International Journal of Mathematics and Soft Computing
Vol.4, No.2. (2014), 213 - 224.

ISSN Print : 2249 – 3328
ISSN Online: 2319 – 5215

Perishable inventory system with (s, Q) policy in supply chain

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Abstract

In this paper we consider a continuous review perishable inventory system in Multi-echelon system, which is a building block for supply chain. A (s, Q) type perishable inventory system with Poisson demand and exponential distributed lead times for items are assumed at DC (middle echelon). A one-for- one type inventory policy is assumed at retailer node (lower echelon). Demands occurring during the stock out periods are assumed to be lost. The DC replenishes their stocks with exponential distributed lead times from warehouse (upper echelon) has abundant supply source. The items are supplied to the DC in packs of Q (= S-s) items from the warehouse. The steady state probability distribution and the operating characteristics are obtained explicitly. The required algorithm is derived and it is implemented using Mat lab. The measures of system performance in the steady state are obtained.

Keywords: **S**upply Chain, Markov process, Inventory control, Optimization.

AMS Subject Classification (2010): 90B05.

1 Introduction

Supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products and the distribution of these finished products to customers. Supply Chain exists in both service and manufacturing organizations, but the complexity of the chain may vary greatly from industry to industry.

Inventory decision is an important component of the supply chain management, because Inventories exist at each and every stage of the supply chain as raw material or semi- finished or finished goods. They can also be as Work-in-process between the stages or stations. Since holding of inventories can cost anywhere between 20% to 40% of their value, their efficient management is critical in Supply Chain operations

The usual objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. The reason is that a key concept of supply chain management is that a company should strive to develop an informal partnership relation with its suppliers and retailers that enable them jointly to maximize their total profit.

Information technology has a substantial impact on supply chains. Scanners collect sales data at the point-of-sale, and Electronic Data Interchange (EDI) allows these data to be shared immediately with all stages of the supply chain.

Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature. As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency.

The first quantitative analysis in inventory studies started with the work of Harris [9].Clark and Scarf [4] had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size, Recent developments in two-echelon models may be found in Q.M. He and E.M. Jewkes[17].Sven Axsäter [1] proposed an approximate model of inventory structure in SC. One of the oldest papers in the field of continuous review multi-echelon inventory system is a basic and seminal paper written by Sherbrooke [19] in 1968. He assumed (S-1, S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Continuous review models of multi-echelon inventory system in 1980's concentrated more on repairable items in a Depot-Base system than as consumable items (see Graves [6,7], Moinzadeh and Lee [15]). All these papers deal with repairable items with batch ordering. Seifbarghy and Jokar [18] analyzed a two echelon inventory system with one warehouse and multiple retailers controlled by continuous review (R, Q) policy. A Complete review was provided by Benita M. Beamon (1998)[2]. the supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic work of Clark and Scarf (1960)[4]. In the case of continuous review perishable inventory models with random life times for the items, most of the models assume instantaneous supply of order [8,13,14]. The assumption of positive lead times further increases the complexity of the analysis of these models and hence there are only a limited number of papers dealing with positive lead times. A continuous review perishable inventory system at Service Facilities was studied by Elango (2001) [5]. A continuous review (s, S) policy with positive lead times in two-echelon Supply Chain was considered by K. Krishnan and C. Elango[12].

The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, both transient and steady state analysis are done. Section 4 deals with the derivation of operating characteristics of the system. In Section 5, the cost analysis for the operation is derived. Numerical examples and sensitivity analysis are provided in section 6 and the last section 7 concludes the paper.

2 The Model description

We consider a supply chain system consisting of a manufacturer, warehousing facility, Two Distribution Centre's (DC) each associated with *n* identical retailers dealing with a single perishable product. These finished products moves from the manufacturer through the network consist of WH, DC, Retailer then the final customer.

