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Total edge Fibonacci - like sequence irregular labeling

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Abstract

In this paper, we define total edge Fibonacci - like sequence irregular labeling $f : V(G) \bigcup E(G) \rightarrow \{1, 2, \ldots, K\}$ of a graph $G = (V, E)$ of vertices and edges of G in such a way that for any two different edges xy and $x'y'$ their weights $f(x) + f(xy) + f(y)$ and $f(x') + f(x'y') + f(y')$ are distinct Fibonacci-like sequence numbers. The total edge Fibonacci - like sequence irregularity strength, tefls(G) is defined as the minimum K for which G has a total edge Fibonacci - like sequence irregular labeling. A graph that admits a total edge Fibonacci - like sequence irregular labeling is called a total edge Fibonacci - like sequence irregular graph. In this paper, we prove P_n and C_n and Book (with 3 and 4 sides) are total edge Fibonacci - like sequence irregular graphs.

Keywords: Total vertex irregular labeling, edge irregular total K-labeling, total edge Fibonacci - like sequence irregular labeling.
AMS Subject Classification(2010): 05C78.

1 Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges and for terms not defined here, we refer to Harary [2].

A total vertex irregular labeling on a graph G with v vertices and e edges is an assignment of integer labels to both vertices and edges so that the weights calculated at vertices are distinct. The weight of a vertex v in G is defined as the sum of the label of v and the labels of all the edges incident with v. That is, $wt(v) = \lambda(v) + \sum_{uv \in E} \lambda(uv)$.

The total vertex irregularity strength of G, denoted by tvs(G), is the minimum value of the largest label over all such irregular assignments.

For a graph $G = (V, E)$, define a labeling $f : V(G) \bigcup E(G) \rightarrow \{1, 2, \ldots, K\}$ to be an edge irregular total K-labeling of the graph G if for every two different edges xy and $x'y'$ of G the edge weights $wt(xy) \neq wt(x'y')$. The total edge irregularity strength, tes(G), is defined as the minimum K for which G has an edge irregular total K-labeling.

The notion of total vertex irregular labeling and total edge irregular labeling were introduced by Bača[1]. Motivated by this paper, we defined the total edge Fibonacci-like sequence irregular labeling.

Definition 1.1. The Fibonacci - like sequence is defined by the linear recurrence relation

$$S_n = \begin{cases} 2 & \text{if } n = 0 \\ 2 & \text{if } n = 1 \\ S_{n-1} + S_{n-2} & \text{if } n \geq 2 \end{cases}$$

The Fibonacci - like sequence we generate is $2, 4, 6, 10, 16, 26, 42, 68, 110, 178, \ldots$.

2 Main Results

Definition 2.1. A total edge Fibonacci-like sequence irregular labeling $f : V(G) \bigcup E(G) \to \{1, 2, \ldots, K\}$ of a graph $G = (V, E)$ is a labeling of vertices and edges of G in such a way that for any different edges xy and $x'y'$ their weights $f(x) + f(xy) + f(y)$ and $f(x') + f(x'y') + f(y')$ are distinct Fibonacci-like sequence numbers where the Fibonacci-like sequence is $S_1 = 2$, $S_2 = 4$, $S_3 = 6$, $S_4 = 10$, $S_5 = 16$, $S_6 = 26$, $S_7 = 42$ etc.

The total edge Fibonacci-like sequence irregularity strength, tefls(G) is defined as the minimum K for which G has total edge Fibonacci-like sequence irregular labeling.

Note that if f is a total edge Fibonacci-like sequence irregular labeling of $G = (V, E)$ with $|V(G)| = p$ and $|E(G)| = q$ then $S_4(= 10) \leq wt(xy) \leq S_{q+3}$ which implies that tefls $\geq \lceil \frac{S_{q+3}}{3} \rceil$.

Theorem 2.2. The path P_n of n vertices admits a total edge Fibonacci-like sequence irregular labeling and tefls(P_n)=$\lceil \frac{S_{n+2}}{3} \rceil$ for any n.

