# Accelerated Algorithm for Solids of Revolution Converting into Ribbon by Spiral Coordinate System 

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#### Abstract

The article considering the problem of quick transformation of solids of revolution into raster layer for Roll Powder Sintering (RPS) additive manufacturing technology. The author describes the algorithm for solving this problem. This work is devoted to the development and the validation of the impetuous algorithm for converting the 3D coordinates of a solid of revolution to spiral coordinates. It is based on the assumption that one point on the figure of revolution is predetermined multitude of points in the spiral coordinate system and these can be defined without calculations because of symmetry properties. Such method provides to accelerate the performance of the coordinate conversion thousand fold and real time forming objects $1.5 \mathrm{~m}^{3}$ in volume with layer thickness $\sim 25 \mu \mathrm{~m}$ by non-powerful computers per hour. The proposed algorithm has been extensively tested and proven by numerical examples of several parts of computer aided design models. It is faster than other coordinates transformations and can be extended to the acceleration converting and processing geometry of 3D objects.


Keywords: High-performance additive manufacturing, High precision additive manufacturing, Inexpensive additive manufacturing.

## 1. Introduction

Additive manufacturing [1-2] is used by multiple engineering subsectors, including motor vehicles, aerospace [3-5], machinery, electronics, and medical products. Although currently dominated additive manufacturing technologies [6-7] (stereolithography (SL) [8], selective laser sintering (SLS), fused deposition modeling (FDM) and laminated object manufacturing (LOM) [9-10]) allow the manufacture of personalized and gradually more complex parts, slow print speed and high costs limits using these devices for mass production [11]. If to decrease the costs of additive manufacturing systems, this technology may improve the interaction of consumers and producers. An inexpensive 3D printer allows the user to produce plastic, ceramic and metal objects at home or in the office. Currently, there are several additive
manufacturing systems that are within the budget of the average consumer, but it is still possible to improve its efficiency.

As describe in [12-14], RPS technology has several advantages, such as using many powder materials with tend to sinter, when heat is applied, higher building speed and higher precision. This technology could make a large contribution to the micro-scale manufacturing [15-16] which requires the thickness range of 5-100 $\mu \mathrm{m}$ [17-20] with positional precision in the range of 0.1 to $10 \mu \mathrm{~m}$ [21], for microactuators based on lead zirconate titanate (PZT) [22] to provide wide deflections and high actuation force [23]. There is another strong problem connected with the layer-based manufacturing technologies for instance layer stairstepping effect [24]. It is possible to use the laser system similar to "Blu-ray" to resolve this difficulty and to achieve smooth surfaces.


Figure. 1 Brief description of RPS technology

Moreover, the objects made with the help of additive manufacturing, due to their properties such as strength, electrical and thermal conductivity, and to the anisotropy caused by the layer-by-layer approach, typically have lower quality compared to their manufactured counterparts, made in a traditional way. Mechanical properties of the objects manufactured by dominated additive manufacturing technologies depend on the printed direction, and the application of these details is usually limited to the models for form/fit testing, functional testing, presentation models, prototypes and non-load bearing products.

RPS [25, 26] technology makes able to resolve these problems because the whole object's powder layers can be simultaneously sintered. Consequently, it does not have the disadvantages of a powerful laser and 2D beam distortion scanning systems. RPS provides much more consistent and predictable mechanical properties, creating objects same sintered parts that are smooth on the outside and solid on the inside.

Evidently, the manufacturing of $1 \mathrm{~cm}^{3}$ by cubic elements with rib of $10 \mu \mathrm{~m}$ involves billion calculations. Transformation of this volume with
layer thickness about $1 \mu \mathrm{~m}$ and dot precision same 'Blu-ray' laser $77,000 \times 77,000 \mathrm{dpi}(0.33 \times 0.33 \mu \mathrm{~m})$ within an hour entails calculating approximately 8 Gpoints per second. Therefore, RPS requires swift conversion algorithm for Cartesian coordinates system into spiral one.

