*International Journal of Mathematics and Soft Computing*
*Vol.4, No.2. (2014), 183 - 196.*

**ISSN Print : 2249 – 3328**
**ISSN Online: 2319 – 5215**

# On *a*-vertex consecutive edge bimagic labeling for switching graphs

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**Abstract**

A bijective function $f: V \cup E \rightarrow \{1, 2, \ldots, n+e\}$ is called $a$-vertex consecutive edge magic total labeling of $G$, if $f(u) + f(uv) + f(v) = k$ for each edge $e \in E$ and $f(V) = \{a+1, a+2, \ldots, a+n\}$ for some $a$, $0 \leq a \leq e$. It is said to be $a$-vertex consecutive edge bimagic total labeling, if the above sum is either $k_1$ or $k_2$. In this paper, we investigate $a$-vertex consecutive edge bimagic labeling for the switching of $P_n\ (n \geq 3)$, $C_n\ (n \geq 4)$, $p_n^+\ (n \geq 2)$, $C_n^+\ (n \geq 3)$, $P_2 + nK_1\ (n \geq 2)$, $B_{n,m}\ (n, m \geq 2)$, $P_n^2\ (n \geq 4)$ and $C_n^2\ (n \geq 6)$ graphs.

**Keywords:** Edge labeling, bimagic labeling, consecutive bimagic labeling.

**AMS Subject Classification (2010):** 05C78.

## 1 Introduction

All graphs considered in this paper are finite and undirected. Let $G = (V, E)$ be a graph with $|V| = n$ and $|E| = e$. In 1963 the notion of magic labeling was introduced by Sedlacek [5] who used the label values as distinct positive real numbers. In 1970, Kotzig and Rosa [4] defined a magic valuation of a graph $G$, as a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, n+e\}$ such that $f(u) + f(uv) + f(v)$ is a constant $k$ for any edge $e \in E$. The concept of bimagic labeling was introduced by Baskar Babujee [1] in 2004. A total edge-magic graph is called $a$-vertex consecutive edge magic labeling, if $f(V) = \{a+1, a+2, \ldots, a+n\}$ for some $a$, $0 \leq a \leq e$.

This concept was introduced by Sugeng and Miller [6]. Further, Baskar Babujee, Vishnupriya and Jagadesh [2] introduced the notion of $a$-vertex consecutive edge bimagic total labeling of a graph $G$, as a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, n+e\}$ such that $f(u) + f(uv) + f(v) = k_1$ or $k_2$ where $k_1$ and $k_2$ are distinct positive integers for each edge $e \in E$ and $f(V) = \{a+1, a+2, \ldots, a+n\}$ for some $a$, $0 \leq a \leq e$. For a survey on graph labeling, we refer to Gallian [3].

**Definition 1.1. [6]** A bijection $f: V \cup E \rightarrow \{1, 2, \ldots, n+e\}$ is called a-vertex consecutive edge magic total labeling of $G$, if $f(u) + f(uv) + f(v)$ is a constant $k$ for each edge $e \in E$ and $f(V) = \{a+1, a+2, \ldots, a+n\}$ for some $a$, $0 \le a \le e$.

**Definition 1.2. [2]** A bijection $f: V \cup E \rightarrow \{1, 2, \ldots, n+e\}$ is called a-vertex consecutive edge bimagic total labeling of $G$, if $f(u) + f(uv) + f(v) = k_1$ or $k_2$ for each edge $e \in E$ and $f(V) = \{a+1, a+2, \ldots, a+n\}$ for some $a$, $0 \le a \le e$. If $a = 0$, then the labeling is called super edge bimagic labeling.

**Definition 1.3. [7]** A vertex switching $G_v$ of a graph $G$ at a vertex $v \in V$ is obtained by removing all edges incident to $v$ and adding the edges joining $v$ to every other vertex which are not adjacent to $v$ in $G$.

**Definition 1.4.** For a simple connected graph $G$ the square of $G$ is denoted by $G^2$ and defined as the graph with the same vertex set as that of $G$ and two vertices in $G^2$ are adjacent if they are at a distance 1 to 2 apart in $G$.

In this paper, we study $a$-vertex consecutive edge bimagic labeling for vertex switching of some special families of graphs.

## 2 Main Results

**Theorem 2.1.** Switching of a pendant vertex in a path graph $P_n$, $(n \ge 3)$ admits an $a$-vertex consecutive edge bimagic total labeling with $a = n-2$.

**Proof:** Let $G_V$ be the graph obtained by switching a pendant vertex v of $G = P_n$. Let the vertices of $P_n$ $(n \ge 3)$ be $V = \{u_1, u_2, ..., u_n\}$. Without loss of generality let us assume that the vertex $u_1$ is switched to obtain $G_v$. We observe that $|V(G_v)| = n$ and $|E(G_v)| = 2(n-2)$. In $G_v$, $E(G_v) = E_1 \cup E_2$ where $E_1 = \{u_i u_{i+1} : 2 \le i \le n-1\}$, $E_2 = \{u_1 u_i : 3 \le i \le n\}$.

We define a bijective function $f : V(G_v) \cup E(G_v) \rightarrow \{1, 2, ..., (3n-4)\}$ as follows:

For $i = 2$ to $n$; when $i \equiv 0 \pmod 2$, let $f(u_i) = n - 2 + \frac{i}{2}$ and

when $i \equiv 1 \pmod 2$, let $f(u_i) = n + \left\lfloor \frac{n}{2} \right\rfloor - 3 + \frac{i+1}{2}$.

For $i = 3$ to $n$; when $i \equiv 1 \pmod 2$, let $f(u_1 u_i) = n - \left\lfloor \frac{n}{2} \right\rfloor + 1 - \frac{i+1}{2}$ and

when $i \equiv 0 \pmod 2$, let $f(u_1 u_i) = n - \frac{i}{2}$.

For $i = 2$ to $n-1$; let $f(u_i u_{i+1}) = 3n - 2 - i$ and let $f(u_1) = 2(n-1)$.

We claim that, $k_1 = 5n + \left\lfloor \frac{n}{2} \right\rfloor - 6$ and $k_2 = 4(n-1)$.