Proof: Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(P_n) = \{e_1, e_2 \ldots, e_{n-1}\}$.
Define $f : V \bigcup E \to \{1, 2, \ldots, \lceil \frac{S_{n+2}}{3} \rceil\}$ by

$$\begin{aligned} f(v_i) &= S_i, \quad i = 1, 2, \ldots, n-2 \\ f(v_{n-1}) &= S_{n+2} - 2\lceil \frac{S_{n+2}}{3} \rceil \\ f(v_n) &= \lceil \frac{S_{n+2}}{3} \rceil \\ f(e_i) &= S_{i+1}, \quad i = 1, 2, \ldots, n-3 \\ f(e_{n-2}) &= S_{n+1} + 2\lceil \frac{S_{n+2}}{3} \rceil - S_{n-2} - S_{n+2} \\ f(e_{n-1}) &= \lceil \frac{S_{n+2}}{3} \rceil. \end{aligned}$$

By this labeling, $wt(e_i) = f(v_i) + f(e_i) + f(v_{i+1})$; $i = 1, 2, \ldots, n-3$

$$\begin{aligned} &= S_i + S_{i+1} + S_{i+1} = S_{i+2} + S_{i+1} = S_{i+3} \\ wt(e_{n-2}) &= f(v_{n-2}) + f(e_{n-2}) + f(v_{n-1}) \\ &= S_{n-2} + (S_{n+1} + 2\lceil \tfrac{S_{n+2}}{3} \rceil - S_{n-2} - S_{n+2}) + (S_{n+2} - 2\lceil \tfrac{S_{n+2}}{3} \rceil) \end{aligned}$$

$$= S_{n+1}$$
$$wt(e_{n-1}) = f(v_{n-1}) + f(e_{n-1}) + f(v_n)$$
$$= (S_{n+2} - 2\lceil \tfrac{S_{n+2}}{3} \rceil) + \lceil \tfrac{S_{n+2}}{3} \rceil + \lceil \tfrac{S_{n+2}}{3} \rceil = S_{n+2}$$

Thus, the weights of $e_1, e_2, \ldots, e_{n-3}, e_{n-2}, e_{n-1}$ are $S_4, S_5, \ldots, S_n, S_{n+1}, S_{n+2}$ respectively. Also, $tefls(P_n) = \lceil \frac{S_{n+2}}{3} \rceil$ for any n. ■

Example 2.3. A total edge Fibonacci-like sequence irregular labeling of P_{10} with tefls $(P_{10}) = \lceil \frac{S_{12}}{3} \rceil = 156$ is given in Figure 1.

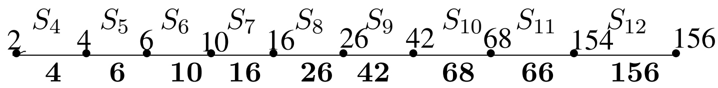

Figure 1: Total edge Fibonacci-like sequence irregular labeling of P_{10}.

Theorem 2.4. The cycle C_n of length n admits a total edge Fibonacci-like sequence irregular labeling and tefls(C_n)=$\lceil \frac{S_{n+3}}{3} \rceil$ for all n except for n=3, 4 and tefls(C_n)=$\lceil \frac{S_{n+3}}{3} \rceil + 2$ for $n = 3, 4$.

Proof: Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(C_n) = \{e_1, e_2, \ldots, e_n\}$.
Case(i): Let $n \geq 5$.
Define $f : V \bigcup E \to \{1, 2, \ldots, \lceil \frac{S_{n+3}}{3} \rceil\}$ by

$$\begin{aligned}
f(v_i) &= S_i, \quad i = 1, 2, \ldots, n-2 \\
f(v_{n-1}) &= S_{n+3} - 2\lceil \frac{S_{n+3}}{3} \rceil \\
f(v_n) &= \lceil \frac{S_{n+3}}{3} \rceil \\
f(e_i) &= S_{i+1}, \quad i = 1, 2, \ldots, n-3 \\
f(e_n) &= S_{n+1} - 2 - \lceil \frac{S_{n+3}}{3} \rceil \\
f(e_{n-2}) &= S_{n+2} + 2\lceil \frac{S_{n+3}}{3} \rceil - S_{n-2} - S_{n+3} \\
f(e_{n-1}) &= \lceil \frac{S_{n+3}}{3} \rceil.
\end{aligned}$$