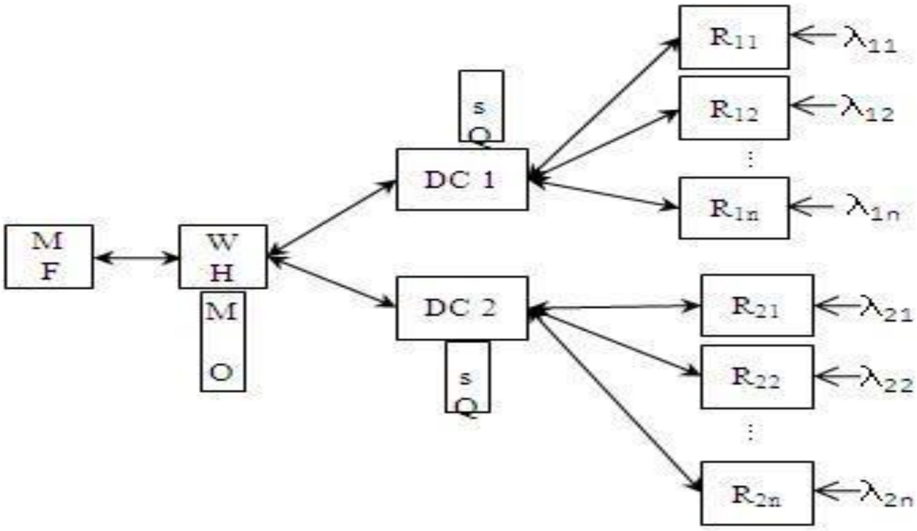

Figure 1: Multi-echelon Inventory System.

A finished product is supplied from MF to WH which adopts (0, M) replenishment policy then the product is supplied to DC's who adopts (s, Q) policy. The demand at retailer node follows a Poisson distribution with rate $\lambda_{ij}\,(i=1,2;\ j=1,2,\cdots,n)$. Scanners collect sales data at retailer nodes and Electronic Data Interchange (EDI) allows these data to be shared to the corresponding DC. With the strong communication network and transport facility a unit of item is transferred from Dc to the related retailer with negligible lead time. That is all the inventory transactions are managed by DC's. Supply to the Manufacturer in packets of Q items is administrated with exponential lead time having parameter $\mu_i > 0\,(i=1,2)$. It is assumed that the items are perishes only at DC with rate γ. The replenishment of items in terms of pockets is made from Manufacturer to WH is instantaneous. Demands occurring during the stock out periods are assumed to be lost. The maximum inventory level at DC node S is fixed and the reorder point is s and the ordering quantity is Q(=S-s) items. The maximum inventory level at Manufacturer is M (M = *n*Q). The optimization criterion is to minimize the total cost incurred at all the locations subject to the performance level constraints.

According to the assumptions the on hand inventory levels at both nodes follows a Markov process.

We fix the following notations for the forthcoming analysis part of our paper.

[R]ij : The element /sub matrix at (*i*,*j*)th position of R.

0 : Zero matrix. I : Identity matrix.

e : A column vector of 1's of appropriate dimension.

Ii(t) : On hand inventory level at time t at location i(i =0,1,2) .

kiD : Fixed ordering cost, regardless of order size at DC node i(i =1,2).

kiR : the ordering cost at retailer node related to DC i(i =1,2).

ki0 : kiD + kiR , ordering cost for integrated DC system i(i =1,2).

k1 : Fixed ordering cost for WH.

hiD : The holding cost per unit of item per unit time at DC i(i =1,2).

hR : the holding cost per unit of item per unit time at retailer nodes.

hi0 : hiD + hiR the holding cost for integrated DC i(i =1,2).

h1 : The holding cost per unit of item per unit time at WH.

gD : gD = g1D + g2D The unit shortage cost at DC.

gR **: The average shortage cost per unit shortage at retailer node.**

g :g= gD + gR The unit shortage cost for integrated DC system.

$$\sum_{i=1}^{k} a_i = a_1 + a_2 + \ldots + a_k \text{ and } \sum_{i=0}^{nQ_*} i = 0 + Q + 2Q + \ldots + nQ .$$