(i) For the edges in $E_1$, when $i \equiv 1 \pmod 2$ we have

$$f(u_i)+f(u_iu_{i+1})+f(u_{i+1})=\left(n+\left\lfloor\frac{n}{2}\right\rfloor-3+\frac{i+1}{2}\right)+(3n-2-i)+\left(n-2+\frac{i+1}{2}\right)=5n+\left\lfloor\frac{n}{2}\right\rfloor-6=k_1$$

and when $i\equiv 0\ (\text{mod } 2)$

$$f(u_i)+f(u_iu_{i+1})+f(u_{i+1})=\left(n-2+\frac{i}{2}\right)+(3n-2-i)+\left(n+\left\lfloor\frac{n}{2}\right\rfloor-3+\frac{i+2}{2}\right)=5n+\left\lfloor\frac{n}{2}\right\rfloor-6=k_1.$$

(ii) For edges in $E_2$, when $i\equiv 1\ (\text{mod } 2)$ we have

$$f(u_1)+f(u_1u_i)+f(u_i)=(2(n-1))+\left(n-\left\lfloor\frac{n}{2}\right\rfloor+1-\frac{i+1}{2}\right)+\left(n+\left\lfloor\frac{n}{2}\right\rfloor-3+\frac{i+1}{2}\right)=4(n-1)=k_2$$

and when $i\equiv 0\ (\text{mod } 2)$

$$f(u_1)+f(u_1u_i)+f(u_i)=(2(n-1))+\left(n-\frac{i}{2}\right)+\left(n-2+\frac{i}{2}\right)=4(n-1)=k_2.$$

Therefore, switching of a pendant vertex in a path $P_n, (n\geq 3)$ admits an *a*-vertex consecutive edge bimagic total labeling with *a* = *n*-2. ■

**Theorem 2.2.** Simultaneous switching of both the pendant vertices in a path graph $P_n$ ,$(n\geq 4)$ admits an *a*- vertex consecutive edge bimagic labeling with $a=2n-5$, if *n* is even and $a=3n-8$, if *n* is odd.

**Proof:** Let *G* = *P*$_n$: *u*$_1$, *u*$_2$,…,*u*$_n$ be a path on $n\geq 4$ vertices and let $G_v'$ be the graph obtained by switching both the pendant vertices $u_1$ and $u_2$ of *G*. We observe that $|V(G_v')|=n$ and $|E(G_v')|=3n-8$.

In $G_v'$, $E(G_v')=E_1\cup E_2\cup E_3\cup E_4$ where $E_1=\{u_iu_{i+1}:2\leq i\leq n-2\}$, $E_2=\{u_1u_i:3\leq i\leq n-1\}$, $E_3=\{u_nu_i:2\leq i\leq n-2\}$ and $E_4=\{u_1u_n\}$.

Define a bijective function $f:V(G_v)\cup E(G_v)\to\{1,2,...,4(n-2)\}$ as given below:

**Case (i):** *n* is even.

For *i = 2 to n*; when $i\equiv 0\ (\text{mod } 2)$ let $f(u_i)=2(n-2)+\frac{i}{2}$ and

when $i\equiv 1\ (\text{mod } 2)$ let $f(u_i)=2(n-3)+\frac{n}{2}+\frac{i+1}{2}$.

For *i = 3 to n*; when $i\equiv 1\ (\text{mod } 2)$ let $f(u_1u_i)=n+\frac{n}{2}-1-\frac{i+1}{2}$ and

when $i\equiv 0\ (\text{mod } 2)$ let $f(u_1u_i)=2n-3-\frac{i}{2}$.

For *i = 2 to n-2*; when $i\equiv 0\ (\text{mod } 2)$ let $f(u_nu_i)=n-2-\frac{i}{2}$ and

when $i\equiv 1\ (\text{mod } 2)$ let $f(u_nu_i)=\frac{n}{2}-\frac{i+1}{2}$.

For *i* = 2 *to* *n*-2; let $f(u_iu_{i+1})=2(2n-3)-i$ and let $f(u_1)=2(n-2)$, $f(u_n)=3n-5$, $f(u_1u_n)=n-2$.

We claim that $k_1 = 8n + \frac{n}{2} - 15$ and $k_2 = 6n - 11$.

(i) For the edges in $E_1$, when $i \equiv 1 \pmod 2$ we have

$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \left(2(n-3) + \frac{n}{2} + \frac{i+1}{2}\right) + \left(2(2n-3) - i\right) + \left(2(n-2) + \frac{i+1}{2}\right) = 8n + \frac{n}{2} - 15 = k_1$$

and when $i \equiv 0 \pmod 2$

$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \left(2(n-2) + \frac{i}{2}\right) + \left(2(2n-3) - i\right) + \left(2(n-3) + \frac{n}{2} + \frac{i+2}{2}\right) = 8n + \frac{n}{2} - 15 = k_1.$$

(ii) For the edges in $E_2$, when $i \equiv 1 \pmod 2$ we have

$$f(u_1) + f(u_1 u_i) + f(u_i) = \left(2(n-2)\right) + \left(n + \frac{n}{2} - 1 - \frac{i+1}{2}\right) + \left(2(n-3) + \frac{n}{2} + \frac{i+1}{2}\right) = 6n - 11 = k_2$$

and when $i \equiv 0 \pmod 2$

$$f(u_1) + f(u_1 u_i) + f(u_i) = \left(2(n-2)\right) + \left(2n - 3 - \frac{i}{2}\right) + \left(2(n-2) + \frac{i}{2}\right) = 6n - 11 = k_2.$$

(iii) For the edges in $E_3$, when $i \equiv 1 \pmod 2$ we have

$$f(u_n) + f(u_n u_i) + f(u_i) = (3n-5) + \left(\frac{n}{2} - \frac{i+1}{2}\right) + \left(2(n-3) + \frac{n}{2} + \frac{i+1}{2}\right) = 6n - 11 = k_2$$

and when $i \equiv 0 \pmod 2$

$$f(u_n) + f(u_n u_i) + f(u_i) = (3n-5) + \left(n - 2 - \frac{i}{2}\right) + \left(2(n-2) + \frac{i}{2}\right) = 6n - 11 = k_2.$$

(iv) For the edges in $E_4$, we have

$$f(u_1) + f(u_n u_1) + f(u_n) = \left(2(n-2)\right) + (n-2) + (3n-5) = 6n - 11 = k_2.$$

**Case (ii):** $n$ is odd.

For $i = 2$ *to* $n$-1; when $i \equiv 0 \pmod 2$, let $f(u_i) = 4(n-2) - \frac{i}{2}$ and

when $i \equiv 1 \pmod 2$ let $f(u_i) = 3n + \frac{n+1}{2} - 7 - \frac{i+1}{2}$.

