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Some graph products on $(r, 2, k)$-regular graphs

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Abstract

A graph G is said to be $(2, k)$-regular if $d_2(v) = k$, for all v in G. A graph G is said to be $(r, 2, k)$-regular if $d(v) = r$ and $d_2(v) = k$, for all v in G. In this paper, we study some graph products in $(2, k)$-regular graph and $(r, 2, k)$- regular graph.

Keywords: $(2, k)$-regular graph, $(r, 2, k)$-regular graph, cartesian product, composition of graphs, join of graphs, strong cartesian product, strong tensor product, strong augumented product in graphs.
AMS Subject Classification(2010): 05C12.

1 Introduction

In this paper, we consider only finite,simple,connected graphs. Notations and terminology that we do not define here can be found in Harary[8] and J.A.Bondy and U.S.R.Murty [4]. The degree of a vertex v is the number of edges incident at v. A graph is regular if all its vertices have the same degree. Consequently, we define the degree of a vertex v as the number of vertices at a distance one from v. For a positive integer d and a vertex v of a graph G, the d^{th} degree of v in G, denoted by $d_d(v)$, is defined as the number of vertices at a distance d away from v[3]. Hence $d_1(v) = d(v)$. The concept related to the F-degree of a vertex was introduced by Kocay [6], when reconstructing degree sequence on graphs. A graph G is said to be F-regular if the F-degrees of all the vertices of G are the same and it is called F-irregular if the F-degrees of the vertices of G are distinct. In a similar way, we defined d_2 of a vertex in graph[10].

For a given graph G, the d_2-degree of a vertex v in G, denoted by $d_2(v)$ means the number of vertices at a distance two away from v. A graph G is said to be d_2 - regular if d_2-degrees of all the vertices of G are the same. A graph G is called d_2-irregular if d_2-degrees of G are distinct, but there is no d_2 -irregular graph. We observe that $(2, k)$-regular[10] and semiregular [2] and d_2-regular graphs are same. A graph G is said to be $(2, k)$-regular[11] (d_2-regular) if $d_2(v) = k$, for all v in G. A graph G is said to be $(r, 2, k)$-regular [13] if $d(v) = r$ and $d_2(v) = k$, for all v in G. In this paper, we study some graph products in $(2, k)$-regular graph and $(r, 2, k)$-regular graph.

2 Preliminaries.

In this section, we recall some results about the d_2 of a vertex in following types of composite graphs[4] $G \times H, G[H], G + H, G \circ H, G\underline{\otimes}H, G \otimes' H$.

Definition 2.1. [4] The cartesian product of two simple graphs G and H is a simple graph $G \times H$ with vertex set $V(G) \times V(H)$, in which (u_1, v_1) is adjacent to (u_2, v_2) if and only if either (1). $u_1 = u_2$ and $v_1v_2 \in E(H)$ or (2). $v_1 = v_2$ and $u_1u_2 \in E(G)$. Also degree of (u_1, v_1) in $G \times H = d(u_1) + d(v_1)$.

Theorem 2.2. [12]Let G and H be connected graphs. Then $d_2(u_1, v_1)$ in $G \times H = d_2(u_1) + d_2(v_1) + d(u_1)d(v_1)$.

Definition 2.3. [4] The composition of two simple graphs G and H is a simple graph $G[H]$ with vertex set $V(G) \times V(H)$, in which (u_1, v_1) is adjacent to (u_2, v_2) if and only if either (1) $u_1u_2 \in E(G)$ or (2) $u_1 = u_2$ and $v_1v_2 \in E(H)$. Also degree of (u_1, v_1) in $G[H] = d(u_1)|V(H)| + d(v_1)$.

Theorem 2.4. [12] Let G and H be connected graphs. Then $d_2(u_1, v_1)$ in $G[H] = d_2(u_1)|V(H)| + |V(H)| - 1 - d(v_1)$.

Definition 2.5. [6] Join of two graphs G and H is a simple graph $G + H$ with vertex set $V(G) \cup V(H)$, in which each vertex of G is adjacent to every vertex of H. Degree of a vertex u in $G + H = d(u) + |V(H)|, u$ in $V(G)$. Degree of a vertex v in $G + H = d(v) + |V(G)|, v$ in $V(H)$.