3 Analysis

Let $I_D(t)$ denote the on hand inventory at warehouse and $I_1(t)$, $I_2(t)$ denote the on hand inventory at DCi (i=1,2) respectively at time t. From the assumptions on the input and output processes, we define $I(t)= \{(I_1(t), I_2(t), I_D(t)) : t \geq 0\}$ and we get $\{I(t) : t \geq 0\} = \{(I_1(t), I_2(t), I_D(t)) : t \geq 0\}$ as a Markov process with state space E = {(i, k, m) / I = S,S-1,…, s,s-1,…,2,1,0; k = S,S-1,… s,s-1, …, 2,1,0; m = nQ, (n-1)Q, ... 2Q, Q}, since E is finite and all its states are aperiodic, recurrent non-null and also irreducible. That is all the states are ergodic. Hence the limiting distribution exists and is independent of the initial state.

The infinitesimal generator of this process A= $(a(i,k,m : j,l,n))_{(i,k,m),(j,l,n)\in E}$ can be obtained from the following arguments.

- The arrival of a demand or perish of an item at DC1 makes a state transition in the Markov process from (i, k, m) to (i-1,k,m) with intensity of transition $\lambda_{1j} + i\gamma$ (j=1,2,...,n).
- The arrival of a demand for an item at DC2 makes a state transition in the Markov process from (i, k, m) to (i,k-1,m) with intensity of transition $\lambda_{2j} + k\gamma$ (j=1,2,...,n).
- Replenishment of inventory at DC1 makes a state transition from ((i, k, m) to (i+Q, k, m-Q) with rate of transition μ1(> 0).
- Replenishment of inventory at DC2 makes a state transition from (i, k, m) to (i, k+Q, m-Q) with rate of transition μ2(> 0).

The infinitesimal generator R is given by $R = \begin{bmatrix} A & B & 0 & \cdots & 0 & 0 \\ 0 & A & B & \cdots & 0 & 0 \\ 0 & 0 & A & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & B \\ B & 0 & 0 & \cdots & 0 & A \end{bmatrix}$

Hence entities of R are given by
$$R_{pq} = \begin{cases} A & p = q; \quad p = nQ,(n-1)Q,\cdots,Q \\ B & p = q+Q; \quad p = nQ,(n-1)Q,\cdots,Q \\ B & p = q; \quad p = nQ \\ 0 & \text{otherwise.} \end{cases}$$

The sub matrices are given by
$$[A]_{pq} = \begin{cases} A_1 & p = q; \quad p = S, S-1,\ldots,s+1 \\ B_1 & p = q+1; \quad p = S, S-1,\ldots,1 \\ C_1 & p = q; \quad p = s, s-1,\ldots,1 \\ D_1 & p = q; \quad p = 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$[B]_{pq} = \begin{cases} M & p = q; \quad p = S, S-1,\ldots,1,0 \\ M & p = q+Q; \quad p = s, s-1,\ldots,1,0 \\ 0 & \text{otherwise} \end{cases}$$

The sub matrices of A and B are

$$[A_1]_{pq} = \begin{cases} -(\lambda_1 + \lambda_2 + i\gamma + k\gamma) & p = q; \quad p, i, k = S, S-1, \dots, s+1 \\ \lambda_2 + k\gamma & p = q+1; \quad p, k = S, S-1, \dots, 1 \\ -(\lambda_1 + \lambda_2 + \mu_2 + i\gamma + k\gamma) & p = q; \quad p, i, k = s, s-1, \dots, 1 \\ -(\lambda_1 + \mu_2 + i\gamma) & p = q; \quad p = 0, i = S, S-1, \dots, s+1 \\ 0 & \text{otherwise} \end{cases}$$

$$[B_1]_{pq} = \begin{cases} \lambda 1 + i\gamma & p = q; \quad p, i, k = S, S-1, \dots, s+1 \\ 0 & \text{otherwise} \end{cases}$$