The main advantages of the proposed impetuous algorithm for converting objects into roll based on the assumption that one point on the figure of revolution is predetermined locus in the spiral coordinate system due to symmetry properties.

Designed method provides to increase velocity of coordinate conversion thousand fold in comparison to other processes which don't use feature of solids of revolution.

## 2. Materials and methods

Figure 1 shows brief description of RPS technology: (1) ribbon perforation where it is necessary, (2) filling ribbon with the component or support powder and its compression for merging the adjacent filled holes to produce a continuous component roll (3), depending on the powder properties, sintering the object's roll, with
conventional, micro-wave, induction or other heating in the atmosphere of various gases and pressures (4), removing the ribbon and support powder mechanically, with the help of air flows or other methods (5), after the sintered object is ready (6).

### 2.1 Advantages of the spiral coordinate system

Objects and cavities in the forms of solids of revolution with a continuous shape for molds and casting are widely used in different industries.

It is well known that Cartesian and cylindrical coordinate systems require two coordinates $(x, y)$ or $(\rho, \varphi)$, respectively, to identify the point position on the plane instead of one coordinate $(l)$ and constant ' $h$ ' (the spiral length and distance between the successive turnings accordingly) in spiral coordinate system.

The proposed additive manufacturing technology using spiral coordinate system derived from idea of the spiral of Archimedes. Obviously, any ribbon (Fig. 2 (a)) can be transformed into a roll
like carpet (Fig. 2 (b)) and, in this way, the third dimension appears. Consequently, two coordinates and constant (height of ribbon) is enough for determining any point's location in threedimensional (3D) space. A spiral turn tends to take a shape of an ideal ring, if ' $h$ ' tends to 0, Fig. 3(a). With these assumptions, a circle in the spiral coordinate system looks like as a line, Fig. 3. (b), and the circle equation is transforms into an inequality:

$$
\begin{equation*}
L_{\text {Turn }} \min \leq l \leq L_{\text {Turn }} \max \tag{1}
\end{equation*}
$$

Via these results, is possible to determine the point's position, whether on a circle or not, with the help of only two comparison operations. It is very strong advantage for transformation solids of revolution to a binary raster layer because, in this case, one point helps to identify all points in the ring and the mathematical complexity of the transformation may be significantly reduced.

(b)

Figure. 2 Conformal coordinates transformation 2D space into 3D one: (a) object roll at the beginning and (b) rewound ribbon in roll.


Figure. 3 Simplified circle transformation to line: (a) roll after rewind and (b) roll before rewind


Figure. 4 A pawn of revolution: (a) general view, (b) cross section, and (c) figure of revolution


Figure. 5 Simplified figure of revolution transformation into spiral coordinates:
(a) figure of revolution and (b) image in the spiral coordinates

Let's consider the conversion of a pawn (Fig. 4 (a)) to a ribbon using the spiral coordinate system. It is a solid of revolution with the cross section (Fig. 4 (b)) and the figure of revolution (Fig. 4 (c)). A special feature of these solids is that the conformal transformation of the whole layer of a 3D object is possible without taking into consideration all the layer's points but only the layer's points located on the row perpendicular to the axis of revolution. Using this advantage, the coordinate conversion performance may be accelerated thousand fold. For that reason, the proposed technique makes it possible to do real-time conversion Cartesian coordinate to the spiral one with non-powerful and non-expensive computers.

Fig. 5 illustrates the simplified pawn shape of revolution (Fig. 5 (a)) transformation into the spiral coordinates (Fig. 5 (b)). To make thing clear, an example three corresponding points of contour of the solid of revolution and a component ribbon are shown. By scanning along the $y$ axis, starting from minimal z point coordinate, the point is mapped into the line on $l$ axis. The point lies closer to the edge of the figure and it is converting to a longer line.