For $i = 2$ *to* $n$-2; when $i \equiv 1 \pmod 2$ let $f(u_n u_i) = n + \frac{n-9}{2} + \frac{i+1}{2}$ and

when $i \equiv 0 \pmod 2$, let $f(u_n u_i) = n - 3 + \frac{i}{2}$.

For $i = 3$ *to* $n$-2; when $i \equiv 1 \pmod 2$ let $f(u_1 u_i) = \frac{n-7}{2} + \frac{i+1}{2}$.

For $i = 6$ *to* $n$-1; when $i \equiv 0 \pmod 2$, let $f(u_1 u_i) = \frac{i}{2} - 2$

and let $f(u_1) = 4(n-2)$, $f(u_n) = 3n - 7$, $f(u_1 u_n) = n - 3$, $f(u_1 u_4) = n + \frac{n-7}{2}$, $f(u_4) = 2(2n-5)$.

We claim that $k_1 = 9n + \frac{n+1}{2} - 22$ and $k_2 = 2(4n-9)$.

(i) For the edges in $E_1$, when $i \equiv 1 \pmod 2$ we have

$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \left(3n + \frac{n+1}{2} - 7 - \frac{i+1}{2}\right) + (2(n-3)+i) + \left(4(n-2) - \frac{i+1}{2}\right) = 9n + \frac{n+1}{2} - 22 = k_1$$

and when $i \equiv 0 \pmod 2$

$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \left(4(n-2) - \frac{i}{2}\right) + (2(n-3)+i) + \left(3n + \frac{n+1}{2} - 7 - \frac{i+2}{2}\right) = 9n + \frac{n+1}{2} - 22 = k_1.$$

(ii) For the edges in $E_2$, when $i \equiv 1 \pmod 2$ we have

$$f(u_1) + f(u_1 u_i) + f(u_i) = (4(n-2)) + \left(\frac{n-7}{2} + \frac{i+1}{2}\right) + \left(3n + \frac{n+1}{2} - 7 - \frac{i+1}{2}\right) = 2(4n-9) = k_2$$

and when $i \equiv 0 \pmod 2$

$$f(u_1) + f(u_1 u_i) + f(u_i) = (4(n-2)) + \left(\frac{i}{2} - 2\right) + \left(4(n-2) - \frac{i}{2}\right) + = 2(4n-9) = k_2.$$

(iii) For the edges in $E_3$, when $i \equiv 1 \pmod 2$ we have

$$f(u_n) + f(u_n u_i) + f(u_i) = (3n-7) + \left(n + \frac{n-9}{2} + \frac{i+1}{2}\right) + \left(3n + \frac{n+1}{2} - 7 - \frac{i+1}{2}\right) = 2(4n-9) = k_2$$

and when $i \equiv 0 \pmod 2$

$$f(u_n) + f(u_n u_i) + f(u_i) = (3n-7) + \left(n + -3 + \frac{i}{2}\right) + \left(4(n-2) - \frac{i}{2}\right) = 2(4n-9) = k_2.$$

(iv) For the edges in $E_4$, we have

$$f(u_1) + f(u_1 u_n) + f(u_n) = (4(n-2)) + (n-3) + (3n-7) = 2(4n-9) = k_2,$$

$$f(u_1) + f(u_1 u_4) + f(u_4) = (4(n-2)) + \left(n + \frac{n-7}{2}\right) + (2(2n-5)) = 9n + \frac{n+1}{2} - 22 = k_1.$$

Therefore, switching of both the pendant vertices in a path $P_n$ admits an *a*-vertex consecutive edge bimagic labeling with $a = 2n-5$, if $n$ even and with $a = 3n-8$, if $n$ is odd. ■

**Example 1:** Simultaneous switching of both the pendant vertices in a path graph $P_8$ is given in Figure 1. It is *a*-vertex consecutive edge bimagic labeling with $a = 11$.

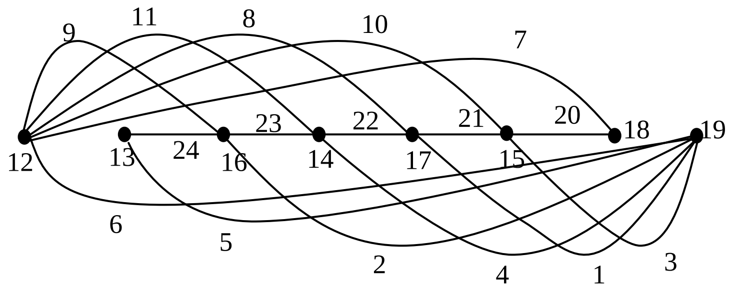


**Figure 1:** $k_1 = 53$, $k_2 = 37$.

**Theorem 2.3.** Switching of a pendant vertex having a support vertex of degree 2 in a comb graph $P_n^+$ ,$(n \geq 2)$ admits an $a$-vertex consecutive edge bimagic total labeling with $a = 2(n-1)$.

**Proof:** Let $G_v$ be the graph obtained by switching a pendant vertex $v$ of $G = P_n^+$. Let the vertices of $P_n^+,(n \geq 2)$ be $V = \{u_1, u_2, ..., u_n\} \cup \{v_1, v_2, ..., v_n\}$. Without loss of generality let us assume that vertex $v_1$ is switched to obtain $G_v$. We observe that $|V(G_v)| = 2n$ and $|E(G_v)| = 4(n-1)$ . In $G_v$, $E(G_v) = E_1 \cup E_2 \cup E_3 \cup E_4$ where $E_1 = \{u_i u_{i+1} : 1 \leq i \leq n-1\}$, $E_2 = \{v_1 u_i : 2 \leq i \leq n\}$, $E_3 = \{v_1 v_i : 2 \leq i \leq n\}$ and $E_4 = \{u_i v_i : 2 \leq i \leq n\}$.