$$[C_1]_{pq} = \begin{cases} -(\lambda_1 + \lambda_2 + \mu_2 + i\gamma + k\gamma) & p = q; \quad p = s, s-1, \dots, 1 \\ \lambda_2 + k\gamma & p = q+1; \quad p = S, S-1, \dots, 1 \\ -(\lambda_1 + \lambda_2 + i\gamma + k\gamma + \mu_1 + \mu_2) & p = q; \quad p = s, s-1, \dots, 1 \\ -(\lambda_1 + i\gamma + \mu_1 + \mu_2) & p = q; \quad p = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$[D_1]_{pq} = \begin{cases} -(\lambda_2 + k\gamma + \mu_1) & p = q; \quad p = 0 \\ \lambda_2 + k\gamma & p = q+1; \quad p = S, S-1, \dots, 1 \\ -(\lambda_2 + k\gamma + \mu_1 + \mu_2) & p = q; \quad p = s, s-1, \dots, 1 \\ -(\mu_1 + \mu_2) & p = q; \quad p = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$[M_1]_{pq} = \begin{cases} \mu_1 & p = q; \quad p = S, S-1, \dots, 1 \\ 0 & \text{otherwise} \end{cases}$$

$$[M_2]_{pq} = \begin{cases} \mu_2 & p = q + Q; \quad p = s, s-1, \dots, 1, 0 \\ 0 & \text{otherwise} \end{cases}$$

3.1 Transient Analysis

Define the transient probability function

$$p_{i,k,m}(j,1,n:t) = \Pr\{(I_1(t), I_2(t), I_D(t)) = (j,1,n) \mid (I_1(0), I_2(0), I_D(0)) = (i,k,m)\}.$$

The transient matrix for $t \geq 0$ is of the form $P(t) = \left(p_{i,k,m}(j,1,n:t)\right)_{(i,k,m)(j,1,n)\in E}$ satisfies the Kolmogorov- forward equation $P'(t) = P(t)R$ (1)

where R is the infinitesimal generator of the process $\{I(t), t \geq 0\}$.

The solutions of (1) can be written in the form $P(t) = P(0)e^{Rt} = e^{Rt}$ where e^{Rt} is the matrix given by $e^{Rt} = I + \sum_{n=1}^{\infty} \frac{R^n t^n}{n}$.

Assume that the eigenvalues of R are all distinct. Then from the spectral theorem of matrices, we have $R=HDH^{-1}$ where H is a non-singular matrix(formed with the right eigenvectors of R) and D is the diagonal matrix having for its diagonal elements the eigenvalues of R. Now , 0 is an eigen value of R and $d_i \neq 0, i = 1, 2, ..., m$ m are the other distinct eigenvalues, then

$$D = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ 0 & d_1 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & d_m \end{bmatrix}$$

We then have

$$D^n = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ 0 & d_1^n & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & d_m^n \end{bmatrix} \text{ and} R^n = HD^nH^{-1}.$$

We have

$$P(t) = I + \sum_{n=1}^{\infty} \frac{\left(HD^nH^{-1}\right)}{n!} = H\left\{I + \sum_{n=1}^{\infty} \frac{D^n t^n}{n!}\right\}H^{-1} = He^{Dt}H^{-1} \text{ where}$$

$$e^{Dt} = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & e^{d_1 t} & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & e^{d_m t} \end{bmatrix}$$

The right-hand side of above equation gives explicit solution of the matrix p(t). Note that even in the general case when the Eigenvalues of R are not necessarily distinct, a canonical representation of $R=SZS^{-1}$ exists.

3.2 Steady state analysis

The structure of the infinitesimal matrix R, reveals that the state space E of the Markov process $\{I(t), t \geq 0\}$ is finite and irreducible. Let the limiting probability distribution of the inventory level process be $m_{vi}k = \lim_{t\to\infty} Pr\{(I_1(t), I_2(t), I_D(t)) = (i, k, m)\}$ where $m_{vi}k$ is the steady state probability that the system be in state (i, k, m), (Cinlar [3]).

