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EFFICIENT ASPECT RATIO OF THE WING WITH AT WINGLETS

ЭФФЕКТИВНОЕ УДЛИНЕНИЕ КРЫЛА С ЗАКОНЦОВКАМИ "AT WINGLETS"

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Abstract. In the article, by the method of the system of equations of steady horizontal flight the effective aspect ratio of the wing with AT winglet wingtips is explored. The total aerodynamic force, created by all four parts of the wingtips is determined. The vector of the total aerodynamic force of the wingtips is represented in the form of components of a linked coordinate system. Equilibrium equations for the steady rectilinear motion of an aircraft with AT winglets type wing tips in a horizontal flight are recorded. From these equations it is obtained that, in the direction of motion, the longitudinal component of the vector of the total aerodynamic force of the tips reduces the force of the drag of the wing, the vertical component is added to the lifting force of the wing and increases it, and the lateral component, due to the symmetry of the wing, is zero. The coefficient of inductive drag of the wing with the tips is written in the form of the difference in the inductive drag of the wing without the tip and the coefficient of the longitudinal component of the total aerodynamic force of the tips. Writing the coefficient of inductive drag of the wing with the tips in the traditional form through the coefficient of lift and aspect ratio of the wing, the expression for the effective wing aspect ratio with the AT winglets, which is longer without aerodynamic termination. An important consequence is that at the constant weight of the aircraft, the product of the effective wing aspect ratio with aerodynamic wingtips to its coefficient of inductive resistance is a constant value, independent of the kind of the wingtips. It is shown that with increasing flight speed, and also by reducing the inductive speed, the aspect ratio of the wing decreases.

Аннотация. В статье методом системы уравнений установившегося горизонтального полета исследуется влияние аэродинамических законцовок типа "AT winglets" на удлинение крыла. Определяется полная аэродинамическая сила, созданная всеми четырьмя частями законцовок. Вектор полной аэродинамической силы законцовок представляется в виде компонент в связанной системе координат. Приведены уравнения равновесия установившегося прямолинейного движения самолета с законцовками крыла типа "AT winglets" в горизонтальном полете. Из этих уравнений получено, что, будучи направленной в сторону движения, продольная компонента вектора полной аэродинамической силы законцовок уменьшает силу лобового сопротивления крыла, вертикальная компонента прибавляется к подъемной силе крыла и увеличивает ее, а боковая компонента, за счет симметрии крыла, равна нулю. Коэффициент индуктивного сопротивления крыла с законцовками записывается в виде разности индуктивного сопротивления крыла без законцовок и коэффициента продольной компоненты полной аэродинамической силы законцовок. Записывая коэффициент индуктивного сопротивления крыла с законцовками в традиционной форме через коэффициент подъемной силы и удлинение крыла, определено выражение эффективного удлинения крыла с законцовками типа “AT winglets”, которое больше удлинения крыла без аэродинамических законцовок. Получено важное следствие о том, что при условии постоянства веса самолета, произведение эффективного удлинения крыла с аэродинамическими законцовками и ее коэффициента индуктивного сопротивления есть величина постоянная, не зависящая от вида законцовок. Показано, что с увеличением скорости полета, а также с уменьшением индуктивной скорости, эффективное удлинение крыла уменьшается.

Keywords: AT winglet, mathematical model, inductive reactance, effective aspect ratio, steady motion.

Ключевые слова: математическая модель, индуктивное сопротивление, эффективное удлинение, установившееся движение.