Theorem 2.6. [12] Let G and H be any two graphs. Then $d_2(u)$ in $G + H$ = $|V(G)| - 1 - d(u)$ for all $u \in V(G)$ and $d_2(v)$ in $G + H = |V(H)| - 1 - d(v)$, for all $v \in V(H)$.

Definition 2.7. [7] Strong cartesian product of two simple graphs G and H is a simple graph $G \circ H$ with vertex set $V(G) \times V(H)$, in which (u_1, v_1) is adjacent to (u_2, v_2) if (1) $u_1 = u_2$ and $v_1v_2 \in E(H)$ or (2) $v_1 = v_2$ and $u_1u_2 \in E(G)$ or (3) $u_1u_2 \in E(G)$ and $v_1v_2 \in E(H)$ and also degree of (u_1, v_1) in $G \circ H = d(u_1) + d(v_1) + d(u_1)d(v_1)$.

Theorem 2.8. [12] Let G and H be two connected graphs. Then $d_2((u_1, v_1))$ in $G \circ H = d_2(u_1) + d_2(v_1) + d_2(u_1)d_2(v_1) + d_2(u_1)d(v_1) + d_2(v_1)d(u_1)$.

Definition 2.9. [7] Strong tensor product of two simple graphs G and H is a simple graph $G \otimes H$ with vertex set $V(G) \times V(H)$, in which (u_1, v_1) is adjacent to (u_2, v_2) if (1) $u_1 = u_2$ and $v_1v_2 \in E(H)$ or (2) $u_1u_2 \in E(G)$ and $v_1v_2 \in E(H)$. Also degree of (u_1, v_1) in $G \otimes H = d(u_1)d(v_1) + d(v_1)$.

Theorem 2.10. [12] Let G and H be connected graphs.Then $d_2((u_1, v_1))$ in $G \otimes H = d_2(u_1) + d_2(v_1) + d_2(u_1)d_2(v_1) + d_2(v_1)d(u_1) + d(u_1)$.

Definition 2.11. [7] Strong augumented product of simple graphs G and H is a simple graph $G \otimes' H$ with vertex set $V(G) \times V(H)$, in which (u_1, v_1) is adjacent to (u_2, v_2) if (1) $v_1 = v_2$ and $u_1u_2 \in E(G)$ or (2) $u_1u_2 \in E(G)$ and $v_1v_2 \in E(H)$. Also degree of (u_1, v_1) in $G \otimes' H = d(u_1)d(v_1) + d(u_1)$.

Theorem 2.12. [12] Let G and H be connected graphs.Then $d_2(u_1, v_1)$ in $G \otimes' H = d_2(u_1) + d_2(v_1) + d_2(u_1)d_2(v_1) + d_2(u_1)d(v_1) + d(v_1)$.

3 Graph products in $(2, k)$- regular graphs.

In this section, we discuss the graph products on $(2, k)$-regular graphs.

Definition 3.1. A graph G is $(2, k)$-regular if $d_2(v) = k$, for all $v \in V(G)$, where $d_2(v)$ is defined as the number of vertices at a distance 2 from v.

Result 3.2. If the Cartesian product of two graphs is $(2, k)$-regular then it is not necessary that they are $(2, k)$-regular.

Example: We know that, P_3 is not a $(2, k)$ - regular graph and K_2 is $(2, 0)$- regular. But the cartesian product $P_3 \times K_2$ is a $(2, k)$- regular graph.

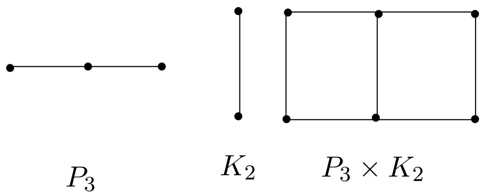

Figure 1: $P_3 \times K_2$ is a $(2, k)$- regular.

Result 3.3. Cartesian product of two $(2, k)$ regular graphs need not be $(2, k)$- regular.

