Dark Energy Cosmological Models with Linear Equation of State in Plane Symmetric Universe

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Abstract. Plane Symmetric cosmological models with linear equation of state (EoS) $p = \alpha\rho - \beta$, where α and β are constants, have been investigated, in General Relativity. The exact solutions of the field equations are obtained by assuming a constant deceleration parameter that leads two different aspects of the volumetric expansion namely a power law and an exponential volumetric expansion. Some physical and geometric properties of the models along with physical acceptability of the solutions have also been discussed in detail.

Keywords: Plane Symmetric universe, linear equation of state, dark energy.

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1 Introduction

Accelerating enlargement of our universe determined by cosmological observations like SNeIa [1, 2], CMB [3, 4] and WMAP [5, 6] can be explained by dark energy [7, 8]. Barrow [9] in his investigation has pointed that the entropy level of the universe makes it probable that its initial state was isotropic and quiescent ($p = \omega\rho$, $\omega \in (-1,0)$) instead of chaotic provided that the equation of state for prime density of matter tends to stiff $\rho = p$ (ρ being the matter density and p the isotropic pressure). The cosmic background radiation is additionally thought of to be a significant experimental proof on that the foremost unremarkably accepted theory regarding the origin of universe i.e. "Big-Bang" cosmology relies. Dark energy cosmology with generalized equation of state for FRW cosmological model has been investigated by Bachichev et al. [10]. Singh and Chaubey [11] have thought of a spatially unvaried and anisotropic Bianchi type-I cosmological model crammed with dark energy with generalized equation of state. Recently, Adhav et. al.[12-15] have studied cosmological models with equation of state normally relativity theory.

Motivating with on high of study work, throughout this paper I tend to acquire plane symmetrical cosmological models with equation of state (EoS) $p = \alpha\rho - \beta$, where α and β are constants, in General Relativity. The exact solutions of the field equations are obtained by assuming a constant deceleration parameter that leads two different aspects of the volumetric expansion namely a power law and an exponential volumetric expansion.Physical and kinematical properties of the model are discussed.

2 Metric and Field Equations

We consider plane-symmetric, which is less restrictive than spherical symmetry and can provide an avenue to study inhomogeneities. Inhomogeneous cosmological models play an important role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. Plane symmetric inflationary model has astrophysical interest since cosmological models play a vital role in the structure formation of the universe. At the present state of evolution, the universe, on the whole, is spherically symmetric and isotropic. But in its early stages of evolution, it could not have had such a smoothed out picture. The plane symmetric metric is in the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2dz^2 \tag{1}$$

where the metric potentials A and B are the functions of time t only.

Katore et. al. [16] investigated Plane symmetric cosmological models with perfect fluid and dark energy. Katore and Shaikh [17] studied Plane symmetric dark energy model in Brans-Dicke theory of gravitation. Statefinder Diagnostic for Modified Chaplygin Gas in Plane Symmetric Universe has been discussed by Katore and Shaikh [18]. Katore and Shaikh [19] have investigated Plane Symmetric cosmological model in the presence of cosmic string and Bulk Viscosity in Saez-Ballester scalar tensor theory of gravitation. Very recently, Katore et. al. [20] obtained Plane Symmetric Inflationary Universe with Massless Scalar Field and Time Varying Lambda.

The energy momentum tensor is given by

$$T_\gamma^\mu = diag\left[T_1^1, T_2^2, T_3^3, T_4^4\right] = diag\left[-p, -p, -p, \rho\right] \tag{2}$$

where, ρ is the energy density of the fluid and p is its pressure.

Here we use linear equation of state [21] as

$$p = \alpha\rho - \beta \tag{3}$$

where, α and β are constants.

The Einstein field equations, in natural units ($8\pi G = 1$ and $c = 1$), are

$$G_{\mu\gamma} = R_{\mu\gamma} - \frac{1}{2} R g_{\mu\gamma} = -T_{\mu\gamma} \tag{4}$$

where, $g_{\mu\lambda} u^\mu u^\gamma = 1$, $u^\mu = (0,0,0,1)$ is the four velocity vector, $R_{\mu\gamma}$ is the Ricci tensor, R is the Ricci scalar, and $T_{\mu\gamma}$ is the energy-momentum Tensor.

Using equations (4), the corresponding field equations for metric (1) with the help of equation (3) and (4) can be written as

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = -\left(\alpha\rho - \beta\right) \tag{5}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -\left(\alpha\rho - \beta\right) \tag{6}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = \rho \tag{7}$$

where a dot here in after denotes ordinary differentiation with respect to cosmic time "t" only.