Introduction

Inductive resistance of the wing is associated with the finiteness of the wing span and inversely proportional to the wing aspect ratio (as known, the wing aspect ratio is determined by the formula $\lambda=l^2/S$, where l is the wing span, and S is its area). With increasing aspect ratio, it decreases and vice versa. For the theoretical wing of the infinite span, there is no inductive impedance at all. Aircraft designers are trying to take advantage of this fact to reduce drag of the wing, which ultimately leads to an increase in aerodynamic quality. However, the increase in wing aspect ratio is limited by certain strength properties of the wing, the capabilities of aerodromes, etc. For the help comes the so-called — effective aspect ratio of the wing, the increase which does not lead to an increase in the wing span, and is associated with the end structures of the wing — wingtips. In practice, there are many kinds of wing tips: AT winglets, vertical wingtips, horizontal, Dreamliner type, double feathers, Whitcomb wings, sharklets, end washers, etc. For theoretical studies of the effect of the vertical wing tips on the aerodynamic characteristics are corrected out in the works [1–3]. By a system of equations of steady horizontal flight the influence of improved wingtips such as AT winglets on the induction wing resistance is investigated in the work [4]. A mathematical model of the problem is created, which allows you to find out the effect of wing tip to the other aerodynamic forces. In this work this model is used to study the effect of the wingtips such as Advantage Technology winglets on the effective wing aspect ratio.

Statement of the Problem

We take the following coordination system. We place the beginning of the coordinates in the middle of the wing, direct the axis of OZ along the span to the right, the axis OY’s directed upwards and OX axis on the undisturbed flow. We define all the forces influencing on the aircraft with winglets type Advantage Technology winglets at a steady level flight. Due to equilibrium of these forces we determine the influence of aerodynamic forces of wingtip on the coefficient of inductive resistance and determine the effective wing aspect ratio.

The solution of the problem

The following forces effect the plane in level flight [5–7]:

–The power of the weight G — always directed vertically down to the center of the earth;

–Lift of the aircraft Y — is perpendicular to the direction of the undisturbed flow;

–Drag force of the aircraft Q — aimed in the direction opposite to the movement of aircraft;

–Thrust P — is generally directed towards the aircraft movement motion, along the axis;

–Full aerodynamic force created by the upper wingtips. The force created by the upper left wingtip is symbolized $\vec{R}^{в}_{л}$, but the force created by the right wingtip is symbolized $\vec{R}^{в}_{n}$;

–Complete aerodynamic forces created by the lower wingtips. They are symbolized respectively, the left $\vec{R}^{н}_{л}$ and the right force $\vec{R}^{в}_{n}$.

The angle between the true velocity and the free–flow speed for a wing with wingtips α_z equals to the angle between the vector of the total aerodynamic force generated by the upper left wingtip $\vec{R}^{в}_{л}$ and the longitudinal axis of the wing, as the sides of these angles are perpendicular to each other. Then the projection of the full aerodynamic force of the left wingtip will have the form:

$$\vec{R}^{в}_{л} = \{R^{в}_{лx}, R^{в}_{лy}, R^{в}_{лz}\},$$

Where

$$R^{в}_{lx} = R^{в}_{l} \cos\varphi \sin\alpha_z$$

Longitudinal force created by the upper left wingtip, φ–angle of wingtip camber (the angle between the vertical plane of aircraft symmetry and the tangent plane to the wingtip surface at the point of its center of pressure):

$$R^{в}_{ly} = R^{в}_{l} \sin\varphi$$

lift force created by the upper left wingtip:

$$R^{в}_{lz} = R^{в}_{l} \cos\varphi \cos\alpha_z$$

the lateral force generated by the upper left wingtip.

Here, $R^{в}_{l}$ is the vector unit of $\vec{R}^{в}_{l}$,

$$R^{в}_{l} = \sqrt{R^{в2}_{lx} + R^{в2}_{ly} + R^{в2}_{lz}}\ .$$

This force is applied to the center of the wingtip pressure. The total aerodynamic force of right upper wingtip differs from it only with the mark of the third component, so it can be written as

$$\vec{R}^{в}_{n} = \{R^{в}_{n} \cos\varphi \sin\alpha_z, R^{в}_{n} \sin\varphi, -R^{в}_{n} \cos\varphi \cos\alpha_z\}.$$

Obviously, $R^{в}_{n} = \sqrt{R^{в2}_{nx} + R^{в2}_{ny} + R^{в2}_{nz}} = R^{в}_{л}$ therefore, the lower indices that indicate the left and right wingtips will be removed in the future.