Let $V = (nQ_V, (n-1)Q_V, (n-2)Q_V, ..., 2Q_V, Q_V, 0_V)$ denote the steady state probability distribution where $jQ_V = (v_S^k, v_{S-1}^k, v_{S-2}^k, ..., v_1^k, V_0^k)v$ for j = 1, 2 ... n and k = 1,2,…, S, for the system under consideration. For each (i, k, m), $m_{vi}k$ can be obtained by solving the matrix equation vR = 0 together with normalizing condition $\sum_{(i,k,m)\in E} m_{v_j} k = 1$.

Assuming $Q_V = a$, we obtain the steady state probabilities $iQ_V = (-1)^k a(BA)^k$, i = 1,2, … n ; k = n-i+1, where $a = e^1 \left[\sum_{i=0}^{n-1} (-1)^i \left(BA^{-1}\right)^i \right]$.

4 Operating characteristics

In this section, we derive some important system performance measures.

4.1 Mean Reorder states

The event β_1, β_2 and β_D are the mean reorder rate at DC1, DC2 and WH and are given by $\beta_1 = (\lambda_1 + (s+1)\gamma) \sum_{q,k} q_{v_{s+1}^k}$; $\beta_2 = (\lambda_2 + (s+1)\gamma) \sum_{q,k} q_{v_k^{s+1}}$ and

$$\beta_D = \left[\mu_1 \sum_{i=0}^{S} \sum_{k=0}^{s} nQ_{v_i^k} + \mu_2 \sum_{i=0}^{S} \sum_{k=0}^{s} nQ_{v_i^k} \right] \tag{2}$$

4.2 Mean Inventory levels

Let denote the mean inventory level in the steady state at node i (i=D,1,2). Thus the mean inventory level at WH is given by $\overline{I_D} = \sum_{q=Q}^{nQ} q \left(\sum_{i,k} q_{v_i^k} \right)$. The mean inventory level at DC node 1 and node 2 are given by $\overline{I_1} = \sum_{i=0}^{S} i \left(\sum_{q,k} q_{v_i^k} \right)$ and $\overline{I_2} = \sum_{k=0}^{S} i \left(\sum_{q,i} q_{v_i^k} \right)$. (3)

4.3 Mean storage rates

The shortage rates at DC1 and DC2 are given by $\alpha_1 = \lambda_1 \sum_{q,k} q_{v_0^k}$ and $\alpha_2 = \lambda_2 \sum_{q,k} q_{v_i^0}$ (4)

5 Cost analysis

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time.

The long run expected cost rate C(s, Q) is given by

$$C(s,Q) = h_D \overline{I_D} + h_1 \overline{I_1} + h_2 \overline{I_2} + k_D \beta_D + k_1 \beta_1 + k_2 \beta_2 + \alpha_1 g_1 + \alpha_2 g_2 \tag{5}$$

Although we have not proved analytically the convexity of the cost function C(s,Q), our experience with considerable number of numerical examples indicates that C(s,Q) for fixed Q appears to be convex in s. In some cases it turned out to be an increasing function of s. For large number cases of

C(s, Q) revealed a locally convex structure. Hence we adopted the numerical search procedure to determine the optimal values s*.

6 Numerical Example and Sensitivity Analysis

In this section we discuss the problem of minimizing the steady state expected cost rate under the following cost structure. We assume $k_1 \geq k_0$, since the setup cost which includes the freight charges could be higher for the larger size order (pockets) compared to that of the small one initiated at retailer nodes. Regarding the holding cost we assume $h_1 \leq h_0$, since the holding cost at distribution node is less than that of the retailer node as the rental charge may be high at retailer node. The results we obtained in the steady state case may be illustrated through the following numerical example.