It appears feasible to modify commercially available printing machine similar to "COMEXI FI2108 CNC" (COMEXI GROUP Company, print width 1670 mm and maximum printing speed 600 $\mathrm{m} / \mathrm{min}$ ) for RPS with help of laser or inkjet [27] perforation, powder and compression systems and thus to achieve object's roll forming performance $\sim 1.5 \mathrm{~m}^{3}$ in volume per hour with layer thickness $\sim 25$ $\mu \mathrm{m}$. As a result, similar modification could exceed volume build rate of the modern generation of SLS printers "ProX ${ }^{\text {™ }}$ SLS 500" (3D Systems Corporation) hundred fold. Subsequent performance acceleration, lower price and shorter fabrication time is possible when several component rolls are sintering simultaneously.

Accuracy of manufacturing includes ribbon reinforcement with not flexible stripes on its edges with the height less than a ribbon. As since high ribbon reeling velocity, wide perforation width and precision of 600 dpi involves transformation area of $10000 \times 1670 \mathrm{~mm}$ with the point size $\sim 42 \times 42 \mu \mathrm{~m}$, i.e. about 9 Gpoints ( $10^{9}$ points) per second, an efficient conversion algorithm is necessary.

### 2.2Converting algorithm for solids of revolution

For transform the solid of revolution to spiral coordinates, it is enough to transform only the coordinates its figure of revolution. This significant feature causes the fact that the quantity of dots for processing greatly decreases. Evidently, in this case, it is enough to convert just a half (or less) of a usual spatial object layer. Such method makes achievable real-time coordinate transformation large objects with high precision with non-powerful computers.

Figure 6 illustrates the algorithm of converting 2D points array to spiral coordinates. The algorithm begins with loading the full contour a solid of revolution file and definition the manufacturing precision, the layer's thickness and the take-up axle radius. Then, according to this shape the program is filling out the figure of revolution (FOR) matrix, taking into account the substance of an object and the scanning offset because of the radius of the object roll axle:

$$
\begin{equation*}
\text { Offset }=\text { FORHeight }+ \text { AxleRadius } \tag{2}
\end{equation*}
$$

where, FORHeight is the height of the figure of revolution, AxleRadius is the radius of object roll spindle.

As described above, one dot of a figure of revolution defines all the points on the circle in spiral coordinate system, and it defines the line's starting point and length more correctly. The point's beginning position is given by:

$$
\begin{equation*}
y+n+\text { YOffset } \tag{3}
\end{equation*}
$$

where $y$ is the distance between the axis of revolution and reference point, $n$ is the point's number and YOffset is the previous line offset. The length of the line can be represented like this:

$$
\begin{equation*}
\text { LineLength }=2 \pi(y+\text { AxleRadius }) \tag{4}
\end{equation*}
$$

where AxleRadius is the radius of object roll spindle. After the length of line for all line's points on radius $=y$ is calculated, using the algorithm starts to define the value of the ribbon array $[x, y]$ element with the substance codes, along with the solid of revolution file, while $x<$ FORWidth, where FORWidth is figure of revolution width. When the line converting is complete, the algorithm increases YOffset accordingly:

$$
\begin{equation*}
\text { YOffset }=\text { YOffset }+ \text { LineLength }-1 \tag{5}
\end{equation*}
$$



Figure. 6 Simplified logic algorithm for conversion 2D points array to spiral coordinates

After that, $y$ is incremented by 1 and, if $y<$ FORHeight, the process return to calculating LineLength for the next points with $y$ equal to the
figure of revolution matrix. Complete transformation of the solid of revolution into a ribbon requires processing while $y$ becomes equal FORHeight. The efficiency of the proposed solution lies in the fact that complicated mathematics becomes unnecessary, and it becomes possible to convert the Cartesian coordinates to spiral ones in real-time with non-powerful computers.

This method helps to accelerate the conversion due to using only one dot for determining the locus of the line on the plane, instead of the locus of circle. The algorithm can be successfully used for fast geometrical modeling and for converting 3D objects into flat ones. Relying upon these results, it can be extended to the acceleration of spatial computations.

## 3. Results and discussion

Obviously, the spiral coordinate system provides certain advantages for the conformal transformation of a solid of revolution points into flatness with the linear radius dependence: the smaller the dot size is, and the bigger the dot radius is, the faster the conversion.