Define a bijective function $f : V(G_v) \cup E(G_v) \to \{1, 2, ..., 2(3n-2)\}$ as follows:

For $i$ = *1 to n*; when $i \equiv 1 \pmod 2$, let $f(u_i) = 2n - 1 + i$ and
when $i \equiv 0 \pmod 2$, let $f(u_i) = 3n - 2 + i$.

For $i$ = *2 to n*; when $i \equiv 0 \pmod 2$, let $f(v_i) = 2n - 1 + i$ and
when $i \equiv 1 \pmod 2$, let $f(u_i) = 3n - 2 + i$.

For $i$ = *2 to n*; when $i \equiv 0 \pmod 2$, let $f(v_1 u_i) = 5n - 1 - i$ and
when $i \equiv 1 \pmod 2$, let $f(v_1 u_i) = 6n - 2 - i$.

For $i$ = *2 to n*; when $i \equiv 0 \pmod 2$, let $f(v_1 v_i) = 6n - 2 - i$ and
when $i \equiv 1 \pmod 2$, let $f(v_1 v_i) = 5n - 1 - i$.

For $i$ = *1 to n-1;* let $f(u_i u_{i+1}) = 2(n - i)$.

For $i$ = *2 to n*; let $f(u_i v_i) = 2(n - i) + 1$ and let $f(v_1) = 2n - 1$.

We claim that $k_1 = 7n - 2$ and $k_2 = 2(5n - 2)$.

(i) For the edges in $E_1$, when $i \equiv 1 \pmod 2$ we have

$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = (2n - 1 + i) + (2(n - i)) + (3n - 2 + i + 1) = 7n - 2 = k_1$

and when $i \equiv 0 \pmod 2$

$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = (3n - 2 + i) + (2(n - i)) + (2n - 1 + i + 1) = 7n - 2 = k_1$.

(ii) For the edges in $E_2$, when $i \equiv 1 \pmod 2$ we have

$f(v_1) + f(v_1 u_i) + f(u_i) = (2n - 1) + (6n - 2 - i) + (2n - 1 + i) = 2(5n - 2) = k_2$

and when $i \equiv 0 \pmod 2$

$f(v_1) + f(v_1 u_i) + f(u_i) = (2n - 1) + (5n - 1 - i) + (3n - 2 + i) = 2(5n - 2) = k_2$.

(iii) For the edges in $E_3$, when $i \equiv 1 \pmod 2$ we have

$f(v_1) + f(v_1 v_i) + f(v_i) = (2n - 1) + (5n - 1 - i) + (3n - 2 + i) = 2(5n - 2) = k_2$

and when $i \equiv 0 \pmod 2$

$f(v_1) + f(v_1 v_i) + f(v_i) = (2n - 1) + (6n - 2 - i) + (2n - 1 + i) = 2(5n - 2) = k_2$.

(iv) For the edges in $E_4$, when $i \equiv 1 \pmod 2$ we have

$f(u_i) + f(u_i v_i) + f(v_i) = (2n-1+i) + (2(n-i)+1) + (3n-2+i) = 7n-2 = k_1$

and when $i \equiv 0 \pmod 2$

$f(u_i) + f(u_i v_i) + f(v_i) = (3n-2+i) + (2(n-i)+1) + (2n-1+i) = 7n-2 = k_1$.

Therefore, switching a pendant vertex having a support vertex of degree 2 in a comb graph $P_n^+$, $(n \ge 2)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a = 2(n-1)$. ■

**Remark:** If we switch any one of the non-pendant vertices in G = $P_n^+ (n \ge 2)$, then the resultant graph becomes disconnected.

**Theorem 2.4.** Switching of a vertex of degree 2 in the graph $P_2 + nK_1, (n \ge 2)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a = n-1$.

**Proof:** Let $G_v$ be the graph obtained by switching a vertex *v* of degree 2 in $G = P_2 + nK_1$. Let the vertices of G be $V = \{u_1, u_2, ..., u_n\} \cup \{u, v\}$. Without loss of generality let us assume that vertex $u_n$ is switched to obtain $G_v$. We observe that $|V(G_v)| = n+2$ and $|E(G_v)| = 3n-2$. In $G_v$, $E(G_v) = E_1 \cup E_2 \cup E_3 \cup E_4$ where $E_1 = \{uu_i : 1 \le i \le n-1\}$, $E_2 = \{u_n u_i : 1 \le i \le n-1\}$, $E_3 = \{vu_i : 1 \le i \le n-1\}$ and $E_4 = \{uv\}$.

Define a bijective function $f : V(G_v) \cup E(G_v) \to \{1, 2, ..., 4n\}$ as follows:

For *i* = 1 to *n;* let $f(u_i) = n+i$. For *i = 1 to n-1;* let $f(uu_i) = 4n+1-i$.

For *i* = 1 to *n*-1*;* let $f(vu_i) = 3n+1-i$. For *i = 1 to n-1;* let $f(u_n u_i) = n-i$ and let $f(u) = n$, $f(v) = 2n$, $f(u_n) = 2n+1$, $f(vu) = 3n+1$.

We claim that $k_1 = 6n+1$ and $k_2 = 4n+1$.

(i) For the edges in $E_1$, we obtain

$f(u) + f(uu_i) + f(u_i) = (n) + (4n+1-i) + (n+i) = 6n+1 = k_1$.

(ii) For the edges in $E_2$, we obtain

$f(u_n) + f(u_n u_i) + f(u_i) = (2n+1) + (n-i) + (n+i) = 4n+1 = k_2$.

(iii) For the edges in $E_3$, we obtain

$f(v) + f(vu_i) + f(u_i) = (2n) + (3n+1-i) + (n+i) = 6n+1 = k_1$.

(iv) For the edges in $E_4$, we obtain

$f(u) + f(uv) + f(v) = (n) + (3n+1) + (2n) = 6n+1 = k_1$.

Therefore, switching of a vertex of degree 2 in $P_2 + nK_1$, $(n \ge 2)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a = n-1$. ■

**Remark:** If a vertex of degree more than 2 is switched in G = $P_2 + nK_1, (n \geq 2)$, then the resultant graph becomes disconnected.

