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Cordial Labeling for Some Bistar Related Graphs

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Abstract

In this paper we prove that square graph and shadow graph of bistar admit cordial labeling. Moreover we prove that splitting graph of bistar as well as degree splitting graph of bistar are cordial graphs.

Keywords: Cordial labeling, square graph, shadow graph, splitting graph, degree splitting graph.
AMS Subject Classification(2010): 05C78.

1 Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. For standard terminology and notations related to graph theory we refer to Gross and Yellen [7].

Various labeling schemes have been introduced so far and explored as well by many researchers. Graph labelings have enormous applications within mathematics as well as to several areas of computer science and communication networks. Some diversified applications of graph labeling have been studied by Yegnanaryanan and Vaidhyanathan [17]. A dynamic survey on different graph labeling problems with an extensive bibliography can be found in Gallian [5].

The concept of cordial labeling was introduced by Cahit [4] as a weaker version of graceful [10] and harmonious [6] labeling. In the same paper Cahit investigated some classes of cordial graphs and a necessary condition for an eulerian graph to be cordial graph. Ho *et al.* [9] have also proved many results on cordial labeling. This concept is explored by many researchers like Andar *et al.* [1, 2], Vaidya and Dani [12, 13]. Lawrence and Koilraj [8] studied cordial labeling for the splitting graph of some standard graphs. Babujee and Shobana [3] introduced the concepts of cordial languages and cordial numbers. Some labeling schemes are also introduced with minor variations in cordial theme like product cordial labeling, total product cordial labeling and prime cordial labeling.

Many results on star and bistar related graphs in the context of various graph labeling problems have been proved by Vaidya and Shah [14–16]. In the present work, we investigate some results on cordial labelings of some bistar related graphs.

We provide a brief summary of the definitions and other information which are useful for the present investigations.

Definition 1.1. The *graph labeling* is an assignment of numbers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a *vertex labeling* (*edge labeling*).

Definition 1.2. A mapping $f : V(G) \to \{0, 1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the label of the vertex v of G under f.

If for an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$ then we introduce following notations,

$$\left.\begin{array}{rl} v_f(i) = & \text{number of vertices of } G \text{ having label } i \text{ under } f \\ e_f(i) = & \text{number of edges of } G \text{ having label } i \text{ under } f^* \end{array}\right\} \text{where } i = 0 \text{ or } 1$$

Definition 1.3. A binary vertex labeling f of a graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is *cordial* if it admits cordial labeling.

Definition 1.4. The *bistar* $B_{n,n}$ is graph obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge.

Definition 1.5. For a simple connected graph G the *square of graph* G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

Definition 1.6. The *shadow graph* $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

Definition 1.7. For a graph G the *splitting graph* $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Definition 1.8. [11] Let $G = (V(G), E(G))$ be a graph with $V = S_1 \cup S_2 \cup S_3 \cup \ldots S_i \cup T$ where each S_i is a set of vertices having at least two vertices of the same degree and $T = V \setminus \cup S_i$. The *degree splitting graph* of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2, w_3, \ldots, w_t$ and joining to each vertex of S_i for $1 \leq i \leq t$.

2 Main Results

Theorem 2.1. $B^2_{n,n}$ is a cordial graph.

Proof: Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i : 1 \leq i \leq n\}$ where u_i, v_i are pendant vertices. Let G be the graph $B^2_{n,n}$ then $|V(G)| = 2n + 2$ and $|E(G)| = 4n + 1$.
We define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows.

$f(u) = 1,$
$f(v) = 0,$
$f(u_i) = 1; \quad 1 \leq i \leq n$
$f(v_i) = 0; \quad 1 \leq i \leq n$
In view of the above defined labeling pattern we have,

$v_f(0) = n + 1 = v_f(1)$, $e_f(0) = 2n$ and $e_f(1) = 2n + 1$.

Thus we proved that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Hence, $B^2_{n,n}$ is a cordial graph. ∎

Illustration 2.2. Cordial labeling of the graph $B^2_{7,7}$ is shown in Figure 1.

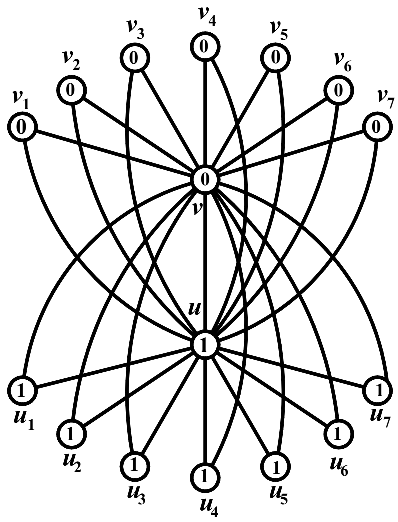

Figure 1: $B^2_{7,7}$ and its cordial labeling.

