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INFINITESIMALES, A DIVERGENCE AND METHODS OF SUMMATION

ИНФИНИТЕЗИМАЛИ, ДИВЕРГЕНЦИЯ И МЕТОДЫ СУММИРОВАНИЯ

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Abstract. Positive definitions made it possible to prove new theorems on well-known objects of analysis: numerical sequences and series. The estimates of the quantity of all primes and the largest of them are obtained.

Аннотация. Позитивные определения позволили доказать новые теоремы о хорошо известных объектах анализа: числовых последовательностях и рядах. Получены оценки количества всех простых чисел и наибольшего из них.

Keywords: C-pair, Euclidian Axiom 8th, continuum-hypothesis, e-divergence, w-convergence, infinite larger number, Prime Number, dogmas.

Ключевые слова. С-пара, е-расходимость и w-сходимость, аксиома 8 Евклида, бесконечно большие числа, простые числа, догмы.

We use well known mathematical texts and follow to Paul Cohen's forecast about continuum-hypothesis (CH) [1: IV.13]: "A point of view which the author feels may eventually come to be accepted is that CH is obviously false". As well as following [2, Chapter 1], we consider the functional series $\sum u(x, t)$ to be meaningless if it diverges for all values of the parameter t .

1. We define new alternative concept using positive properties of subject matter.

Definition 1. The pair (m, k) of natural variables $m \in A, k \in B$ is said to be C-pair, if there exists such number C that every pair of neighboring in $E = A \cup B$ elements $m \in A$ and $k \in B$ holds an inequality $|m - k| < C$.

Definition 2. The number sequence (a) is named *e-divergent* one if there are such two infinite subsequences $\xi_1, \xi_2 \subset N, \xi_1 \cap \xi_2 = \emptyset$, which hold:

$$\exists(\delta > 0, n^*(\delta) \in N): \forall(m, k) \in (\xi_1, \xi_2) m, k > n^*(\delta) |a_m - a_k| \geq \delta.$$

Definition 3. The number sequence (a) is said to be *w-convergent* if the following condition holds

$$\forall \varepsilon > 0 \exists n(\varepsilon) \in N: (\forall n \geq n(\varepsilon) |a_{n+1} - a_n| < \varepsilon).$$

2. Using new notions we prove alternative Theorems which contradict some traditional

dogmas [3]. At the first we prove [4, (6.2.7)]

Theorem 1. $\varphi(N) = N \Rightarrow \lim_{n \in N} (\varphi(n):n) = 1$.

With Theorem 1 we prove [4, Th. 6.2.5; 5, Th. 6.2.3]

Theorem 2. *There does not exist any bijection between set N of natural numbers and its own subset A .*

Then we divide all injective mappings $\varphi: N \rightarrow N$ onto six not crossed classes.

In common case we prove Euclidian 8th Axiom as [4, Th. 3.8]

Theorem 3. $B \subset A \Rightarrow \{\forall \varphi: A \rightarrow B \exists (a, q) \in (A, A): a \neq q \& \varphi(a) = \varphi(q)\}$.

Theorem 3 has brief form: $B \subset A \Rightarrow \neg(A \sim B)$, which confirms the Paul Cohen's forecast: some false hypothesis had implicated an incorrect Problem (Continuum Hypothesis); thus we prove following statement which contrary this dogma:

Theorem 4. *The infinite sets are divided into classes of equivalence as well as the finite sets to within of one element.*

Ignoring Theorem 4 often leads to either incorrect formulations or false statements. We give the simplest, but the traditional passage with divergent series as an illustration to what has been said

Example 1. $s \triangleq 1 - 1 + 1 - 1 + \dots \Rightarrow s = 1 - (1 - 1 + 1 - 1 + \dots) \Rightarrow_{tr}$

$$s = 1 - (s) \Rightarrow s = 1/2. \quad (1)$$

Really we have

$$s_k \triangleq 1 - 1 + 1 - 1 + \dots + (-1)^{k+1} \Rightarrow s = 1 - (1 - 1 + 1 - 1 + \dots + (-1)^k) \Rightarrow_{Alt} s_k = 1 - (s_{k-1}) \Rightarrow s_k \neq 1/2 \Rightarrow_{Alt} s \triangleq s_\infty \neq 1/2.$$

By analogy with (1) there is

Example 2: Let $s_k(x) \triangleq 1 + x + \dots + x^k = 1 + x(1 + x + \dots + x^{k-1})$. Then

$$s_k(x) = 1 + x(s_{k-1}(x) \pm x^k) = 1 + xs_k(x) - x^{k+1} \Rightarrow_{Alt} s(x) \triangleq s_\infty(x) \neq 1/(1-x), \quad \text{at } |x| \geq 1.$$

3. Now we need following statements:

Theorem 5. *There exist a set of unlimited with finite number Cauchy sequence (a), everyone of them converges to corresponding infinity large number (ILN (a)), [4, Th. 7.1.3])*

Theorem 6. *Unlimited differentiated in the ∞ function $f: R \rightarrow R$ converges to corresponding ILN $\Omega(f)$ if and only if $f'(\infty) = 0$. [4, Th. 7.2.1]*

Theorem 7. *Any permutation of alternative series addends does not change its convergence. [4, Th. 8.2.1])*

Example 3. With Theorem 6 we have proved [6, P. 229–230] the convergence of three sequences: $\{ln(n)\}$, $\{\sin(lnn)\}$, $\{\cos(lnn)\}$: $(ln\infty) \triangleq \Omega_e$, $\{\sin(\varphi_0)\} \triangleq a$, $\cos(\varphi_0) \triangleq b$ at $\varphi_0 \triangleq \text{mod}_{0,5\pi}(\Omega_e)$ at $ab \neq 0$. Thus, $\lim(ctg(lnn)) = ctg(\varphi_0) = b/a$.

Example 4. Now following [7, formulae (1)] we consider

$$V \triangleq \lim (\sin\varphi(n+1)/\sin(n\varphi)). \quad (2)$$

Let $n = \ln t$, then $t = e^n$. Then we have $V = \cos(\varphi) + \sin\varphi (\cos(\varphi_0)/\sin(\varphi_0))$.

A brief solution of this problem is presented in the note [7]. The interested reader will find a great many details and the history of the investigation of continued fractions and their applications in a thorough monograph [8].

4. Now let \mathbf{P} be the set of all prime numbers p_k , $k \in \pi \subset N$. Let farther $\mathbf{P}(x) \triangleq \{p_k : p_k \leq x > 1\}$. Now let $\pi(x) \triangleq |\mathbf{P}(x)|$ by [9], then $\lim_{x \rightarrow \infty} \pi(x) = |\mathbf{P}|$, what is generally accepted. That is obvious that the graph $y = \pi(x)$ of function $\pi(x)$ has a consecutive form and the function $\pi(x)$ is a step-function with $\forall k \pi(p_{k+1}) - \pi(p_k) = 1$. Let $g(x)$ be a differentiable function which has following complementary properties: $\forall k \in \pi g(p_k) = \pi(p_k)$. A limiting equality $\lim g'(x) = 0$ holds

Theorem 8. There exists the ILN $\Omega(\pi)$ which defined the $|\mathbf{P}|$ as the quantity of all Prime Numbers.

Consequence of Theorem 8. There exists the max ILN $(\pi) \triangleq \lim_{n \in \pi} (p_n)$.

A finely by Theorem 7–8 we prove that Hardy–Littlewood’s Hypothesis [9: 1.2.4] has the positive decision.

Note in small print. Sometimes a traditional mathematical thinking happens in the captivity either of a formula or any dogma.

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