**Theorem 2.5.** Switching of a pendant vertex in a bistar $B_{n,m}, (n, m \geq 2)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a = m + n$.

**Proof:** Let $G_v$ be the graph obtained by switching a pendant vertex v of $G = B_{n,m}$. Let the vertices of a bistar $G = B_{n,m}$ $(n, m \geq 2)$ be $V(G) = \{u_1, u_2, ..., u_n\} \cup \{v_1, v_2, ..., v_m\} \cup \{u, v\}$ where the vertex u is adjacent to $u_i, 1 \leq i \leq n$ and v is adjacent to $v_j, 1 \leq j \leq m.$ Without loss of generality let us assume that vertex $u_n$ is switched to obtained $G_v$. We observe that $|V(G_v)| = n + m + 2$ and $|E(G_v)| = 2(n + m)$. In $G_v$, $E(G_v) = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ where $E_1 = \{uu_i : 1 \leq i \leq n-1\}$, $E_2 = \{u_n u_i : 1 \leq i \leq n-1\}$, $E_3 = \{vv_j : 1 \leq j \leq m\}$, $E_4 = \{uv, vu_n\}$ and $E_5 = \{u_n v_j : 1 \leq j \leq m\}$.

Define a bijective function $f : V(G_v) \cup E(G_v) \rightarrow \{1, 2, ..., 3(n + m) + 2\}$ as follows:

For *i = 1 to n;* let $f(u_i) = n + 2m + 1 + i$. For *j = 1 to m;* let $f(v_j) = n + m + 1 + j$.

For *i = 1 to n-1;* let $f(uu_i) = n + m + 1 - i$. For *j = 1 to m;* let $f(vv_j) = m + 1 - j$.

For *i = 1 to n-1;* let $f(u_n u_i) = 3(n + 1) + 2m - i$. For *j = 1 to m;* let $f(u_n v_j) = 3(n + m + 1) - j$.

and let $f(v) = 2(n + m) + 1$, $f(uv) = m + 1$, $f(u_n) = 2(n + m + 1)$, $f(vu_n) = 2(n + m) + 3$.

We claim that $k_1 = 3(n + 1) + 4m$ and $k_2 = 6(n + m + 1)$

(i) For the edges in $E_1$, we obtain

$$f(u) + f(uu_i) + f(u_i) = (n + m + 1) + (n + m + 1 - i) + (n + 2m + 1 + i) = 3(n + 1) + 4m = k_1.$$

(ii) For the edges in $E_2$, we obtain

$$f(u_i) + f(u_n u_i) + f(u_n) = 2(n + m + 1) + (3(n + 1) + 2m - i) + (n + 1 + 2m + i) = 6(n + m + 1) = k_2.$$

(iii) For the edges in $E_3$, we obtain

$$f(v) + f(vv_j) + f(v_j) = (2(n + m) + 1) + (n + m + 1 + j) + (m + 1 - j) = 3(n + 1) + 4m = k_1.$$

(iv) For the edges in $E_4$, we obtain

$$f(u) + f(uv) + f(v) = (n + m + 1) + (m + 1) + (2(n + m) + 1) = 3(n + 1) + 4m = k_1 \text{ and}$$

$$f(v) + f(u_n v) + f(u_n) = (2(n + m) + 1) + (2(n + m) + 3) + (2(n + m + 1)) = 6(n + m + 1) = k_2.$$

(v) For edges in $E_5$, we obtain

$$f(u_n) + f(u_n v_j) + f(v_j) = (2(n + m + 1)) + (3(n + m + 1) - j) + (n + m + 1 + j) = 6(n + m + 1) = k_2.$$

Therefore, switching of a pendant vertex in a bistar $B_{n,m}, (n, m \geq 2)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a = m + n.$ ■

**Remark:** If we switch one of the non-pendant vertices in G = $B_{n,m}$ $(n, m \geq 2)$, then the resultant graph becomes disconnected.

**Theorem 2.6.** Switching of a vertex in a cycle $C_n$ ,$(n \geq 4)$ admits an $a$-vertex consecutive edge bimagic total labeling with $a = n-2$ if $n$ is even and $a = n-3$ if $n$ is odd.

**Proof:** Let $G_v$ be the graph obtained by switching a vertex v of $G = C_n$. Let the vertices of $C_n$ $(n \geq 4)$ be $V = \{u_1, u_2, ..., u_n\}$. Without loss of generality let us assume that vertex $u_n$ is switched to obtain $G_v$. We observe that $|V(G_v)| = n$ and $|E(G_v)| = 2n-5$.

In $G_v$, $E(G_v) = E_1 \cup E_2 \cup E_3$ where $E_1 = \{u_i u_{i+1} : 1 \leq i \leq n-2\}$, $E_2 = \{u_n u_i : 2 \leq i \leq n-2\}$ and $E_3 = \{u_{n-1} u_{n-2}\}$. A bijective function $f : V(G_v) \cup E(G_v) \to \{1, 2, ..., 3n-5\}$ is given below:

**Case (i):** $n$ is odd.

For $i$ = 1 to $n$-1; when $i \equiv 1 \pmod 2$, let $f(u_i) = 2n - 3 - \frac{i+1}{2}$ and

when $i \equiv 0 \pmod 2$, let $f(u_i) = n + \frac{n+1}{2} - 3 - \frac{i}{2}$.

*For* i = 2 to $n$-2; when $i \equiv 1 \pmod 2$, let $f(u_n u_i) = \frac{i+1}{2} - 1$ and

when $i \equiv 0 \pmod 2$, let $f(u_n u_i) = \frac{n-3}{2} + \frac{i}{2}$.

For i = 1 to $n$-2; let $f(u_i u_{i+1}) = 2n - 4 + i$ and let $f(u_n) = 3n - 5$.

We claim that $k_1 = 5n + \frac{n+1}{2} - 11$ and $k_2 = 5n - 9$.