For the following example, we assume that, S = 6, M = 12, $\lambda_1 = 2$, $\lambda_2 = 5$, $\mu_D = 0.4$, $\mu_1 = 0.3$, $\mu_2 = 0.7$, $h_D = 1.25$, $h_1 = 3.8$, $h_2 = 0.9$, $k_D = 0.6$, $k_1 = 0.7$, $k_2 = 0.7$, $g_1 = 1.25$, $g_2 = 3$ and $\gamma = 1.2$.

The cost for different reorder levels are given by

Table 1. Total expected cost rate as a function s and Q.

s	Q	C(s,Q)
0	6	7.5002
1	5	7.4979
2	4	7.4843
3*	3*	7.4581*
4	2	7.5047
5	1	7.5083

For the inventory capacity S, the optimal reorder level‘s*’ and optimal cost C(s,Q) are indicated by the symbol ’*’.

The graphical representation of the long run expected cost rate C(s*,Q*) is given below.

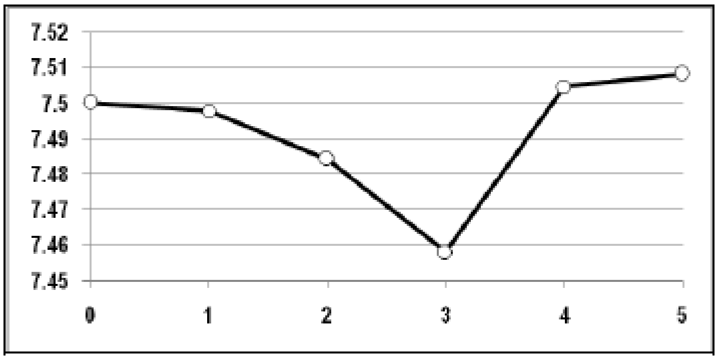

Figure 2: The graphical representation of the long run expected cost rate C(s*,Q*).

6.1 Sensitivity Analysis

Table 2 presents a numerical study to exhibit the sensitivity of the system on the effect of varying demand rates λ_1 and λ_2 with fixed reorder at s=3.

Table 2. The total expected cost vs. demand rates (λ_1 and λ_2)

$\lambda_1 \rightarrow$	2	3	4	5	6
5	7.4588	7.7172	7.9818	8.2479	8.5146
6	8.8049	9.0576	9.3181	9.5806	9.8438
7	10.1493	10.3974	10.6550	10.9151	11.1760
8	11.4918	11.7360	11.9914	12.2497	12.5089
9	12.8327	13.0734	13.3270	13.5839	13.8418

It is observed that the total expected cost C(s, Q) is increasing with the different demand rates. Hence the demand rate is a very important parameter of this system.

7 Concluding remarks

In this paper we analyzed a continuous review perishable inventory system in a supply chain. The structure of the chain allows vertical movement of goods from distribution center to retailers. A (s, Q) type inventory system with Poisson demand and exponential distributed lead times for items are assumed at DC (middle echelon). And one-for-one type inventory policy is assumed at retailer node (lower echelon). Demands occurring during the stock out periods are assumed to be lost. The DC replenishes their stocks with exponential distributed lead times from warehouse (upper echelon) has abundant supply source. The items are supplied to the DC in packs of Q (= S-s) items from the warehouse. The model deals with lost sales at DC and the supply from manufacturer is in terms of pockets. It would be interesting to analyze the problem discussed in this article where the life time of items are constant. Naturally, with the inclusion of constant life time of each items, the problem will be more challenging. Another important extension could be made by relaxing the assumption of exponentially distributed lead times to a class of arbitrarily distributed lead times using techniques from renewal theory and semi-regenerative processes. Once this is done, the general model can be used to generate various special cases. For example, three different lead time distributions one with coefficient of variation greater than one, one with coefficient of variation less than one and another with coefficient of variation equal to one can be compared. Cost analysis can then be carried out for (s, Q), (S, S-1) and lot-for-lot models using each of the three different lead time distributions to determine which policy is optimal for any given lead time distribution.

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