Example:1

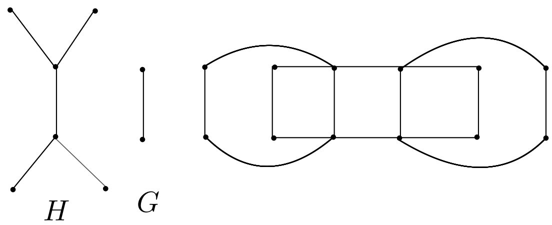

Figure 2: $H \times G$ is not $(2, k)$-regular.

Example:2

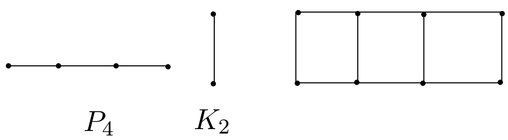

Figure 3:$P_4 \times K_2$ is not $(2, k)$-regular.

Result 3.4. Lexicographic product of two (2,k)- regular graphs need not be $(2, k)$- regular.

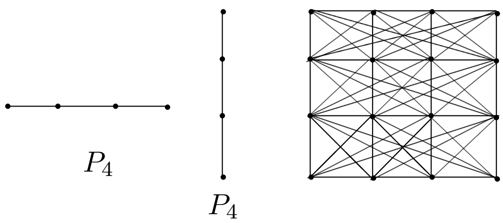

Figure 4: $P_4[P_4]$ is not $(2, k)$-regular.

Result 3.5. Join of two $(2, k)$-regular graphs need not be $(2, k)$- regular.

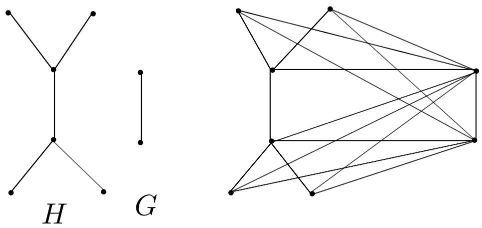

Figure 5: $H \vee G$ is not $(2, k)$-regular.

Result 3.6. Strong Cartesian product of two $(2, k)$- regular graphs need not be $(2, k)$-regular.

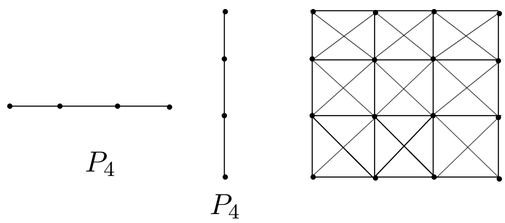

Figure 6: $P_4 \circ P_4$ is not $(2, k)$-regular.

Result 3.7. Strong tensor product of two $(2, k)$- regular graphs need not be $(2, k)$ regular.

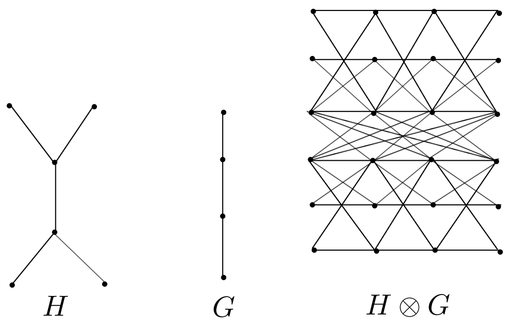

Figure 7: $H \otimes G$ is not $(2, k)$- regular.

Result 3.8. Augmented tensor product of two $(2, k)$- regular graphs need not be $(2, k)$- regular.

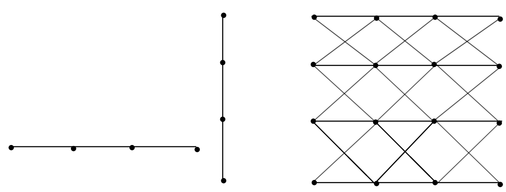

Figure 8: $G \otimes' H$ is not $(2, k)$- regular.