(i) For the edges in $E_1$, when $i \equiv 1 \pmod 2$, we have

$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \left(2n - 3 - \frac{i+1}{2}\right) + (2n - 4 + i) + \left(n + \frac{n+1}{2} - 3 - \frac{i+1}{2}\right) = 5n + \frac{n+1}{2} - 11 = k_1$$

and when $i \equiv 0 \pmod 2$

$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \left(n + \frac{n+1}{2} - 3 - \frac{i}{2}\right) + (2n - 4 + i) + \left(2n - 3 - \frac{i+2}{2}\right) = 5n + \frac{n+1}{2} - 11 = k_1.$$

(ii) For the edges in $E_2$, when $i \equiv 1 \pmod 2$, we have

$$f(u_n) + f(u_n u_i) + f(u_i) = (3n-5) + \left(\frac{i+1}{2} - 1\right) + \left(2n - 3 - \frac{i+1}{2}\right) = 5n - 9 = k_2,$$

and when $i \equiv 0 \pmod 2$

$$f(u_n) + f(u_n u_i) + f(u_i) = (3n-5) + \left(\frac{n-3}{2} + \frac{i}{2}\right) + \left(n + \frac{n+1}{2} - 3 - \frac{i}{2}\right) = 5n - 9 = k_2.$$

**Case (ii):** $n$ is even.

For $i$ = 1 to $n$-2; when $i \equiv 1 \pmod 2$, let $f(u_i) = 2n - 3 - \frac{i+1}{2}$ and

when $i \equiv 0 \pmod 2$, let $f(u_i) = n + \frac{n}{2} - 2 - \frac{i}{2}$.

For $i$ = 2 to $n$-2; when $i \equiv 0 \pmod 2$, let $f(u_n u_i) = \frac{n}{2} - 2 + \frac{i}{2}$ and

when $i \equiv 1 \pmod 2$, let $f(u_n u_i) = \frac{i+1}{2} - 1$.

For $i$ = 1 to $n$-3*;* let $f(u_iu_{i+1})=2(n-1)+i$ and let $f(u_n)=2(n-1),\ f(u_{n-1})=2n-3,$
$f(u_{n-2})=n-1,\ f(u_{n-1}u_{n-2})=n-2.$

We claim that $k_1=5n+\frac{n}{2}-8$ and $k_2=2(2n-3)$.

(i) For the edges in $E_1$, when $i\equiv 1\,(\text{mod } 2)$ we have

$$f(u_i)+f(u_iu_{i+1})+f(u_{i+1})=\left(2n-3-\frac{i+1}{2}\right)+(2(n-1)+i)+\left(n+\frac{n}{2}-2-\frac{i+1}{2}\right)=5n+\frac{n}{2}-8=k_1$$

and when $i\equiv 0\,(\text{mod } 2)$

$$f(u_i)+f(u_iu_{i+1})+f(u_{i+1})=\left(n+\frac{n}{2}-2-\frac{i}{2}\right)+(2(n-1)+i)+\left(2n-3-\frac{i+2}{2}\right)=5n+\frac{n}{2}-8=k_1.$$

(ii) For the edges in $E_2$, when $i\equiv 1\,(\text{mod } 2)$ we have

$$f(u_n)+f(u_nu_i)+f(u_i)=(2(n-1))+\left(\frac{i+1}{2}-1\right)+\left(2n-3-\frac{i+1}{2}\right)=2(2n-3)=k_2$$

and when $i\equiv 0\,(\text{mod } 2)$

$$f(u_n)+f(u_nu_i)+f(u_i)=(2(n-1))+\left(\frac{n}{2}-2+\frac{i}{2}\right)+\left(n+\frac{n}{2}-2-\frac{i}{2}\right)=2(2n-3)=k_2.$$

(iii) For the edges in $E_3$, we have

$$f(u_{n-1})+f(u_{n-1}u_{n-2})+f(u_{n-2})=(2n-3)+(n-1)+(n-2)=2(2n-3)=k_2.$$

Therefore, switching of a vertex in a cycle $C_n$ ,$(n\geq 4)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a$ = $n$-2, if $n$ is even and $a$ = $n$-3, if $n$ is odd. ∎

**Theorem 2.6.** Switching of a pendant vertex in $C_n^+$ ,$(n\geq 3)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a=2(n-1)$, if $n$ is even and $a=2n-1$, if $n$ is odd.

**Proof:** Let $G_v$ be the graph obtained by switching a pendant vertex $v$ of $G=C_n^+$. Let the vertices of $C_n^+$ ,$(n\geq 3)$ be $V=\{u_1,u_2,...,u_n\}\cup\{v_1,v_2,...,v_n\}$. Without loss of generality let us assume that vertex $v_1$ is switched to obtain $G_v$. We observe that $|V(G_v)|=2n$ and $|E(G_v)|=4n-3$. In $G_v$, $E(G_v)=E_1\cup E_2\cup E_3\cup E_4$ where $E_1=\{u_iu_{i+1},u_1u_{n1},:1\leq i\leq n-1\}$, $E_2=\{u_iv_i:2\leq i\leq n\}$, $E_3=\{v_1u_i:2\leq i\leq n\}$ and $E_4=\{v_1v_i:2\leq i\leq n\}$.

Define a bijective function $f:V(G_v)\cup E(G_v)\to\{1,2,...,6n-3\}$ as follows:

**Case (i):** $n$ is odd.

For $i$ = *1 to n;* when $i\equiv 1\,(\text{mod } 2)$, let $f(u_i)=2n+i$ and
when $i\equiv 0\,(\text{mod } 2)$, let $f(u_i)=3n-1+i.$

For $i$ = *2 to n;* when $i\equiv 1\,(\text{mod } 2)$, let $f(v_i)=3n-1+i$ and
when $i\equiv 0\,(\text{mod } 2)$, let $f(v_i)=6n-1+i$

For $i$ = *1 to n-1;* let $f(u_iu_{i+1})=2n-2i$ and let $f(u_1u_n)=2n-1$ $f(v_1)=2n$

For $i$ = *2 to n;* when $i\equiv 1\,(\text{mod } 2)$, let $f(v_1v_i)=5n-i$ and when $i \equiv 0 \pmod 2$, let $f(v_1 v_i) = 6n - 1 - i$

For *i = 2 to n;* let $f(u_i v_i) = 2n + 1 - 2i$ .