By this labeling, $wt(e_i) = f(v_i) + f(e_i) + f(v_{i+1})$; $i = 1, 2, \ldots, n-3$

$$= S_i + S_{i+1} + S_{i+1} = S_{i+2} + S_{i+1} = S_{i+3}$$
$$wt(e_{n-2}) = f(v_{n-2}) + f(e_{n-2}) + f(v_{n-1})$$
$$= S_{n-2} + (S_{n+2} + 2\lceil \tfrac{S_{n+3}}{3} \rceil - S_{n-2} - S_{n+3}) + (S_{n+3} - 2\lceil \tfrac{S_{n+3}}{3} \rceil)$$
$$= S_{n+2}$$
$$wt(e_{n-1}) = f(v_{n-1}) + f(e_{n-1}) + f(v_n)$$
$$= (S_{n+3} - 2\lceil \tfrac{S_{n+3}}{3} \rceil) + \lceil \tfrac{S_{n+3}}{3} \rceil + \lceil \tfrac{S_{n+3}}{3} \rceil = S_{n+3}$$
$$wt(e_n) = f(v_n) + f(e_n) + f(v_1)$$

$$= \lceil \frac{S_{n+3}}{3} \rceil + (S_{n+1} - 2 - \lceil \frac{S_{n+3}}{3} \rceil) + 2 = S_{n+1}$$

Thus, the weights of edges $e_1, e_2, \ldots, e_{n-3}, e_{n-2}, e_{n-1}, e_n$ are $S_4, S_5, \ldots, S_{n+2}, S_{n+3}, S_{n+1}$ respectively and also tefls(C_n)=$\lceil \frac{S_{n+3}}{3} \rceil$ for any $n \geq 5$.

Case(ii): For n=3 and 4 the labelings are as follows.

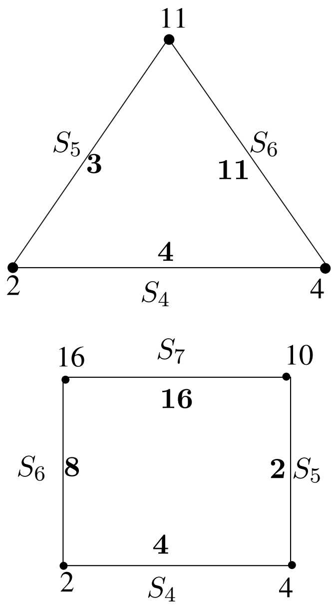

Figure 2: Total edge Fibonacci-like sequence irregular labeling of $C_3 and C_4$.

Thus, we proved that the cycle C_n has total edge Fibonacci - like sequence irregular labeling and found tefls(C_n) for all n. ∎

Example 2.5. A total edge Fibonacci-like sequence irregular labeling of C_8 with tefls $(C_8) = \lceil \frac{S_{11}}{3} \rceil$=96 is given in Figure 3.

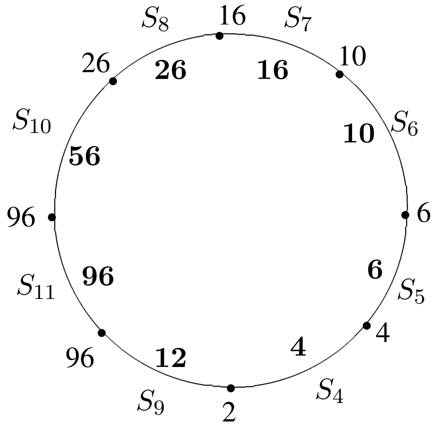

Figure 3: Total edge Fibonacci-like sequence irregular labeling of C_8.

Theorem 2.6. Triangular book admits a total edge Fibonacci - like sequence irregular labelings with tefls $\leq \lfloor \frac{S_{2n+4}}{2} \rfloor - 1$.

Proof: Let $V = \{u, v, u_1, u_2, \ldots, u_n\}$ be the vertex set and $E = \{e = uv, x_i = uu_i, y_i = vu_i; i = 1, 2, \ldots, n\}$be the edge set. Then $|V| = n + 2$ and $|E| = 2n + 1$.
Define $f : V \bigcup E \rightarrow \{1, 2, \ldots, \lfloor \frac{S_{2n+4}}{2} \rfloor - 1\}$ by $f(u) = 3$, $f(v) = 5$, $f(u_1) = 3$, $f(u_i) = \lfloor \frac{S_{2i+4}}{2} \rfloor - 1$; $i = 2, 3, \ldots, n$, $f(e) = 8$, $f(x_1) = 20$, $f(x_i) = S_{2i+4} - \lfloor \frac{S_{2i+4}}{2} \rfloor - 2$; $i = 2, 3, \ldots, n$ and $f(y_1) = 2$, $f(y_i) = S_{2i+3} - \lfloor \frac{S_{2i+4}}{2} \rfloor - 4$; $i = 2, 3, \ldots, n$.

