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INFLUENCE OF WINGTIP TYPE AT WINGLETS ON THE AERODYNAMIC QUALITY OF WINGS

ВЛИЯНИЕ ЗАКОНЦОВОК ТИПА "AT WINGLETS" НА АЭРОДИНАМИЧЕСКОЕ КАЧЕСТВО КРЫЛА

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Abstract. In this article by the method of equations of steady horizontal flight the impact AT winglets wingtips on the wing aerodynamic quality is explored. Determine the total aerodynamic force, created by all four parts of the endings. The vector of the total aerodynamic force of the endings is found in the form of components along the associated coordinate system. Equilibrium equations for the forces acting on the aircraft are recorded, with the linear motion of the aircraft with wing tips of the AT winglets type in a horizontal flight established. From these equations it is obtained that, in the direction of motion, the longitudinal component of the vector of the total aerodynamic force of the tips reduces the drag of the wing, and the vertical component increases the lifting force and this increases the aerodynamic quality of the wing. The components of the vector of the total aerodynamic force in the transverse direction from the left and right ends of the wing counterbalance each other By virtue of symmetry. Naturally, provided that the weight of the aircraft is constant and the speed of a straight horizontal flight, the angle of attack of the aircraft with the wing with endings like AT winglets should be reduced in order to observe the equality of the lift and the weight of the aircraft. A decrease in the angle of attack, in turn, leads to a decrease in the drag force. The effect of the aerodynamic wing tips on the maximum value of aerodynamic quality is studied. The expression for the aerodynamic lift coefficient at the most advantageous angle of attack is found. It is shown that the aerodynamic wing tips reduce the coefficient of lift of the wing without endings at the most advantageous angle of attack.

Аннотация. В статье методом системы уравнений установившегося горизонтального полета исследуется влияние двухсторонних аэродинамических законцовок типа "АТ winglets" на аэродинамическое качество крыла. Определяется полная аэродинамическая сила, созданная всеми четырьмя частями законцовок. Вектор полной аэродинамической силы законцовок найден в виде компонент в связанной системе координат. Приведены уравнения равновесия сил, действующих на самолет, при установившемся прямолинейном движении самолета с законцовками крыла типа "AT winglets" в горизонтальном полете. Из этих уравнений получено, что направляясь в сторону движения, продольная компонента вектора полной аэродинамической силы законцовок уменьшает лобовое сопротивление крыла, а компонента увеличивает подъемную силу аэродинамическое качество крыла. Компоненты вектора полной аэродинамической силы в поперечном направлении от левого и правого концов крыла уравновешивают друг друга в силу симметрии. Естественно, при условии постоянства веса самолетов и скорости прямолинейного горизонтального полета угол атаки полета самолета с крылом с законцовками типа "AT winglets" должен уменьшается, чтобы соблюдалось равенство подъемной силы и веса самолета. А уменьшение угла атаки, в свою очередь, приводит к уменьшению силы лобового сопротивления. Исследовано влияние аэродинамических законцовок крыла на максимальное значение аэродинамического качества. Найдено выражение аэродинамического коэффициента подъемной силы при наивыгоднейшем угле атаки. Показано, что аэродинамические законцовки крыла уменьшают коэффициент подъемной силы крыла без законцовок при наивыгоднейшем угле атаки.

Keywords: AT winglet, mathematical model, aerodynamic quality of an effective aspect ratio, steady motion.

Ключевые слова: AT winglet, математическая модель, аэродинамическое качество эффективное удлинение, установившееся движение.

Introduction

The inductive resistance of the wing related to the finite wing span and can be reduced by various constructive methods. One of these methods is the installation of winglets, which prevent air from flowing through the wingtip, perpendicular to the main-flow of air. Theoretical wings of infinite magnitude don't have such flows and the inductive resistance does not appear. In another way it is mentioned that in the case of a two-dimensional wing flow (infinite span) the inductive reactance does not occur but in the case of a three-dimensional wing flow (finite span) such resistance arises. Thus, the problem of decreasing of the inductive resistance wing is equivalent to a three-dimensional approximation of unseparated flow over the wing to the two-dimensional flow. For theoretical studies of the effect of the vertical wing tips on the aerodynamic characteristics are corrected out in the works [1–3]. By a system of equations of steady horizontal flight the influence of improved wingtips such as AT winglets on the induction wing resistance is investigated in the work [4]. A mathematical model of the problem is created, which allows you to find out the effect of wing tip to the other aerodynamic forces. In this work this model is used to study the effect of the wingtips such as Advantage Technology winglets [5] on the wing aerodynamic quality. It must be noted that modern aerodynamics is advanced developed science, which was founded by brilliant scientist N. Ye. Zhukovsky, S. A. Chaplygin, L. Prandtlem, T. Karman and many others.