3 Solutions of the Field Equations

Using equations (5) and (6), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0 \tag{8}$$

which on integration gives

$$\frac{A}{B} = k_2 \exp\left[k_1 \int \frac{dt}{V}\right] \tag{9}$$

where k_1 and k_2 are constants of integration.

In view of $V = A^2 B$, we write A, B in the explicit form

$$A = D_1 V^{\frac{1}{3}} \exp\left(X_1 \int \frac{1}{V} dt\right) \tag{10}$$

$$B = D_2 V^{\frac{1}{3}} \exp\left(X_2 \int \frac{1}{V} dt\right) \tag{11}$$

where $D_i (i = 1, 2)$ and $X_i (i = 1, 2)$ satisfy the relation $D_1^2 D_2 = 1$ and $2X_1 + X_2 = 0$.

Since field equations (5)–(7) are three equations having four unknowns and are highly nonlinear, an extra condition is needed to solve the system completely. Here we have used two different volumetric expansion laws

$$V = at^b \tag{12}$$

and

$$V = \alpha_1 e^{\beta_1 t} \tag{13}$$

where a, b, α_1, β_1 are constants. In this way, all possible expansion histories, the power law expansion, (12), and the exponential expansion, (13), have been covered.

4 Isotropization

The anisotropy of the expansion can be parameterized after defining the directional Hubble parameters and the mean Hubble parameter of the expansion. The directional Hubble parameters within the directions for the Plane symmetric metric outlined in (1) are also outlined as follows:

$$H_x = H_y = \frac{\dot{A}}{A},\, H_z = \frac{\dot{B}}{B} \tag{14}$$

The mean Hubble parameter, H, is given by

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{15}$$

where R is the mean scale factor and $V = R^3 = A^2 B$ is the spatial volume of the universe.

The anisotropy parameter of the expansion Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{16}$$

in the x, y, z directions, respectively. The mean anisotropic parameter of the expansion Δ has a very crucial role in deciding whether the model is isotropic or anisotropic. It is the measure of the deviation from isotropic expansion, the universe expands isotropically when $\Delta = 0$.

Let us introduce the dynamical scalars, such as expansion parameter (θ) and the shear (σ^2) as usual

$$\theta = 3H \tag{17}$$

$$\sigma^2 = \frac{3}{2}\Delta H^2 \tag{18}$$

5 Model for Power Law

Using (12) in (10) and (11), we obtain the scale factors as follows:

$$A = D_1 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{\frac{X_1}{a(1-b)} t^{1-b}\right\} \tag{19}$$

and

$$B = D_2 a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left\{\frac{X_2}{a(1-b)} t^{1-b}\right\} \tag{20}$$

Metric (1) with the help of (19) and (20) can be written as

$$ds^2 = dt^2 - D_1^2 a^{2/3} t^{2b/3} \exp\left\{2\left[\frac{X_1}{a(1-b)} t^{1-b}\right]\right\}\left(dx^2 + dy^2\right) - D_2^2 a^{2/3} t^{2b/3} \exp\left\{2\left[\frac{X_2}{a(1-b)} t^{1-b}\right]\right\} dz^2 \tag{21}$$

Using equations (19) and (20) in equation (7), we get the energy density as

$$\rho = \left\{\frac{b^2}{3t^2} + \frac{2b(2X_1 + X_2)}{3at^{b+1}} + \frac{X_1^2 + 2X_1X_2}{a^2 t^{2b}}\right\} \tag{22}$$

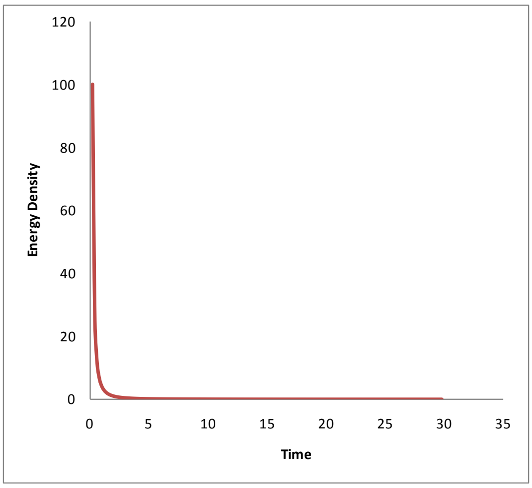

Figure 1. Energy density vs time.