Then, the right and left upper part of the wingtips together create a force with components:

$$2\vec{R}^{в} = \vec{R}^{в}_{n} + \vec{R}^{в}_{л} = \{2R^{в} \cos\varphi \sin\alpha_z, 2R^{в} \sin\varphi, 0\}.$$

Now we define the forces created by the lower part of wingtips (projections) (Figure). Since, under the wing the air pressure is much higher than in the environment, it can be assumed that the lower left wingtip pressure force is applied to the center of pressure of the wingtip, normal to its surface. Lower wingtip camber is indicated by the letter ϕ, and the twist angle of the center of pressure is indicated by the letter β. Then, with the same above mentioned argumentation, we can write:

$$2\vec{R}^{н} = \vec{R}_{п}^{н} + \vec{R}_{л}^{н} = \{2R^{н}\cos\phi\sin\beta,\ 2R^{н}\sin\phi, 0\}.$$

Figure. Airbus A319 wingtip

Thus

$$R_{lx}^{в} = R_{l}^{в}\cos\varphi\sin\alpha_{z}$$

the longitudinal is force generated by the lower parts of wingtip, but

$$2R_{y}^{н} = 2R^{н}\sin\phi$$

lift force is created by them.

Because of symmetry, the lateral forces created by the left and right wingtips, balance each other.

The amount of power generated by all four parts of the wingtips Advantage Technology winglets, is indicated by the vector $\vec{R}_{z}$. Thus

$$\vec{R}_{z} = \{R_{zx}, R_{zy}, R_{zz}\},$$

Where

$$R_{zx} = 2R^{в}\cos\varphi\sin\alpha_{z} + 2R^{н}\cos\phi\sin\beta$$

the longitudinal component of the vector of total aerodynamic wingtip force, which obviously reduces any drag force

Since the drag force consists of the sum of the profile and inductive resistances and the profile resistance is almost unchanged, the force R_{zx} reduces the inductive drag of the wing of the finite span without the win.

Force R_{zx} can be represented through the high–speed head,

$$R_{zx} = \overline{C}_{zx} q_{\infty} S_z = C_{zx} q_{\infty} S,$$

where $q_{\infty} = \rho V_{\infty}^2 / 2$ is velocity head, S is the wing area without the tips, and C_{zx} is the coefficient of the longitudinal force of the tip.The coefficient of inductive drag of the wing with the tips is written in the form of the difference in the inductive drag of the wing without the tip and the force factor R_{zx} is :

$$C_{zxi} = \frac{C_y^2}{\pi\lambda}(1+\delta) - C_{zx}$$

We express this expression in the following form

$$C_{zxi} = \frac{C_y^2}{\pi\lambda}(1+\delta)\left[1 - \frac{\pi\lambda}{C_y^2(1+\delta)} C_{zx}\right]$$

or

$$C_{zxi} = \frac{C_y^2}{\pi\tilde{\lambda}}(1+\delta)$$

where

$$\tilde{\lambda} = \lambda\left[1 - \frac{\pi\lambda}{C_y^2(1+\delta)} C_{zx}\right]^{-1}$$

is the effective aspect ratio of the wing. As can be seen the magnitude of the effective aspect ratio makes it possible to record the inductive drag of the wing with the tips in the usual form. It is easy to see the inequality $\lambda < \tilde{\lambda}$, which shows a decrease in the inductive drag of the wing under the influence of the wingtips.