Statement of the Problem

We take the following coordination system. We place the beginning of the coordinates in the middle of the wing, direct the axis of OZ along the span to the right, the axis OY's directed upwards and Ox axis on the undisturbed flow. We define all the forces influencing on the aircraft with winglets type Advantage Technology winglets at a steady level flight. Due to equilibrium of these forces we determine the influence of aerodynamic forces of wingtip on the aerodynamic wing quality.

The solution of the problem

The following forces effect the plane in level flight [6–8]:

- -The power of the weight G always directed vertically down to the center of the earth;
- -Lift of the aircraft Y is perpendicular to the direction of the undisturbed flow;
- -Drag force of the aircraft Q-aimed in the direction opposite to the movement of aircraft;
- -Thrust P is generally directed towards the aircraft movement motion, along the axis;

-Full aerodynamic force created by the upper wingtips. The force created by the upper left wingtip's symbolized \vec{R}_n^s , but the force created by the right wingtip's symbolized \vec{R}_n^s ;

-Complete aerodynamic forces created by the lower wingtips. They are symbolized respectively, the left \vec{R}_n^n and the right force \vec{R}_n^s .

The angle between the true velocity and the free-flow speed for a wing with wingtips α_z equals to the angle between the vector of the total aerodynamic force generated by the upper left wingtip \vec{R}_{π}^{s} and the longitudinal axis of the wing, as the sides of these angles are perpendicular to each other. Then the projection of the full aerodynamic force of the left wingtip will have the form:

$$\vec{R}_{\scriptscriptstyle A}^{\scriptscriptstyle B} = \left\{ R_{\scriptscriptstyle AX}^{\scriptscriptstyle B}, \, R_{\scriptscriptstyle AY}^{\scriptscriptstyle B}, \, R_{\scriptscriptstyle AZ}^{\scriptscriptstyle B} \right\},\,$$

where

$$R_{lx}^{e} = R_{l}^{e} \cos \varphi \sin \alpha_{z}$$

Longitudinal force created by the upper left wingtip, ϕ -angle of wingtip camber (the angle between the vertical plane of aircraft symmetry and the tangent plane to the wingtip surface at the point of its center of pressure),

$$R_{lv}^{e} = R_{l}^{e} \sin \varphi$$

lift force created by the upper left wingtip,

$$R_{lz}^{\scriptscriptstyle g} = R_{l}^{\scriptscriptstyle g} \cos \varphi \cos \alpha_{z}$$

the lateral force generated by the upper left wingtip.

Here, R_l^s is the vector unit of \vec{R}_l^s ,

$$R_{l}^{e} = \sqrt{R_{lx}^{e2} + R_{ly}^{e2} + R_{lz}^{e2}}.$$

This force is applied to the center of the wingtip pressure. The total aerodynamic force of right upper wingtip differs from it only with the mark of the third component, so it can be written as

$$\vec{R}_n^s = \left\{ R_n^s \cos \varphi \sin \alpha_z, R_n^s \sin \varphi, -R_n^s \cos \varphi \cos \alpha_z \right\}.$$

Obviously, $R_n^6 = \sqrt{R_{nx}^{62} + R_{ny}^{62} + R_{nz}^{62}} = R_n^6$ therefore, the lower indices that indicate the left and right wingtips will be removed in the future.

Then, the right and left upper part of the wingtips together create a force with components:

$$2\vec{R}^{s} = \vec{R}_{n}^{s} + \vec{R}_{n}^{s} = \left\{ 2R^{s} \cos\varphi \sin\alpha_{z}, 2R^{s} \sin\varphi, 0 \right\}$$
 (1)

Now we define the forces created by the lower part of wingtips (projections). Since, under the wing the air pressure is much higher than in the environment, it can be assumed that the lower left

wingtip pressure force is applied to the center of pressure of the wingtip, normal to its surface. Lower wingtip camber is indicated by the letter ϕ , and the twist angle of the center of pressure is indicated by the letter β . Then, with the same above mentioned argumentation, we can write

$$2\vec{R}^{H} = \vec{R}_{n}^{H} + \vec{R}_{n}^{H} = \left\{ 2R^{H} \cos \phi \sin \beta, 2R^{H} \sin \phi, 0 \right\}. \tag{2}$$

Thus

$$R_{lr}^{e} = R_{l}^{e} \cos \varphi \sin \alpha$$

the longitudinal is force generated by the lower parts of wingtip, but

$$2R_v^{\scriptscriptstyle H} = 2R^{\scriptscriptstyle H} \sin \phi$$

lift force is created by them.

Because of symmetry, the lateral forces created by the left and right wingtips, balance each other.

