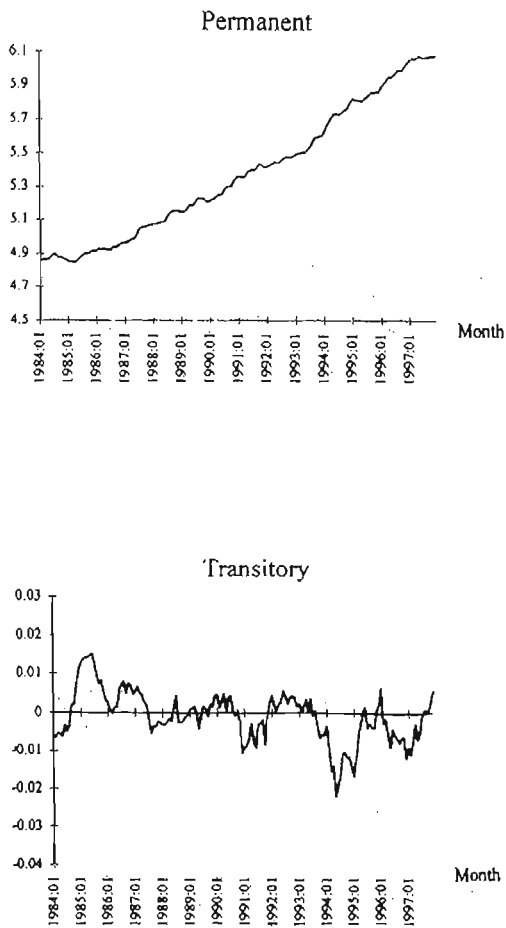
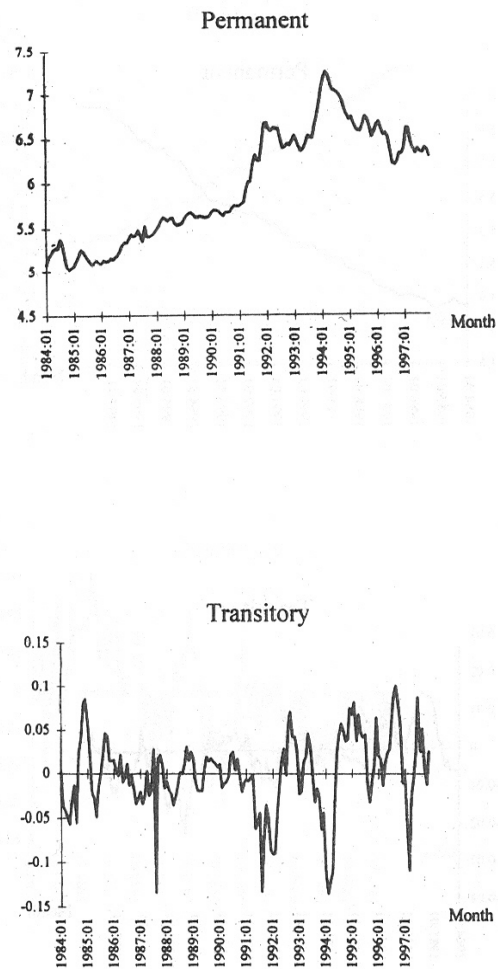


Figure 1

(d) Wholesale Price Index
 Permanent and Transitory Components of Price Indices

**Figure 1**

(e) Share Price Index
Permanent and Transitory Components of Price Indices

Except for the CPI, a strong first order autoregressive process is evident in all the price indices. Furthermore, in all the price variables except the share price index, stochastic seasonality is present as indicated by the significant parameter associated with the MA(12) terms. In the share price index a moving average effect at lag five is found to be present. This process has apparently no particular interpretation.

Using the estimated ARIMA models, we have derived the series of conditional forecasts. These series are then used to derive the permanent and transitory components for each of the five time series. The results of this decomposition are presented in Figure 1 [(a) to (d)].

A number of interesting results are observed from Figure 1. Most of the changes in the selected time series are permanent. The permanent component does not show a smooth trend, as is the case with conventional procedures. The stochastic trend is quite visible, though the deterministic trend dominates the permanent variation in price indices. It is also apparent that the temporary components contain cyclical, seasonal and irregular variations. The series of market and official exchange rates are dominated by cycles. As expected, the size of irregular variations is highest in stock price index. This is followed by CPI and open market exchange rate. The irregular variations are relatively smaller in WPI and the official exchange rate.

The permanent movement in the market exchange rate follows a continuous upward trend, showing some high jumps (for example, in mid 1993, mid 1995 and mid 1996). Between 1986 to 1989 the movements are rather smooth. The timing of these jumps matches the major devaluation of the rupee in the three years. The transitory movements show a cycle of volatility, which is of low intensity. The highest upward temporary variations took place in mid 1986 and around 1995, while the downward temporary variation was prominent during early 1985, mid 1989, mid 1993 and late 1996.