4 Graph products in $(r, 2, k)$- regular graph

Definition 4.1. A graph G is $(r, 2, k)$- regular if $d(v) = r, d_2(v) = k$, for all $v \in V(G)$ where $d_2(v)$ is defined as the number of vertices at a distance 2 from v.

Result 4.2. Consider the cartesian product $G \times H$ and let $(u, v) \in G \times H$, then $d(u, v) = d(u) + d(v)$ and $d_2(u, v) = d_2(u) + d_2(v) + d(u)d(v)$.

Theorem 4.3. Let G be a connected $(r_1, 2, k_1)$-regular graph and H be a connected $(r_2, 2, k_2)$-regular graph. Then the cartesian product $G \times H$ is $(r_1 + r_2, 2, k_1 + k_2 + r_1r_2)$-regular.

Proof: Let G be a connected $(r_1, 2, k_1)$-regular graph. That is, $d(u) = r_1$ and $d_2(u) = k_1$, for all $u \in V(G)$. Let H be a connected $(r_2, 2, k_2)$-regular graph. That is, $d(v) = r_2$ and $d_2(v) = k_2$, for all $v \in V(H)$. Let $(u, v) \in V(G \times H)$. Consider $d(u, v) = d(u) + d(v) = r_1 + r_2$, for all $(u, v) \in G \times H$ and $d_2(u, v) = d_2(u) + d_2(v) + d(u)d(v) = k_1 + k_2 + r_1r_2$, for all $(u, v) \in G \times H$. Then, $G \times H$ is $(r_1 + r_2, 2, k_1 + k_2 + r_1r_2)$-regular . ■

Example 4.4.

1. $K_n \times K_n$ is $(2n-2, 2, (n-1)^2)$- regular, since $d(u, v) = d(u) + d(v) = n-1+n-1 = 2n-2$ and $d_2(u, v) = d_2(u) + d_2(v) + d(u)d(v) = 0 + 0 + (n-1)(n-1). = (n-1)^2$.
2. For $n \geq 5$, $C_n \times C_n$ is $(4, 2, 8)$-regular, since $d(u, v) = d(u) + d(v) = 2 + 2 = 4$ and $d_2(u, v) = d_2(u) + d_2(v) + d(u)d(v) = 2 + 2 + 2 \times 2 = 8$.
3. Cartesian product of two $(r, 2, r(r-1))$-regular graphs is $(2r, 2, 3r^2 - 2r)$-regular, since $d(u, v) = d(u)+d(v) = r+r = 2r$ and $d_2(u, v) = d_2(u)+d_2(v)+d(u)d(v) = r(r-1)+r(r-1)+r\times r = r^2 - r + r^2 - r + r^2 = 3r^2 - 2r$.

Result 4.5. Consider the composition of graph $G[H]$ and let $(u, v) \in G[H]$, then $d(u, v) = d(u)|V(H)|+ d(v)$ and $d_2(u, v) = d_2(u)|V(H)| + |V(H)| - 1 - d(v)$.

Theorem 4.6. Let G be a connected $(r_1, 2, k_1)$-regular graph of order n_1 and H be a connected $(r_2, 2, k_2)$-regular graph of order n_2. Then the composition $G[H]$ is $(r_1n_2 + r_2, 2, (n_2(1 + k_2)) - (1 + r_2))$-regular.

Proof: Let G be a connected $(r_1, 2, k_1)$-regular graph of order n_1. That is, $d(u) = r_1$ and $d_2(u) = k_1$, for all $u \in V(G)$. Let H be a connected $(r_2, 2, k_2)$-regular graph of order n_2. That is, $d(v) = r_2$ and $d_2(v) = k_2$, for all $v \in V(H)$. Let $(u, v) \in V(G[H])$. Consider $d(u, v) = d(u)|V(H)| + d(v) = r_1n_2+r_2$, for all $(u, v) \in G[H]$ and $d_2(u, v) = d_2(u)|V(H)|+|V(H)|-1-d(v) = k_1n_2+n_2-1-r_2$, for all $(u, v) \in G[H]$. Therefore, $G[H]$ is $(r_1n_2 + r_2, 2, (n_2(1 + k_1)) - (1 + r_2))$- regular. ■

Example 4.7.