Theorem 2.3. $D_2(B_{n,n})$ is a cordial graph.

Proof: Consider two copies of $B_{n,n}$. Let $\{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $\{u', v', u'_i, v'_i : 1 \leq i \leq n\}$ be the corresponding vertex sets of each copy of $B_{n,n}$. Let G be the graph $D_2(B_{n,n})$ then $|V(G)| = 4(n + 1)$ and $|E(G)| = 4(2n + 1)$.
We define a vertex labeling $f : V(G) \rightarrow \{0, 1\}$ as follows.
$f(u) = 0,$
$f(u') = 1,$
$f(v) = 0,$
$f(v') = 1,$
$f(u_i) = 0; \quad 1 \leq i \leq n$
$f(v_i) = 0; \quad 1 \leq i \leq n$
$f(u'_i) = 1; \quad 1 \leq i \leq n$
$f(v'_i) = 1; \quad 1 \leq i \leq n$
In view of the above defined labeling pattern we have,

$v_f(0) = 2(n+1) = v_f(1)$ and $e_f(0) = 4n+2 = e_f(1)$.

Thus we proved that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Hence, $D_2(B_{n,n})$ is a cordial graph. ∎

Illustration 2.4. Cordial labeling of the graph $D_2(B_{5,5})$ is shown in Figure 2.

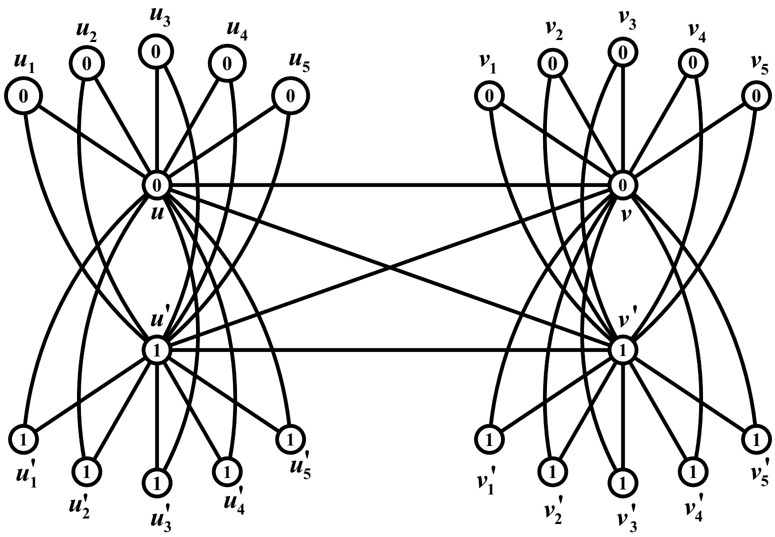

Figure 2 : $D_2(B_{5,5})$ and its cordial labeling.

Theorem 2.5. $S'(B_{n,n})$ is a cordial graph.

Proof: Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i : 1 \leq i \leq n\}$ where u_i, v_i are pendant vertices. In order to obtain $S'(B_{n,n})$, add u', v', u'_i, v'_i vertices corresponding to u, v, u_i, v_i where, $1 \leq i \leq n$. If $G = S'(B_{n,n})$ then $|V(G)| = 4(n+1)$ and $|E(G)| = 6n+3$.

We define a vertex labeling $f : V(G) \rightarrow \{0, 1\}$ as follows.

$f(u) = 0,$

$f(u') = 1,$

$f(v) = 0,$

$f(v') = 1,$

$f(u_i) = 0; \quad 1 \leq i \leq n$

$f\,(u'_i) = 0; \quad 1 \leq i \leq n$

$f(v_i) = 1; \quad 1 \leq i \leq n$

$f\,(v'_i) = 1; \quad 1 \leq i \leq n$

In view of the above labeling pattern we have,

$v_f(0) = 2n+2 = v_f(1)$ and $e_f(0) = 3n+1, e_f(1) = 3n+2.$

Thus we proved that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Hence, $S'(B_{n,n})$ is a cordial graph. ∎

Illustration 2.6. Cordial labeling of the graph $S'(B_{6,6})$ is shown in Figure 3.