As expected, the permanent movements in the official exchange rate are almost identical to the ones in the market exchange rate, except for some variations between 1985 and 1986. The transitory component of the official exchange rate is less volatile when compared to the transitory component of the market exchange rate. This result follows from the observation that the official exchange rate in Pakistan is not based on pure float, but is managed. That is, while in the medium to long run, the official exchange rate is determined by market forces, the day-to-day changes in the market force are not reflected in official determination of the exchange rate. This practice is adopted in order to produce stability in the exchange rate by minimizing the effect of unexpected shocks, especially the ones resulting from speculative activities and misperceived expectations. The open market exchange rate, on the other hand, is a spot rate and it takes into account all types of market sentiments, whether these are real or based on perceptions. In the open

market speculation becomes a potential source of capital gains and, therefore, the scope of rent seeking activities is high.

The presence of cycles in the temporary movements in the exchange rates highlights the typical time profile of spot markets. They indicate the presence of both the stabilizing and destabilizing agents in the market. If, for example, following a 'news item' some agents perceive that the exchange value of the rupee will decline in the near future, they will attempt to buy foreign currency with the rent-seeking motive. The destabilizing agents will typically follow the tide. The resulting 'band-wagon effect' would aggravate pressure on the market and will result in an early decrease in the value of the rupee. But sooner or later, a rational assessment of the market fundamentals would reverse the tide. Similar arguments apply in case of perception in the opposite direction. The speculators who take the lead in producing sanity are the stabilizing agents. Thus the presence of both the stabilizing and destabilizing agents in the market can result in a cat-and-mouse race between euphoria and sanity, thereby producing the cycles of volatility.

Although the official exchange rate is determined by the monetary authority, it cannot remain alien to market sentiments. In order to maintain a healthy balance of foreign exchange reserves, the authorities have to remain competitive with the open market. Likewise any cyclical variations in the official exchange rate will produce similar variations in the open market exchange rate. Thus the cycles of volatility in the open market and the official exchange rates follow similar time profiles.

Coming to the price indices of goods and services, we observe that almost all changes in the CPI during our period of analysis, have been permanent. The permanent movements in CPI are smooth and follow an upward trend, especially after 1991. The transitory movements, though very small, show quite an irregular pattern, especially until early 1990s. Unlike the case of exchange rates, the temporary variations in the CPI are mostly irregular and do not contain regular cycles. This is so because in most cases retail prices of goods and services are not determined on spot and are therefore not influenced by speculative activities. We also observe that the temporary variations in CPI have become less volatile from 1992 onwards. A possible explanation is the devaluation of the rupee at more or less regular intervals during this period. Another reason could be that in this period changes in CPI have been greatly affected by the government's budget policies and in each year prices of utilities have increased in a set and predictable pattern.

The permanent component of WPI is also smooth but its upward trend is slightly more prominent as compared to the trend in CPI, especially after 1994. As with the CPI, the temporary component of WPI also does not follow a cycle. The timings of ups and downs in the irregular variations in the WPI are the same as in CPI. But the transitory component of WPI is relatively more smooth as compared to the temporary component in CPI. Furthermore the temporary component of WPI does

not contain as much irregular variation as the temporary component of CPI. A possible explanation for this pattern of the two price indices may be the stabilizing effects of government fiscal and pricing policies and private price contracts which are more direct at the wholesale level than at the retail level. The retail prices are also generally more sensitive to 'news'. Thus, the wholesale prices do not in general, fluctuate as freely as the retail prices.

Finally, the stock prices are usually volatile and this can also be seen in the permanent as well as temporary components of the stock price index. A sizeable portion of permanent variations in stock prices is stochastic in nature. This is an overwhelming evidence to suggest that the conventional practice of estimating trend through deterministic regression equations is a gross simplification. The results show that until 1991 the permanent changes in stock prices were mostly deterministic. Therefore, the predicted trends in the stock market were relatively precise. However, during the 1990s incentives were given to foreign investors and the process of privatization was initiated; which was continued by the successive governments. Thus with the opening of the market and other financial reforms, the market has become exposed to permanent as well as temporary shocks, especially the external shocks. That the permanent shocks constitute the stochastic portion of the trend, is shown by erratic variations in the permanent components from the year 1992 onwards, while the temporary shocks are noticeable from the transitory component.

The above result means that during the 1980s the stock market trends were quite precise, with the opening of the market, even the long run trend, reflected an increase in volatility resulting from the increased exposure of the market to economic and political news. Thus the incidence of permanent shocks on the stock market has greatly increased with the growth of the market.

The transitory components of both the market and the official exchange rate are also shown together in Figure 2. The temporary portion of the market exchange rate shows larger fluctuations as compared to the temporary fluctuations in the official exchange rate. The reason is that the market rate changes from day to day while the official rate is subject to government policies, which aim at producing tranquillity in the exchange rate, besides following other objectives.