Since

$$\frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)} < 1$$

then a square bracket with negative degree is the sum of a geometric progression

$$1 + \frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)} + \left(\frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)}\right)^2 + \left(\frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)}\right)^3 + \cdots$$

Substituting this into the expressions for the effective aspect ratio and leaving only the first two terms, we have

$$\tilde{\lambda} = \lambda\left[1 + \frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)}\right]$$

or, taking into account the coefficient of inductive drag of the wing without wingtips,

$$\tilde{\lambda} = \lambda\left(1 + \frac{C_{zx}}{C_{xi}}\right)$$

Taking into account the following formula

$$C_{zx} = 2C_{zx}^{в}\cos\varphi\sin\alpha_z + 2C_{zx}^{н}\cos\phi\sin\beta,$$

we get this

$$\tilde{\lambda} = \lambda\left(1 + \frac{2C_{zx}^{в}\cos\varphi\sin\alpha_z + 2C_{zx}^{н}\cos\phi\sin\beta}{C_{xi}}\right)$$

This is an approximate expression of the effective aspect ratio of a wing with double tips. Let us calculate the error of this formula. It is easy to see that the discarded part of the geometric progression, beginning with the third term, can be summed up

$$\left(\frac{C_{zx}}{C_{xi}}\right)^2\left[1 + \frac{C_{zx}}{C_{xi}} + \left(\frac{C_{zx}}{C_{xi}}\right)^2 + \left(\frac{C_{zx}}{C_{xi}}\right)^3 \cdots\right] = \frac{C_{zx}^2}{C_{xi}^2}\frac{1}{1 - \frac{C_{zx}}{C_{xi}}} = \frac{C_{zx}^2}{C_{xi}(C_{xi} - C_{zx})}$$

Hence we see that the sum of the discarded terms of the series is of the same order of smallness $O(C_{zx}^2)$. This expression is the order of accuracy of the formula for effective aspect ratio.

We transform the expression for the effective aspect ratio

$$\tilde{\lambda} = \lambda\left[1 - \frac{\pi\lambda}{C_y^2(1+\delta)}C_{zx}\right]^{-1} = \lambda\frac{1}{1 - \frac{C_{zx}}{C_{xi}}} = \lambda\frac{C_{xi}}{C_{xi} - C_{zx}} = \lambda\frac{C_{xi}}{C_{zxi}}$$

Thus

$$\tilde{\lambda} = \lambda\frac{C_{xi}}{C_{zxi}}$$

or

$$\tilde{\lambda}C_{zxi} = \lambda C_{xi}$$

Thus in case the weight of the aircraft is constant, the product of the effective aspect ratio of the wing with the tips to its coefficient of inductive resistance is constant. As can be seen, with increasing effective lengthening of the wing, its inductive resistance decreases and vice versa.

We have to mention the following simple proof of this result. From the expression of the coefficient of inductive resistance for all wings:

$$C_{zxi} = \frac{C_y^2}{\pi\tilde{\lambda}}(1+\delta)$$

we get the equality:

$$\tilde{\lambda} C_{zxi} = \frac{C_y^2}{\pi}(1+\delta)$$

which is constant at constant weight of the aircraft.

Having regard to that the tangents of the twist angles are equal to the ratios of the inductive velocities of the transverse flows to the velocity of the unperturbed main stream, i. e. $tg\alpha = \frac{V_i^{в}}{V_\infty} \approx \alpha$, $tg\beta = \frac{V_i^{н}}{V_\infty} \approx \beta$, the expression for the effective elongation can be written in the following form:

$$\tilde{\lambda} = \lambda\left(1 + \frac{2C_{zx}^{в} V_i^{в} \cos\varphi + 2C_{zx}^{н} V_i^{н} \cos\phi}{V_\infty C_{xi}}\right)$$

From this it is clear that with an increase in the flight speed, and also by a decrease in inductive speed, the effective aspect ratio decreases. In case $\varphi = \phi = 0$, it has the maximum value, and in case $\varphi = \phi = \frac{\pi}{2}$ it turns into the usual wing aspect ratio without the wingtips.

Conclusions

1. Aerodynamic forces created by AT winglet wingtips while flying are determined.
2. In steady horizontal flight system of algebraic equations containing all the forces acting on the aircraft is recorded, which is a mathematical model of the problem.
3. It is shown that the wing tips reduce the inductive drag of the wing and increase the effective aspect ratio of the wing.
4. It is shown that, in case of the constant aircraft weight, the product of the effective aspect ratio of the wing with its wingtips to its coefficient of inductive resistance is constant.

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