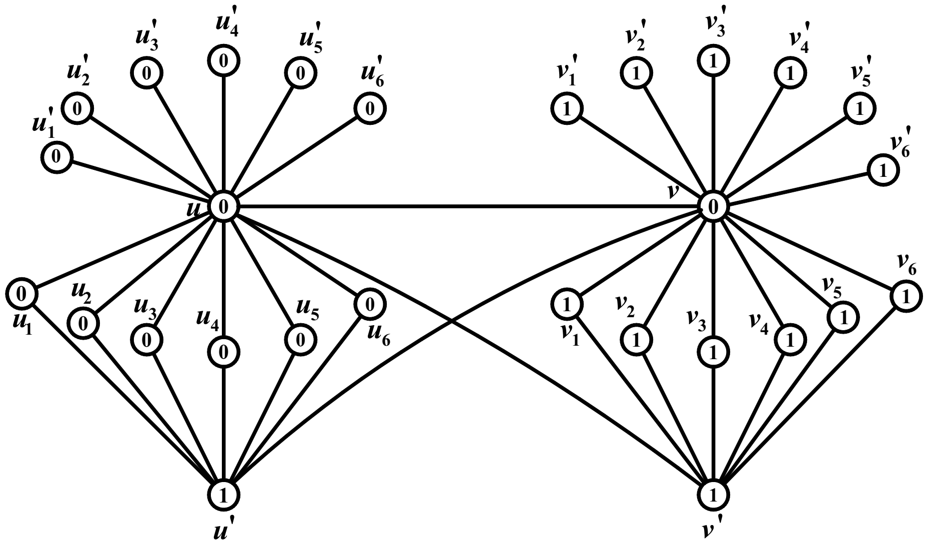

Figure 3: $S'(B_{6,6})$ and its cordial labeling.

Theorem 2.7. $DS(B_{n,n})$ is a cordial graph.

Proof: Consider $B_{n,n}$ with $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$, where u_i, v_i are pendant vertices. Here $V(B_{n,n}) = V_1 \cup V_2$, where $V_1 = \{u_i, v_i : 1 \leq i \leq n\}$ and $V_2 = \{u, v\}$.

Now in order to obtain $DS(B_{n,n})$ from G, we add w_1, w_2 corresponding to V_1, V_2. Then $|V(DS(B_{n,n})| = 2n+4$ and $E(DS(B_{n,n})) = \{uv, uw_2, vw_2\} \cup \{uu_i, vv_i, w_1u_i, w_1v_i : 1 \leq i \leq n\}$ so $|E(DS(B_{n,n})| = 4n+3$.

We define a vertex labeling $f : V(DS(B_{n,n})) \to \{0, 1\}$ as follows.

$f(u) = 0,$

$f(v) = 0,$

$f(w_1) = 1,$

$f(w_2) = 1,$

$f(u_i) = 0; \quad 1 \leq i \leq n$

$f(v_i) = 1; \quad 1 \leq i \leq n$

In view of the above defined labeling pattern we have,

$v_f(0) = n + 2 = v_f(1)$ and $e_f(0) = 2n + 1, e_f(1) = 2n + 2.$

Thus we proved that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Hence, $DS(B_{n,n})$ is a cordial graph. ∎

Illustration 2.8. Cordial labeling of the graph $DS(B_{5,5})$ is shown in Figure 4.

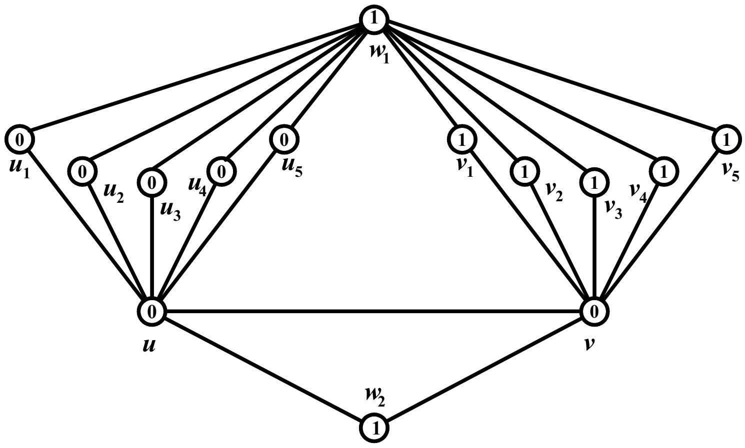

Figure 4: $DS(B_{5,5})$ and its cordial labeling.

3. Concluding remarks

Here we have contributed some new results on cordial labeling for the larger graphs obtained from bistar by means of various graph operations. Similar results can be obtained for different graph families and this is an open area of research.

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