The amount of power generated by all four parts of the wingtips Advantage Technology winglets, is indicated by the vector \vec{R}_z . Thus

$$\vec{R}_z = \left\{ R_{zx}, R_{zy}, R_{zz} \right\},\,$$

where

$$R_{zx} = 2R^{s} \cos \varphi \sin \alpha_{z} + 2R^{H} \cos \varphi \sin \beta \tag{3}$$

The longitudinal component of the vector of total aerodynamic wingtip force, which obviously reduces any drag force, and, of course, is added to the force of traction motors,

$$R_{zy} = 2R^{s} \sin \varphi + 2R^{H} \sin \varphi, \qquad (4)$$

the vertical component of the vector of total aerodynamic wingtip force, which is added to the lift of the wing, and the lateral component of the vector of the total aerodynamic force due to the symmetry of wingtips equals to zero $R_{zz} = 0$.

The coefficients of these forces, as usual, are indicated with the letter \mathcal{C} with the same indices. Then we can write:

$$R_{zx} = C_{zx} \frac{\rho V_{\infty}^2}{2} S$$
, $C_{zx} = 2C^{\theta} \cos \varphi \sin \alpha + 2C^{\theta} \cos \varphi \sin \beta$

$$R_{zy} = C_{zy} \frac{\rho V_{\infty}^2}{2} S, \quad C_{zy} = 2C^e \sin \varphi + 2C^H \sin \varphi$$
 (5)

and where C^s and C^H are the coefficients and of R^s and R^H , respectively, determined in a similar manner, C_{zy} is the coefficient of lift and C_{zx} is the coefficient of the longitudinal wingtip force, S is the wing area without wingtips.

Now, considering the formulas (3) and (4), we can write the equilibrium equations of steady motion of the aircraft with wingtips type AT winglets in level flight in the form of

$$Y + R_{zy} = G$$

$$Q - R_{zx} = P$$
(6)

The left sides of these equations correspond to the lift and the resistance of any wing windshield with wingtips. The ratio of these forces represents the aerodynamic quality of the wing with wingtips,

$$K_z = \frac{Y + R_{zy}}{Q - R_{zx}} = \frac{C_y + C_{zy}}{C_x - C_{zx}},$$

Apparently, the presence of wing tips increases the numerator and reduces denominator of this fraction for the wing without wingtip and as a result increases the aerodynamic wing quality. A reduced part of the drag coefficient is equal to

$$\Delta C_x = -C_{zx}$$

account for the inductive reactance, i.e., $\Delta C_x = \Delta C_{zxi}$. Thus wingtip increase lift and reduce the inductive reactance of the wing, and hence aerodynamic efficiency increases.

If the angles of camber equals to zero, $\sin \phi = \sin \phi = 0$, then $C_{zy} = 0$ and the lift does not change, and the inductive reactance is reduced as much as possible. Herewith aerodynamic quality assume the form:

$$K_z = \frac{Y}{Q - R_{zx}} = \frac{C_y}{C_x - C_{zx}} = \frac{C_y}{C_x - 2C^6 \sin \alpha - 2C^4 \sin \beta}$$

and as we can easily see it increases. Let's imagine the last expression in the form of a geometrical progression sum

$$K_z = \frac{C_y}{C_x} \cdot \frac{1}{1 - \frac{C_{zx}}{C_x}}$$

Here, the second fraction represents the effect of the wingtip on the aerodynamic quality of the wing. Let's expand this fraction in geometric progression:

$$K_z = \frac{C_y}{C_x} \cdot \left[1 + \frac{C_{zx}}{C_x} + \left(\frac{C_{zx}}{C_x} \right)^2 + \left(\frac{C_{zx}}{C_x} \right)^3 + \cdots \right]$$

Since

$$\frac{C_{zx}}{C_x} = \frac{2C^s \sin \alpha + 2C^h \sin \beta}{C_x} << 1,$$

the number is rapidly converging and the calculation of aerodynamic quality can be satisfied with linear (the first two) members within square brackets

$$K_z = \frac{C_y}{C_x} \left(1 + \frac{C_{zx}}{C_x} \right) = \frac{C_y}{C_x} \left(1 + \frac{2C^6 \sin \alpha + 2C^4 \sin \beta}{C_x} \right).$$

Increase of the aerodynamic quality is expressed by the formula:

$$\Delta K_z = \frac{C_y}{C_x^2} \Big(2C^{\theta} \sin \alpha + 2C^{\mu} \sin \beta \Big)$$

and relative growth is expressed by formula:

$$\frac{\Delta K_z}{K} = \frac{1}{C_x} \left(2C^{\theta} \sin \beta + 2C^{H} \sin \beta \right)$$

If the corners of the collapse are equal $\phi=\phi=\pi/2$, then $\sin\phi=\sin\phi=1$, $\cos\phi=\cos\phi=0$, and lift is increased to the maximum and the drag inductive resistance does not change, and the aerodynamic efficiency becomes:

$$K_z = \frac{Y + R_{zy}}{Q} = \frac{C_y + 2C^{\theta} + 2C^{H}}{C_x}$$

This event corresponds to adding wingtips to the ends of the wing and its elongation.