By this labeling,
$$\begin{aligned} wt(e) &= f(u) + f(e) + f(v) \\ &= 3 + 8 + 5 = 16 = S_5 \\ wt(x_1) &= f(u) + f(x_1) + f(u_1) \\ &= 3 + 20 + 3 = 26 = S_6 \\ wt(y_1) &= f(v) + f(y_1) + f(u_1) = 5 + 2 + 3 = 10 = S_4 \\ wt(x_i) &= f(u) + f(x_i) + f(u_i) \text{ ; } i = 2, 3, \ldots, n \\ &= 3 + (S_{2i+4} - \lfloor \tfrac{S_{2i+4}}{2} \rfloor - 2) + (\lfloor \tfrac{S_{2i+4}}{2} \rfloor - 1) = S_{2i+4}. \end{aligned}$$

Thus, the weights of $x_2, x_3, \ldots, x_n$ are $S_8, S_{10}, \ldots, S_{2n+4}$.
$$\begin{aligned} wt(y_i) &= f(v) + f(y_i) + f(u_i) \text{ ; } i = 2, 3, \ldots, n \\ &= 5 + (S_{2i+3} - \lfloor \tfrac{S_{2i+4}}{2} \rfloor - 4) + (\lfloor \tfrac{S_{2i+4}}{2} \rfloor - 1) \\ &= S_{2i+3} \end{aligned}$$

The weights of $y_2, y_3, \ldots, y_n$ are $S_7, S_9, \ldots, S_{2n+3}$.

Hence we have, the weights of edges $e, x_1, y_1, x_2, y_2, \ldots, x_n, y_n$ are $S_4, S_5, S_6, S_7, \ldots, S_{2n+3}$, S_{2n+4}. Also, tefls $\leq \lfloor \frac{S_{2n+4}}{2} \rfloor - 1$. ∎

Example 2.7. A total edge Fibonacci - like sequence irregular labelingThe following of a triangular book is given in Figure 4. Further, tefls $\leq \lfloor \frac{S_{14}}{2} \rfloor - 1 = 609$.

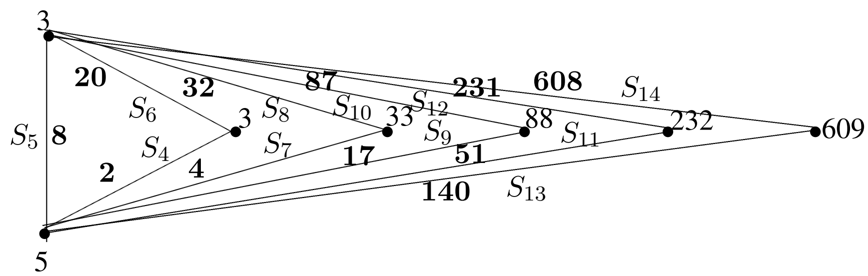

Figure 4: Total edge Fibonacci-like sequence irregular labeling of a triangular book.

Theorem 2.8. Quadrilateral book admits total edge Fibonacci - like sequence irregular labelings with tefls = $\lceil \frac{S_{3n+4}}{3} \rceil$.

Proof: Let $V = \{u, v, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ be the vertex set and $E = \{e = uv, e_i = u_iv_i, x_i = uu_i, y_i = vv_i\ ; i = 1, 2, \dots, n\}$ be the edge set. Then $|V| = 2n+2$ and $|E| = 3n+1$.
Define $f : V \bigcup E \to \{1, 2, \dots, \lceil \frac{S_{3n+4}}{3} \rceil\}$ by

$$\begin{aligned}
f(u) &= 2, \quad f(v) = 4, \quad f(u_1) = 2, \\
f(u_i) &= \lceil \frac{S_{3i+4}}{3} \rceil, \quad i = 2, 3, \dots, n \\
f(v_1) &= 6 \\
f(v_i) &= S_{3i+4} - 2\lceil \frac{S_{3i+4}}{3} \rceil, \quad i = 2, 3, \dots, n \\
f(e) &= 4, \quad f(e_1) = 16 \\
f(e_i) &= \lceil \frac{S_{3i+4}}{3} \rceil, \quad i = 2, 3, \dots, n \\
f(x_1) &= 4 \\
f(x_i) &= S_{3i+3} - 2 - \lceil \frac{S_{3i+4}}{3} \rceil, \quad i = 2, 3, \dots, n \\
f(y_1) &= 6 \\
f(y_i) &= S_{3i+2} + 2\lceil \frac{S_{3i+4}}{3} \rceil - S_{3i+4} - 4.
\end{aligned}$$