Using equation (28) and (3), we obtain the pressure as

$$p = \alpha\left\{\frac{b^2}{3t^2} + \frac{2b(2X_1 + X_2)}{3at^{b+1}} + \frac{X_1^2 + 2X_1X_2}{a^2t^{2b}}\right\} - \beta \tag{23}$$

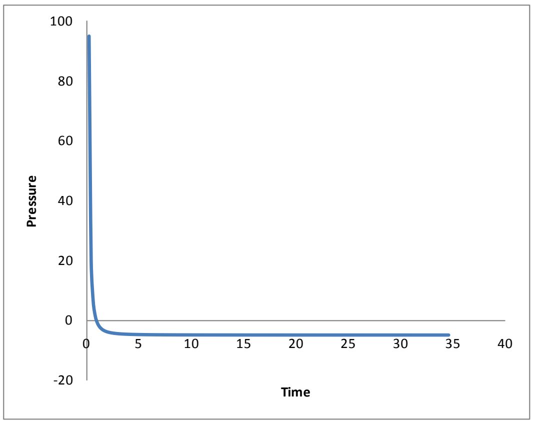

Figure 2. Pressure vs time.

The directional Hubble parameters as defined in (14) are found as

$$H_x = H_y = \frac{b}{3t} + \frac{X_1}{at^b} \tag{24}$$

$$H_z = \frac{b}{3t} + \frac{X_2}{at^b} \tag{25}$$

From equation (15), the mean Hubble's parameter, H, is given by

$$H = \frac{b}{3t} \tag{26}$$

Using the directional and mean Hubble's parameter in (16), we obtain

$$\Delta = \frac{3X^2}{a^2b^2t^{2(b-1)}} \tag{27}$$

From (17) and (18), the dynamical scalars are given by

$$\theta = \frac{b}{t} \tag{28}$$

The shear Scalar

$$\sigma^2 = \frac{X^2}{2a^2 t^{2b}} \tag{29}$$

where $X^2 = 2X_1^2 + X_2^2 =$ constant.

The deceleration parameter

$$q = \frac{3}{b} - 1 \tag{30}$$

The deceleration parameter is always negative for $b > 3$ indicating accelerating universe. It is observed that the Hubble parameter H, expansion scalar θ, shear scalar σ are very large near $t \sim 0$ and finally tend to zero as $t \to \infty$.

6. Model for Exponential Law

Using (13) in (10) and (11), we obtain the scale factors as follows:

$$A = D_1 \alpha_1^{\frac{1}{3}} e^{\frac{\beta_1 t}{3}} \exp\left\{ \frac{-X_1}{\alpha_1 \beta_1} e^{-\beta_1 t} \right\} \tag{31}$$

and

$$B = D_2 \alpha_1^{\frac{1}{3}} e^{\frac{\beta_1 t}{3}} \exp\left\{ \frac{-X_2}{\alpha_1 \beta_1} e^{-\beta_1 t} \right\} \tag{32}$$

It is clear that, the scale factor admit constant values at time $t = 0$, afterwards they evolve with time without any type of singularity and finally diverge to infinity. This is consistent with big bang scenario which resembles with Katore and Shaikh [22].

Metric (1) with the help of (31) and (32) can be written as

$$ds^2 = dt^2 - D_1^2 \alpha_1^{\frac{2}{3}} e^{\frac{\beta_1 t}{3}} \exp\left\{ -2\left[\frac{X_1}{\alpha_1 \beta_1} e^{-\beta_1 t} \right] \right\} \left(dx^2 + dy^2 \right) - D_2^2 \alpha_1^{\frac{2}{3}} e^{\frac{\beta_1 t}{3}} \exp\left\{ -2\left[\frac{X_2}{\alpha_1 \beta_1} e^{-\beta_1 t} \right] \right\} dz^2 \tag{33}$$

Using equations (31) and (32) in equation (7), we get the energy density as

$$\rho = \left\{ \frac{\beta_1^2}{3} + \frac{2\beta_1 (2X_1 + X_2)}{3\alpha_1 e^{\beta_1 t}} + \frac{X_1^2 + 2X_1 X_2}{\alpha_1^2 e^{2\beta_1 t}} \right\} \tag{34}$$

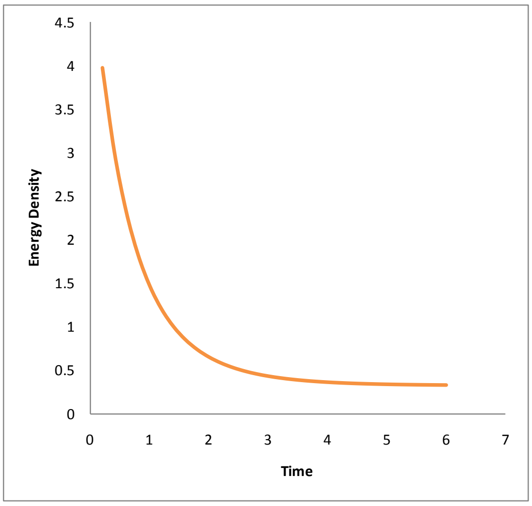

Figure 3. Energy density vs time.