The maximum aerodynamic efficiency

In the formula of the aerodynamic efficiency let's represent a drag coefficient as the sum of the profile and inductive resistances and let's calculate the maximum aerodynamic quality according to the coefficient of lift. According to the rules of mathematical analysis, first we have to find a fixed point, and then at that point check extreme of the aerodynamic qualities. The stationary point is found from the condition of the first derivative of the aerodynamic qualities:

$$\frac{dK_{z}}{dC_{y}} = \frac{d}{dC_{y}} \frac{C_{y} + C_{zy}}{C_{xp} + AC_{y}^{2} - C_{zx}}$$

Here

$$A = \frac{1}{\pi \lambda} (1 + \delta).$$

Calculating the derivative of the fraction, we find out:

$$\frac{dK_{z}}{dC_{y}} = \frac{C_{xp} + AC_{y}^{2} - C_{zx} - 2AC_{y}(C_{y} + C_{zy})}{(C_{xp} + AC_{y}^{2} - C_{zx})^{2}}$$

Introducing the fraction in the following form:

$$\frac{dK_z}{dC_y} = -A \frac{C_y^2 + 2C_y \cdot C_{zy} - \frac{C_{xp}}{A} + \frac{1}{A}C_{zx}}{\left(C_{xp} + AC_y^2 - C_{zx}\right)^2}$$

or

$$\frac{dK_z}{dC_y} = -A \frac{\left(C_y + C_{zy}\right)^2 - \frac{C_{xp} - C_{zx}}{A} - C_{zy}^2}{\left(C_{xp} + AC_y^2 - C_{zx}\right)^2}$$

and equating the numerator to zero, we find out a fixed point (of course, we take the positive root as the lift coefficient cannot be negative)

$$C_{y} = -C_{zy} + \sqrt{C_{zy}^{2} + \frac{C_{xp} - C_{zx}}{A}}$$

In these formulas, it is necessary to take into account the following expression

$$C_{zx} = 2C^{e} \cos \varphi \sin \alpha + 2C^{H} \cos \varphi \sin \beta$$

$$C_{zy} = 2C^{\theta} \sin \varphi + 2C^{H} \sin \varphi$$

Analysis of the last two equations shows that in the left side from the found root derivative $\frac{dK_z}{dC_y} > 0$, and in the right side $\frac{dK_z}{dC_y} < 0$. This means that in a determined point C_y has a maximum

aerodynamic efficiency. The corresponding angle of attack where the maximum aerodynamic efficiency is maximum K_z , is called the most advantageous. The last expression C_y is the aerodynamic lift coefficient at the most favorable angle of attack. We represent it in the following form:

$$C_{ynv} = -2C^{\epsilon} \sin \varphi - 2C^{\mu} \sin \varphi + \sqrt{4(C^{\epsilon} \sin \varphi + C^{\mu} \sin \varphi)^{2} + \frac{C_{xp}}{A} - \frac{2}{A}(C^{\epsilon} \cos \varphi \sin \alpha + C^{\mu} \cos \varphi \sin \beta)}$$

If the corners of the collapse ϕ and ϕ equal to zero, i.e., wingtips are perpendicular to the wing, then this expression becomes simpler

$$C_{ynv} = \sqrt{\frac{\pi \lambda C_{xp}}{1+\delta} - \frac{2\pi \lambda}{1+\delta} \left(C^{\theta} \sin \alpha + C^{H} \sin \beta \right)}$$

If the angles of wingtip twist α and β also equal to zero, then the result is the same as the result for the wing without wingtip. It is evident that at least the twist of high or low wingtips reduce wing lift coefficient without wingtips at the most favorable angle of attack. If $\phi = 0$, but $\phi \neq 0$, but, it turns out the result for the wing with vertical wingtips obtained in the works [1–3]. Also we have to note that the aerodynamic coefficient of lift at the most favorable angle of attack is proportional to the square root of the wing aspect ratio without wingtips. Substituting this in the expression K_{ϕ} it is possible to obtain the maximum aerodynamic efficiency.

Conclusions

- 1. Aerodynamic forces created by AT winglet wingtips while flying are determined.
- 2. In steady horizontal flight system of algebraic equations containing all the forces acting on the aircraft is recorded, which is a mathematical model of the problem.
 - 3. It is shown that the wingtips increase the aerodynamic wing quality.
 - 4. The maximum aerodynamic efficiency is determined.
 - 5. The aerodynamic coefficient of lift at the most favorable angle of attack is determined.

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