For *i = 2 to n;* when $i \equiv 1 \pmod 2$, let $f(v_1 u_i) = 6n - 1 - i$ and

when $i \equiv 0 \pmod 2$, let $f(v_1 u_i) = 5n - i$.

We claim that $k_1 = 7n$ and $k_2 = 10n - 1$

(i) For the edges in $E_1$, when $i \equiv 1 \pmod 2$ we have,

$f(u_i) + f(u_{i+1}) + f(u_i u_{i+1}) = (2n + i) + (3n - 1 + i + 1) + (2n - 2i) = 7n = k_1$

and when $i \equiv 0 \pmod 2$ we have,

$f(u_i) + f(u_{i+1}) + f(u_i u_{i+1}) = (3n - 1 + i) + (2n + i + 1) + (2n - 2i) = 7n = k_1$

and $f(u_1) + f(u_n) + f(u_1 u_n) = (2n + 1) + (3n) + +(2n - 1) = 7n = k_1$.

(ii) For the edges in $E_2$, when $i \equiv 1 \pmod 2$ we obtain

$f(u_i) + f(v_i) + f(u_i v_i) = (2n + i) + (3n - 1 + i) + (2n + 1 - 2i) = 7n = k_1$

and when $i \equiv 0 \pmod 2$ we obtain,

$f(u_i) + f(v_i) + f(u_i v_i) = (3n - 1 + i) + (2n + i) + (2n + 1 - 2i) = 7n = k_1$.

(iii) For the edges in $E_3$, when $i \equiv 1 \pmod 2$ we obtain

$f(v_1) + f(u_i) + f(v_1 u_i) = (2n) + (2n + i) + (6n - 1 - i) = 10n - 1 = k_2$

and when $i \equiv 0 \pmod 2$ we obtain, $f(v_1) + f(u_i) + f(v_1 u_i) = (2n) + (3n - 1 + i) + (5n - i) = 10n - 1 = k_2$.

(iv) For the edges in $E_4$, when $i \equiv 1 \pmod 2$ we obtain

$f(v_1) + f(v_i) + f(v_1 v_i) = (2n) + (3n - 1 + i) + (5n - i) = 10n - 1 = k_2$

and when $i \equiv 0 \pmod 2$ we obtain, $f(v_1) + f(v_i) + f(v_1 v_i) = (2n) + (2n + i) + (6n - 1 - i) = 10n - 1 = k_2$.

**Case (ii):** *n* is even.

For *i* = 1 to *n;* when $i \equiv 1 \pmod 2$, let $f(u_i) = 2n - 1 + i$ and

when $i \equiv 0 \pmod 2$, let $f(u_i) = 3n - 2 + i$.

For i = 2 *to n;* when $i \equiv 1 \pmod 2$, let $f(v_i) = 3n - 2 + i$ and

when $i \equiv 0 \pmod 2$, let $f(v_i) = 2n - 1 + i$

For i = 1 to *n*-1*;* let $f(u_i u_{i+1}) = 2n - 2i$ and let $f(u_1 u_n) = 4n - 1$ $f(v_1) = 2n - 1$

For i = 2 *to n;* when $i \equiv 1 \pmod 2$, let $f(v_1 v_i) = 5n - i$ and

when $i \equiv 0 \pmod 2$, let $f(v_1 v_i) = 6n - 1 - i$

For *i = 2 to n;* let $f(u_i v_i) = 2n + 1 - 2i$ .

For *i = 2 to n;* when $i \equiv 1 \pmod 2$, let $f(v_1 u_i) = 6n - 1 - i$ and

when $i \equiv 0 \pmod 2$, let $f(v_1 u_i) = 5n - i$.

We show that $k_1 = 7n - 2$ and $k_2 = 10n - 3$.

(i) For the edges in $E_1$, when $i \equiv 1 \pmod 2$ we have

$f(u_i)+f(u_{i+1})+f(u_iu_{i+1})=(2n-1+i)+(3n-2+i+1)+(2n-2i)=7n-2=k_1$ and

when $i \equiv 0 \pmod 2$ we have

$f(u_i)+f(u_{i+1})+f(u_iu_{i+1})=(3n-2+i)+(2n-1+i+1)++(2n-2i)=7n-2=k_1$

and $f(u_1)+f(u_n)+f(u_1u_n)=(2n)+(4n-2)++(4n-1)=10n-3=k_2$.

(ii) For the edges in $E_2$, when $i \equiv 1 \pmod 2$ we obtain

$f(u_i)+f(v_i)+f(u_iv_i)=(2n-1+i)+(3n-2+i)+(2n+1-2i)=7n-2=k_1$ and

when $i \equiv 0 \pmod 2$ we obtain,

$f(u_i)+f(v_i)+f(u_iv_i)=(3n-2+i)+(2n-1+i)+(2n+1-2i)=7n-2=k_1$.

(iii) For the edges in $E_3$, when $i \equiv 1 \pmod 2$ we obtain

$f(v_1)+f(u_i)+f(v_1u_i)=(2n-1)+(2n-1+i)+(6n-1-i)=10n-3=k_2$

and when $i \equiv 0 \pmod 2$ we obtain

$f(v_1)+f(u_i)+f(v_1u_i)=(2n-1)+(3n-2+i)+(5n-i)=10n-3=k_2$.

(iv) For the edges in $E_4$, when $i \equiv 1 \pmod 2$ we obtain

$f(v_1)+f(v_i)+f(v_1v_i)=(2n-1)+(3n-2+i)+(5n-i)=10n-3=k_2$

and when $i \equiv 0 \pmod 2$ we obtain

$f(v_1)+f(v_i)+f(v_1v_i)=(2n-1)+(2n-1+i)+(6n-1-i)=10n-3=k_2$.

Therefore, switching of a pendant vertex in $C_n^+, (n \geq 3)$ admits an *a*-vertex consecutive edge bimagic total labeling with *a* = 2(*n*-1), if *n* is even and *a* = 2*n*-1, if *n* is odd. ■

**Remark:** If any one of the non-pendant vertices is switched in $C_n^+$ , $(n \geq 3)$, then the graph will be disconnected.

**Theorem 2.7.** Switching of a vertex in $C_n^2, (n \geq 6)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a = n-2$.