For casting and molds, it is enough just to invert the raster ribbon point's value and the ribbon perforation will take the reverse image shown in Fig.

7 ((a) object, (b) object's cross section, (c) perforated object's ribbon filled with powder). Therefore, RPS additive manufacturing technology could considerably improve the injection-molded part design industry because it is flexible and it is possible to rapidly build design changes with immediate feedback on its design.

Existing algorithms of conformal transformation of three-dimensional Cartesian coordinates to twodimensional space by spiral coordinate system [28, 29] either uses equations for all object's points and requires expensive, powerful accelerators to make real-time transformation to achieve high performance or these algorithms transform just a single binary raster layer for such parts as pinions, blisk etc. [30, 31].

Hence, a quick transformation of such CAD solids and cavities models into a binary raster layers for RPS is an urgent problem.

This research concentrates on finding accelerating algorithm to build solids of revolution fast and precise by RPS. Table 1 shows the dots' quantity for the regular solid and the solid of revolution transformation to spiral coordinates with different diameters, precision and layer thickness.


Figure. 7 A pitcher and de Laval's nozzle spiral-ribbon transformations, length compressed scale: (a) solid of revolution, (b) cross section, and (c) image in the spiral coordinates

Table 1. Calculating the dots' quantity for the transformation of regular solids and solids of revolution

| Object <br> diameter | Precision |  | Layer thickness | Dots quantity for transformation on layer |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| m | dpi | $\mu \mathrm{m}$ | $\mu \mathrm{m}$ | Usual solid | Solid of <br> revolution | Ratio |
| 1000 | 77000 | $\sim 0.33$ | 1 | 7217784 | 1516 | 4762 |
| 10000 | 9600 | $\sim 3$ | 3 | 11219278 | 1890 | 5937 |
| 100000 | 1200 | $\sim 21$ | 25 | 17530122 | 2362 | 7421 |



Figure. 8 Edited screenshot of the "Transformation of a solid of revolution to spiral coordinates" software: (1) button "Load a Solid of revolution picture", (2) picture box for the loaded file, (3) dimensions of the loaded file (pixels, mm), (4) area of input transformation parameters (take up axle diameter, ribbon thickness, stripe's length), (5) button "Start spiral transformation", (6) area of calculated and scaled stripes, (7) picture box of the figure of revolution and a half of the take up axle (lilac bar on the left), and (8) progress bar of the transformation

Fig. 8 shows the interface of "Transformation of a solid of revolution to spiral coordinates" software for converting 2D points array of figure of revolution into stripes.

## 4. Conclusion

The article describes the algorithm of converting solids of revolution into a flat ribbon in the course RPS additive manufacturing technology. The proposed method is applicable for converting a
spatial object into flatness. As shown by the numerical examples, it is possible to increase the performance of the coordinate conversion thousand fold.

The key idea of the author is to use special feature of solids of revolution, which provide the conformal transformation of the whole layer of a 3D object based on the layer's points located in one row perpendicular to the axis of revolution. This technique can be extended to the simplification conformal transformation of three-dimensional Cartesian coordinates to two-dimensional space by spiral coordinate system.

The precise and accelerated algorithm for the transformation of solids of revolution into a flat ribbon, using the spiral coordinate system, has been designed and implemented. The experimental validation of the algorithm is given with the examples of a pawn, a pitcher and de Laval's nozzle. Proposed method acceptable for forming $1.5 \mathrm{~m}^{3}$ objects for molds and casting with precision of 600 dpi, layer thickness $\sim 25 \mu \mathrm{~m}$ and reeling velocity 10 $\mathrm{m} / \mathrm{s}$ with non-powerful computers within about an hour.

Implemented algorithm shows a great potential for its applicability for successfully using for fast geometrical modeling and for the acceleration of spatial computations.

There is a large application field including complex geometries. Spiral coordinate system provides to describe 2D and 3D spaces by one and two dimensions accordingly against two and three in conventional systems.

## Disclosure statement

No potential conflict of interest was reported by the authors.

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