1. $K_n[K_n]$ is $(n^2-1, 2, 0)$- regular, since $d(u, v) = d(u)|V(H)|+d(v) = (n-1)n+(n-1) = n^2-1$ and $d_2(u, v) = d_2(u)|V(H)| + |V(H)| - 1 - d(v) = 0 + n - 1 - (n - 1) = 0$.
2. For $n \geq 5$, $C_n[C_n]$ is $(2n + 2, 2, 3n - 3)$-regular. $d(u, v) = d(u)|V(H)| + d(v) = 2n + 2$ and $d_2(u, v) = d_2(u)|V(H)| + |V(H)| - 1 - d(v) = 2n + n - 1 - 2 = 3n - 3$.

3. Composition of two $(r, 2, r(r-1))$- regular graphs of order $n2^{r-2}$ is $(rn2^{r-2} + r, 2, (r^2 - r + 1)n2^{r-2} - (1 + r)$-regular graph, since $d(u, v) = d(u)|V(H)| + d(v) = rn2^{r-2} + r$ and $d_2(u, v) = d_2(u)|V(H)| + |V(H)| - 1 - d(v) = (r^2 - r + 1)n2^{r-2} - (1 + r)$.

Result 4.8. Consider the join $G + H$ of two graphs G and H. Let $u \in G$ and $v \in H$. Then $d(u)$ in $G+H = |V(H)|+d(u)$ and $d(v)$ in $G+H = |V(G)|+d(v)$ and $d_2(u)$ in $G+H = |V(G)|-1-d(u)$ and $d_2(v)$ in $G + H = |V(H)| - 1 - d(v)$.

Result 4.9. The join of two $(r, 2, k)$-regular graphs need not be a $(r, 2, k)$-regular graph.

Illustration 4.10. Join of C_4 and K_2 is not a $(r, 2, k)$- regular graph.

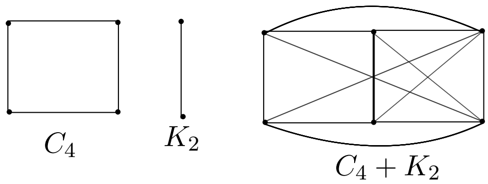

Figure 9: $C_4 + K_2$ is not $(r, 2, k)$- regular.

Theorem 4.11. Let G be a connected $(r_1, 2, k_1)$- regular graph of order n_1 and H be a connected $(r_2, 2, k_2)$- regular graph of order n_2. Then the join $G + H$ is a $(r, 2, k)$-regular graph only when $r_1 - r_2 = n_1 - n_2$.

Proof: Let G be a connected $(r_1, 2, k_1)$-regular graph of order n_1. That is, $d(u) = r_1$ and $d_2(u) = k_1$, for all $u \in V(G)$. Let H be a connected $(r_2, 2, k_2)$-regular graph of order n_2. That is, $d(v) = r_2$ and $d_2(v) = k_2$, for all $v \in V(H)$.

The join graph $G + H$ is regular only when $d(u)$ in $G + H = d(v)$ in $G + H$. That is, for all $u \in V(G)$ and for all $v \in V(H)$, $|V(H)| + d(u) = |V(G)| + d(v)$ implies that $n_2 + r_1 = n_1 + r_2$. Join $G + H$ is regular only when $n_1 - n_2 = r_1 - r_2$.

The join graph $G + H$ is d_2- regular ($(2, k)$-regular) only when $d_2(u)$ in $G + H = d_2(v)$ in $G + H$. That is, for all $u \in V(G)$ and for all $v \in V(H)$, $|V(G)| - 1 - d(u) = |V(H)| - 1 - d(v)$ implies that $n_1 - 1 - r_1 = n_2 - 1 - r_2$. Join $G + H$ is $(2, k)$- regular only when $n_1 - n_2 = r_1 - r_2$. Join $G + H$ is $(r, 2, k)$ -regular only when $r_1 - r_2 = n_1 - n_2$. ■

Example 4.12.