**Proof:** Let $G_v$ be the graph obtained by switching a vertex v of $G = C_n^2$. Let the vertices of $C_n^2$ $(n \geq 6)$ be $V(G) = \{u_1, u_2, ..., u_n\}$. Without loss of generality let us assume that vertex $u_1$ is switched to obtain $G_v$. We observe that $|V(G_v)| = n$ and $|E(G_v)| = 3(n-3)$ . In $G_v$, $E(G_v) = E_1 \cup E_2 \cup E_3 \cup E_4$ where $E_1 = \{u_iu_{i+1} : 3 \leq i \leq n-1\}$, $E_2 = \{u_iu_{i+2} : 3 \leq i \leq n-2\}$ and $E_3 = \{u_1u_i : 4 \leq i \leq n-2\}$ and $E_4 = \{u_2u_3, u_2u_4, u_2u_n\}$.

Define a bijective function $f : V(G_v) \cup E(G_v) \rightarrow \{1, 2, ..., 4n-9\}$ as given below:

For *i = 3 to n;* let $f(u_i) = n-2+i$. For *i = 3 to n-1;* let $f(u_iu_{i+1}) = 4n-3-2i$.

For *i = 3 to n-2;* let $f(u_iu_{i+2}) = 4(n-1)-2i$. For *i = 4 to n-2;* let $f(u_1u_i) = n-i$ and let $f(u_1) = n$, $f(u_2) = n-1$, $f(u_3) = n+1$, $f(u_2u_3) = n-2$, $f(u_n) = 2(n-1)$, $f(u_2u_n) = 1$, $f(u_2u_4) = n-3$.

We show that $k_1 = 6(n-1)$ and $k_2 = 3n-2$.

(i) For the edges in $E_1$, we have

$f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = (n-2+i) + (4n-3-2i) + (n-2+i+1) = 6(n-1) = k_1$.

(ii) For the edges in $E_2$, we have

$f(u_i) + f(u_iu_{i+2}) + f(u_{i+2}) = (n-2+i) + (4(n-1)-2i) + (n-2+i+2) = 6(n-1) = k_1$.

(iii) For the edges in $E_3$, we have

$f(u_1) + f(u_1u_i) + f(u_i) = (n-2+i) + (n-i) + (n) = 3n-2 = k_2$.

(iv) For the edges in $E_4$, we have

$f(u_2) + f(u_2u_3) + f(u_3) = (n-1) + (n+1) + (n-2) = 3n-2 = k_2$,

$f(u_2) + f(u_2u_4) + f(u_4) = (n-1) + (n-3) + (n+2) = 3n-2 = k_2$,

$f(u_2) + f(u_2u_n) + f(u_n) = (n-1) + (1) + 2(n-1) = 3n-2 = k_2$.

Therefore, switching of a vertex in $C_n^2$, $(n \geq 6)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a = n-2$. ∎

**Example 2:** Switching of a vertex in $C_9^2$ is given in Figure 2. It admits an *a*-vertex consecutive edge bimagic labeling with *a* = 7.

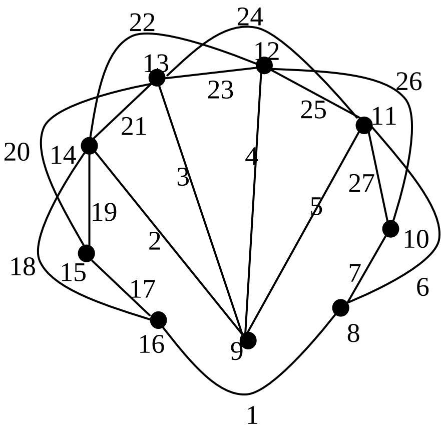


**Figure 2:** $k_1 = 48$, $k_2 = 25$.

**Remark:** If we switch one of the vertices in $C_n^2$, $(n = 3, 4$ *or* $5)$, then the graph will be disconnected.

**Theorem 2.8.** Switching of a vertex of degree 2 in $P_n^2$, $(n \geq 4)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a = 2n-5$.

**Proof:** Let $G_v$ be the graph obtained by switching a vertex v of $G = P_n^2$. Let the vertices of $P_n^2$, $(n \geq 4)$ be $V = \{u_1, u_2, ..., u_n\}$. Without loss of generality let us assume that vertex $u_1$ is switched to obtain $G_v$. We observe that $|V(G_v)| = n$ and $|E(G_v)| = 3n-8$. In $G_v$, $E(G_v) = E_1 \cup E_2 \cup E_3$ where $E_1 = \{u_iu_{i+1} : 2 \leq i \leq n-1\}$, $E_2 = \{u_iu_{i+2} : 2 \leq i \leq n-2\}$ and $E_3 = \{u_1u_i : 4 \leq i \leq n\}$.

Define a bijective function $f: V(G_v) \cup E(G_v) \to \{1, 2, ..., 4n-8\}$ as given below:

For *i = 2 to n;* let $f(u_i) = 2n-5+i.$ For *i = 2 to n-1;* let $f(u_i u_{i+1}) = 2(n-i)-1.$

For *i = 4 to n;* let $f(u_1 u_i) = 4(n-1)-i.$ For *i = 2 to n-2;* let $f(u_i u_{i+2}) = 2(n-i-1)$

and let $f(u_1) = 2n-4.$

We claim that $k_1 = 2(3n-5)$ and $k_2 = 8n-13$.

(i) For the edges in $E_1$, we have

$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = (2n-5+i) + (2(n-i)-1) + (2n-5+i+1) = 2(3n-5) = k_1.$$

(ii) For the edges in $E_2$, we have

$$f(u_i) + f(u_i u_{i+2}) + f(u_{i+2}) = (2n-5+i) + (2(n-i-1)) + (2n-5+i+2) = 2(3n-5) = k_1.$$

(iii) For the edges in $E_3$, we have

$$f(u_1) + f(u_1 u_i) + f(u_i) = (2n-4) + (4(n-1)-i) + (2n-5+i) = 8n-13 = k_2.$$

Therefore, switching of a vertex of degree 2 in $P_n^2$, $(n \geq 4)$ admits an *a*-vertex consecutive edge bimagic total labeling with $a = 2n-5$. ■

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