By this labeling, $wt(e) = f(u) + f(e) + f(v)$

$$\begin{aligned}
&= 2 + 4 + 4 = 10 = S_4 \\
wt(e_1) &= f(u_1) + f(e_1) + f(v_1) \\
&= 20 + 16 + 6 = 42 = S_7 \\
wt(x_1) &= f(u) + f(x_1) + f(u_1) \\
&= 2 + 4 + 20 = 26 = S_6 \\
wt(y_1) &= f(v) + f(y_1) + f(v_1) \\
&= 4 + 6 + 6 = 16 = S_5 \\
wt(e_i) &= f(u_i) + f(e_i) + f(v_i)\ ; i = 2, 3, \dots, n \\
&= \lceil \tfrac{S_{3i+4}}{3} \rceil + \lceil \tfrac{S_{3i+4}}{3} \rceil + (S_{3i+4} - 2\lceil \tfrac{S_{3i+4}}{3} \rceil) \\
&= S_{3i+4}
\end{aligned}$$

That is, the weights of $e_2, e_3, \dots, e_n$ are $S_{10}, S_{13}, \dots, S_{3n+4}$.

$$\begin{aligned}
wt(x_i) &= f(u) + f(x_i) + f(u_i)\ ; i = 2, 3, \dots, n \\
&= 2 + (S_{3i+3} - 2 - \lceil \tfrac{S_{3i+4}}{3} \rceil) + \lceil \tfrac{S_{3i+4}}{3} \rceil = S_{3i+3}.
\end{aligned}$$

That is, the weights of $x_2, x_3, \dots, x_n$ are $S_9, S_{12}, \dots, S_{3n+3}$

$$\begin{aligned}
wt(y_i) &= f(v) + f(y_i) + f(v_i)\ ; i = 2, 3, \dots, n \\
&= 4 + (S_{3i+2} + 2\lceil \tfrac{S_{3i+4}}{3} \rceil - S_{3i+4} - 4) + (S_{3i+4} - 2\lceil \tfrac{S_{3i+4}}{3} \rceil) \\
&= S_{3i+2}
\end{aligned}$$

That is, the weights of $y_2, y_3, \dots, y_n$ are $S_8, S_{11}, \dots, S_{3n+2}$.

Hence, the weights of edges $e, y_1, x_1, e_1, y_2, x_2, e_2, \dots, y_n, x_n, e_n$ are $S_4, S_5, S_6, \dots, S_{3n+2}, S_{3n+3}, S_{3n+4}$ respectively and tefls $= \lceil \frac{S_{3n+4}}{3} \rceil$. ■

Example 2.9. A total edge Fibonacci - like sequence irregular labeling of a quadrilateral book is given in Figure 5. Also, tefls = $\lceil \frac{S_{16}}{3} \rceil = 1065$.

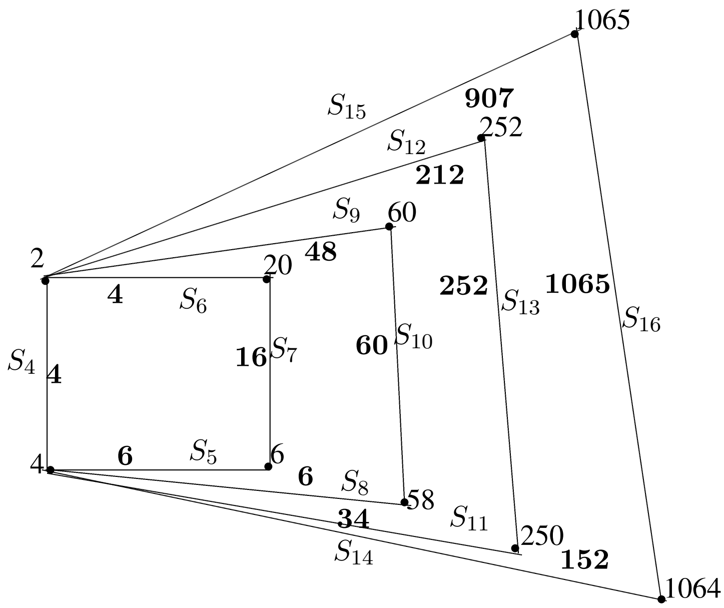

Figure 5: Total edge Fibonacci-like sequence irregular labeling of a quadrilateral book.

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