Figure 3 shows that the transitory movements in CPI are more volatile than in WPI. One of the reason for this result is that the wholesale prices are subject to greater government control than the retail prices. Furthermore price contracts that produce some stability in prices are also more effective at the wholesale level.

V. Concluding Remarks

In this paper, we have decomposed the major price indices in Pakistan into permanent and transitory (or temporary) components. The price indices considered are official and open market exchange rates, the consumer and wholesale price

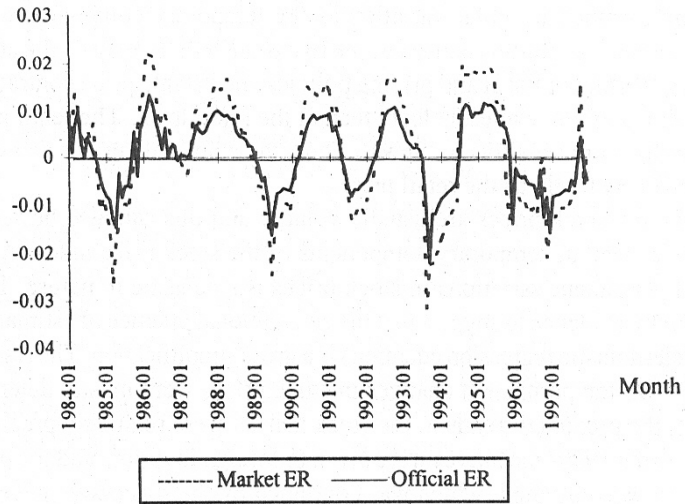


Figure 2
 Transitory Components of
 Market and Official Exchange Rates

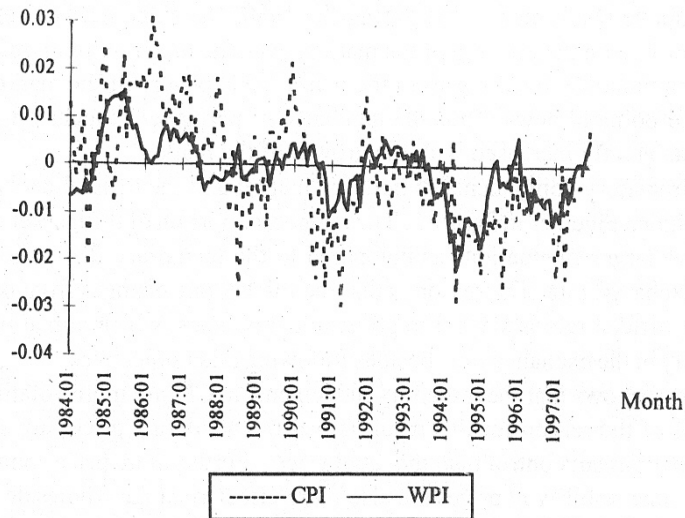


Figure 3
 Transitory Components of
 Wholesale and Consumer Price Indices

indices and the share price index. The decomposition approach, following Beveridge and Nelson (1981), relies on the deviation of long run stochastic forecasts, based on an ARIMA model, from the benchmark deterministic forecasts.

The results show that most changes in price indices are permanent and a major portion of the permanent components is deterministic in nature, though the stochastic portion is also significant, especially in the stock price index. The transitory component of the price indices is dominated by cycles of volatility. A comparative analysis of the permanent and transitory components reveals some interesting patterns. While the permanent components of all the price indices follow similar upward trends, the transitory variations are also closely inter-linked. For example, the transitory variations in both the exchange rates are similar in nature but open market exchange rate has always been more volatile than the official rate. Similarly, the transitory variations in both the consumer and wholesale price indices follow the same time pattern, but the transitory variations in consumer price index follow a relatively more volatile path. An interesting result that defies the conventional wisdom on time series decomposition, is that the permanent portion of the stock price series appears quite 'irregular' in the sense that it does not follow a smooth time path.

These results have a number of implications. First, although the price indices have been subject to a substantial amount of fluctuations, not all these fluctuations can be attributed to uncertainty. A major portion of these fluctuations is permanent and therefore predictable from past experience and can be attributed to risk. Therefore the usual argument that inflation causes uncertainty through increased volatility, is only partially true because part of the volatility is permanent and, hence predictable.

Another conclusion is that the transitory variations in consumer price index are greater than those present in the wholesale price index, though the difference is not too large. This means that at the retail level the rent-seeking opportunities in the wake of fluctuating prices are rather limited. Although a similar pattern holds for the open market versus official exchange rate, the incidence of rent seeking is quite prevalent because the market is thin and trading takes place on a spot basis. Finally, as expected, the share price index is the most volatile among the price indices considered. A useful result is that most of the volatility in share prices is permanent. It follows, therefore that a forecasting strategy based on the Beveridge-Nelson approach can remove the bulk of uncertainty for the investors, but the risk cannot be totally avoided.

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