1. The join graph $K_n + K_n$ is $(2n - 1, 2, 0)$-regular, since $d(u) = 2n - 1$ and $d_2(u) = 0$, for all $u \in K_n$.

2. The join graph $C_n + C_n$ is $(n + 2, 2, n - 3)$-regular, since $d(u) = n + 2$, for all u in C_n and $d_2(u) = n - 3$, for all u in C_n.

3. Since $d(u) = n2^{r-2} + r$ and $d_2(u) = n2^{r-2} - r - 1$, for all u in $(r, 2, r(r-1))$-regular graph, we have the join of two $(r, 2, r(r-1))$-regular graphs is $(n2^{r-2} + r, 2, n2^{r-2} - r - 1)$-regular.

Result 4.13. Consider the strong cartesian product $G \circ H$ and let $(u, v) \in G \circ H$, then $d(u, v) = d(u) + d(v) + d(u)d(v)$ and $d_2(u, v) = d_2(u) + d_2(v) + d_2(u)d_2(v) + d_2(u)d(v) + d(u)d_2(v)$.

Theorem 4.14. Let G be a connected $(r_1, 2, k_1)$-regular graph and H be a connected $(r_2, 2, k_2)$-regular graph. Then the strong cartesian product $G \circ H$ is $(r_1 + r_2 + r_1r_2, 2, k_1 + k_2 + k_1k_2 + k_1r_2 + k_2r_1)$-regular.

Proof: Let G be a connected $(r_1, 2, k_1)$-regular graph. That is, $d(u) = r_1$ and $d_2(u) = k_1$, for all $u \in V(G)$. Let H be a connected $(r_2, 2, k_2)$-regular graph. That is, $d(v) = r_2$ and $d_2(v) = k_2$, for all $v \in V(H)$.

Let $(u, v) \in V(G \circ H)$. Consider $d(u, v) = d(u) + d(v) + d(u)d(v) = r_1 + r_2 + r_1r_2$, for all $(u, v) \in G \circ H. d_2(u, v) = d_2(u) + d_2(v) + d_2(u)d_2(v) + d_2(u)d(v) + d(u)d_2(v) = k_1 + k_2 + k_1k_2 + k_1r_2 + k_2r_1$, for all $(u, v) \in G \circ H$.Therefore, $G \circ H$ is $(r_1 + r_2 + r_1r_2, 2, k_1 + k_2 + k_1k_2 + k_1r_2 + k_2r_1)$-regular. ■

Example 4.15.

1. $K_n \circ K_n$ is $((n+1)(n-1), 2, 0)$-regular, since $d(u, v) = n - 1 + n - 1 + (n-1)^2 = n - 1(1 + 1 + n - 1) = (n-1)(n+1)$ and $d_2(u, v) = 0$.

2. For $n \geq 5$, $C_n \circ C_n$ is $(8, 2, 16)$-regular, since $d(u, v) = 2 + 2 + 4 = 8$ and $d_2(u, v) = 16$.

3. Since $d(u, v) = r(r + 2)$ and $d_2(u, v) = (r(r-1))(r(r+1) + 2)$, strong cartesian product of two $(r, 2, r(r-1))$-regular graphs is a $((2 + r)r, 2, r(r-1)(r(r+1) + 2))$-regular graph.

Result 4.16. Consider the strong tensor product $G \underline{\otimes} H$ of the graphs G and H and let $(u, v) \in G \underline{\otimes} H$. Then, $d(u, v) = d(u)d(v) + d(v)$ and $d_2(u, v) = d_2(u) + d_2(v) + d_2(u)d_2(v) + d_2(v)d(u) + d(u)$.

Theorem 4.17. Let G be a connected $(r_1, 2, k_1)$-regular graph and H be a connected $(r_2, 2, k_2)$-regular graph. Then the strong tensor product $G \underline{\otimes} H$ is $(r_1r_2 + r_2, 2, k_1 + k_2 + k_1k_2 + k_2r_1 + r_1)$-regular.