Using equation (34) and (3), we obtain the pressure as

$$p = \alpha\left\{\frac{\beta_1^2}{3} + \frac{2\beta_1(2X_1 + X_2)}{3\alpha_1 e^{\beta_1 t}} + \frac{X_1^2 + 2X_1X_2}{\alpha_1^2 e^{2\beta_1 t}}\right\} - \beta \tag{35}$$

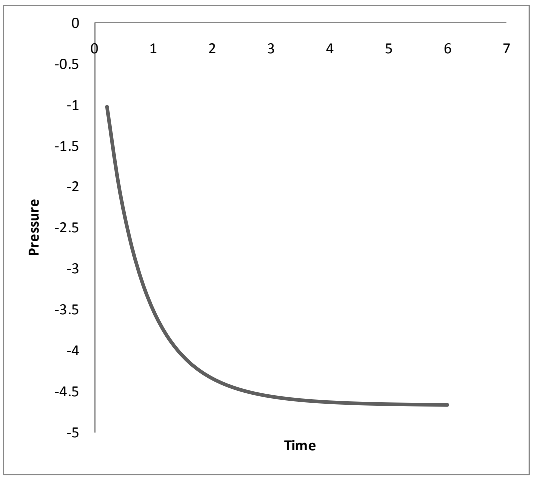

Figure 4. Pressure vs time.

The directional Hubble parameters defined in (14) are found as

$$H_x = H_y = \frac{\beta_1}{3} + \frac{X_1}{\alpha_1 e^{\beta_1 t}} \tag{36}$$

$$H_z = \frac{\beta_1}{3} + \frac{X_2}{\alpha_1 e^{\beta_1 t}} \tag{37}$$

From (15), the mean Hubble's parameter, H, is given by

$$H = \frac{\beta_1}{3} \tag{38}$$

The anisotropy parameter of the expansion, Δ, is

$$\Delta = \frac{3X^2 e^{-2\beta_1 t}}{\alpha_1^2 \beta_1^2} \tag{39}$$

The expansion scalar, θ, is found as

$$\theta = \beta_1 \tag{40}$$

The shear scalar, σ^2, is found as

$$\sigma^2 = \frac{X^2 e^{-2\beta_1 t}}{2\alpha_1^2} \tag{41}$$

The deceleration parameter

$$q = -1 \tag{42}$$

where $X^2 = 2X_1^2 + X_2^2 =$ constant.

For this model $q = -1$ and $\frac{dH}{dt} = 0$, hence, it provides the best values of the Hubble parameter and also the quickest rate of growth of the universe. Thus, this model could represent the inflationary era within the early universe and also the terribly late times of the universe. It is observed that the anisotropy parameter measures a constant value at $t = 0$ while it vanishes at infinite time of the universe which indicates that the universe expands isotropically at later times. The shear scalar

$\sigma \to 0,\ as\ \ t \to \infty$. The shear scalar is finite at $t = 0$. The expansion scalar for these scale factors exhibits the constant value.

7 Conclusion

Plane Symmetric cosmological models with linear equation of state (EoS) $p = \alpha\rho - \beta$, where α and β are constants, have been studied, in General Relativity. The exact solution of the field equations have been obtained by assuming two different volumetric expansion laws in a way to cover all possible expansion: namely exponential and power law expansion.

In power law model, at $t = 0$, both the scale factors vanish, start evolving with time and finally as $t \to \infty$ they diverge to infinity. This is consistent with the big bang model.

In exponential model, it is clear that, the scale factor admit constant values at time $t = 0$, afterwards they evolve with time without any type of singularity and finally diverge to infinity. This is consistent with big bang scenario. The universe accelerates with the highest rate q = -1, which is consistent with the present day observations.

It is observed that, the fluid work as dark energy (negative pressure p) depending on the particular values of α and β was shown in Figure 2 and 4 for both the models. Also, the models reduce to Strange Quark Matter (SQM) for $\alpha = \frac{1}{3}$ and $\beta = \frac{4}{3}B_c$, where B_c is Bag constant or vacuum energy density of Bag Model of quark matter. It is interesting to note that the results obtained resemble with the investigations of Adhav et. al. [12,14].

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