Proof: Let G be a connected $(r_1, 2, k_1)$ - regular graph. That is, $d(u) = r_1$ and $d_2(u) = k_1$, for all $u \in V(G)$. Let H be a connected $(r_2, 2, k_2)$ - regular graph. That is, $d(v) = r_2$ and $d_2(v) = k_2$, for all $v \in V(H)$. Let $(u, v) \in V(G \underline{\otimes} H)$. Consider $d(u, v) = d(u)d(v) + d(v) = r_1r_2 + r_2$,
$for\ all\ (u, v) \in G \underline{\otimes} H$. $d_2(u, v) = d_2(u) + d_2(v) + d_2(u)d_2(v) + d_2(v)d(u) + d(u) = k_1 + k_2 + k_1k_2 + k_2r_1 + r_1$, for all $(u, v) \in G \underline{\otimes} H$. Therefore, $G \underline{\otimes} H$ is $(r_1r_2 + r_2, 2, k_1 + k_2 + k_1k_2 + k_2r_1 + r_1)$-regular. ■

Example 4.18.

1. Since $d(u, v) = n - 1 + (n-1)^2 = n - 1(1 + n - 1) = n(n-1)$ and $d_2(u, v) = n - 1$, we have $K_n \underline{\otimes} K_n$ is $(n(n-1), 2, n-1)$ - regular.

2. Since $d(u, v) = 6$ and $d_2(u, v) = 14$, we have $C_n \underline{\otimes} C_n$ is $(6, 2, 14)$-regular for $n \geq 5$.

3. Since $d(u, v) = r(r+1)$ and $d_2(u, v) = r(r-1)(r(r-1)+2+r)+r$, the strong tensor product of two $(r, 2, r(r-1))$-regular graphs is $((1+r)r, 2, r(r-1)(r(r-1)+2+r)+r)$-regular.

Result 4.19. Consider strong augumented product $G \otimes' H$ and let $(u, v) \in G \otimes' H$. Then $d(u, v) = d(u)d(v) + d(u)$ and $d_2(u, v) = d_2(u) + d_2(v) + d_2(u)d_2(v) + d_2(u)d(v) + d(v)$.

Theorem 4.20. Let G be a connected $(r_1, 2, k_1)$-regular graph and H be a connected $(r_2, 2, k_2)$-regular graph. Then the strong augumented product $G \otimes' H$ is $(r_1r_2 + r_1, 2, k_1 + k_2 + k_1k_2 + k_1r_2 + r_2)$-regular.

Proof: Let G be a connected $(r_1, 2, k_1))$-regular graph. That is, $d(u) = r_1$ and $d_2(u) = k_1$, for all $u \in V(G)$. Let H be a connected $(r_2, 2, k_2)$-regular graph. Then, $d(v) = r_2$ and $d_2(v) = k_2$, for all $v \in V(H)$. Let $(u, v) \in V(G \otimes' H)$. Consider $d(u, v) = d(u)d(v) + d(u) = r_1r_2 + r_1$, for all $(u, v) \in G \otimes' H$.
$d_2(u, v) = d_2(u) + d_2(v) + d_2(u)d_2(v) + d_2(u)d(v) + d(v) = k_1 + k_2 + k_1k_2 + k_1r_2 + r_2$, for all $(u, v) \in G \otimes' H$. Therefore, $G \otimes' H$ is $(r_1r_2 + r_1, 2, k_1 + k_2 + k_1k_2 + k_1r_2 + r_2)$-regular. ■

Example 4.21.

1. Since $d(u, v) = n-1+(n-1)^2 = n-1(1+n-1) = n(n-1)$ and $d_2(u, v) = n-1$, $K_n \otimes' K_n$ is $(n(n-1), 2, n-1)$-regular.

2. For $n \geq 5$, $C_n \otimes' C_n$ is $(6, 2, 14)$-regular graph, as $d(u, v) = 6$ and $d_2(u, v) = 14$.

3. Since $d(u, v) = r(r+1)$ and $d_2(u, v) = r(r-1)(r(r-1)+2+r)+r$, the strong augumented product of two $(r, 2, r(r-1))$-regular graphs is $((1+r)r, 2, r(r-1)(r(r-1)+2+r)+r)$-regular.

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