International Journal of Mathematics and Soft Computing
Vol.4, No.2. (2014), 11 - 18.

ISSN Print : 2249 – 3328
ISSN Online: 2319 – 5215

On skolem difference mean labeling of some trees

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Abstract

A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,3,\ldots,p+q$ in such a way that for each edge $e = uv$, let $f^*(e) = \left\lceil \frac{|f(u) - f(v)|}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from 1, 2, 3,…, q. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph. In this paper, we prove that Tp-tree T, $T \odot \overline{K_n}$, $T @ P_n$ and $T @ 2P_n$ where T is a Tp-tree are extra skolem difference mean graphs.

Keywords: Skolem difference mean labeling, extra skolem difference mean labeling.

AMS Subject Classification (2010): 05C78.

1 Introduction

By a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The disjoint union of m copies of the graph G is denoted by mG. The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. A vertex of degree one is called a pendant vertex. The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.

Let T be a tree and u_0 and v_0 be two adjacent vertices in $V(T)$. Let there be two pendant vertices u and v in T such that the length of $u_o - u$ path is equal to the length of $v_o - v$ path. If the edge u_ov_o is deleted from T and u, v are joined by an edge uv, then such a transformation of T is called an elementary parallel transformation (or an ept) and the edge u_ov_o is called a transformable edge. If by a sequence of ept's T can be reduced to a path, then T is called a Tp-tree (transformed tree) and any such sequence regarded as a composition of mappings (ept's) denoted by P, is called a parallel transformation of T. The path, the image of T under P is denoted as $P(T)$. Let T be a Tp- tree with m

vertices. Let $T @ P_n$ be the graph obtained from *T* and *m* copies of P_n by identifying one pendant vertex of i^{th} copy of P_n with i^{th} vertex of *T*, where P_n is a path of length $n-1$. Let $T @ 2P_n$ be the graph obtained from *T* by identifying the pendant vertices of two vertex disjoint paths of equal lengths $n-1$ at each vertex of the Tp- tree *T*. Terms and notations not defined here are used in the sense of Harary [1].

A graph $G=(V,E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,3,\ldots, p+q$ in such a way that for each edge *e* = *uv*, let $f^*(e) = \frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting labels of the edges are distinct and are from 1, 2, 3,…, *q*. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph.

The concept of skolem difference mean labeling is introduced by K. Murugan and A. Subramanian [2] in 2011. They studied the skolem difference mean labeling of *H*– graphs. In [3], some standard results on skolem difference mean labeling were proved.

Definition 1.1. Let $G=(V,E)$ be a skolem difference mean graph with p vertices and q edges. Let one of the skolem difference mean labeling of G satisfies the condition that all the labels of the vertices are odd, and then we call this skolem difference mean labeling an extra skolem difference mean labeling and the graph G as extra skolem difference mean graph.

The extra skolem difference mean labeling of P_6 is given in Figure 1.

Figure 1: extra skolem difference mean labeling of P_6.

2 Extra Skolem Difference Mean Labeling

In this paper we prove that that Tp-tree T, $T \odot \overline{K_n}$, $T @ P_n$ and $T @ 2P_n$ where T is a *Tp*-tree are extra skolem difference mean graphs.

Theorem 2.1. Every T_p- tree *T* is an extra skolem difference mean graph.

Proof: Let *T* be a T_P - tree with *n* vertices. By the definition of a T_P - tree there exists a parallel transformation *P* of *T* such that for the path *P*(*T*) we have (i) $V(P(T)) = V(T)$ and (ii) $E(P(T)) = (E(T) \setminus E_d) \cup E_P$, where E_d is the set of edges deleted from *T* and E_P is the set of edges newly added through the sequence $P = (P_1, P_2, \ldots, P_k)$ of the epts *P* used to arrive at the path *P*(*T*). Clearly E_d and E_P have the same number of edges.

Now, denote the vertices of $P(T)$ successively as $v_1, v_2, v_3, ..., v_n$ starting from one pendant vertex of $P(T)$ right up to the other.

Define $f : V(T) \to \{1,2,3,..., p+q = 2n-1\}$ as follows:

$$f(v_i) = \begin{cases} i & \text{if } i \text{ is odd, } 1 \le i \le n \\ 2n+1-i & \text{if } i \text{ is even, } 1 \le i \le n \end{cases}$$

is the skolem difference mean labeling of the path $P(T)$.

Let $v_i v_j$ be an edge of T for some indices i and j, $1 \le i < j \le n$ and let P_1 be the ept that deletes this edge and adds the edge $v_{i+t}\, v_{j-t}$ where t is the distance from v_i to v_{i+t} and also the distance from v_j to v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts. Since $v_{i+t}v_{j-t}$ is an edge of the path $P(T)$, it follows that $i + t + 1 = j - t$ which implies $j = i + 2t + 1$. The induced label of the edge $v_i v_j$ is given by,

$$f^*(v_i v_j) = f^*(v_i v_{i+2t+1}) = \left\lceil \frac{|f(v_i) - f(v_{i+2t+1})|}{2} \right\rceil = |i + t - n| \qquad \text{...(1)}$$

and $$f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = \left\lceil \frac{|f(v_{i+t}) - f(v_{i+t+1})|}{2} \right\rceil = |i + t - n| \qquad \text{...(2)}$$

Therefore, from (1) and (2), $f^*(v_i v_j) = f^*(v_{i+t}v_{j-t})$. Hence, f is an extra skolem difference mean labeling of T_P -tree T. ■

The extra skolem difference mean labeling of a T_P-tree with 14 vertices is given in Figure 2.

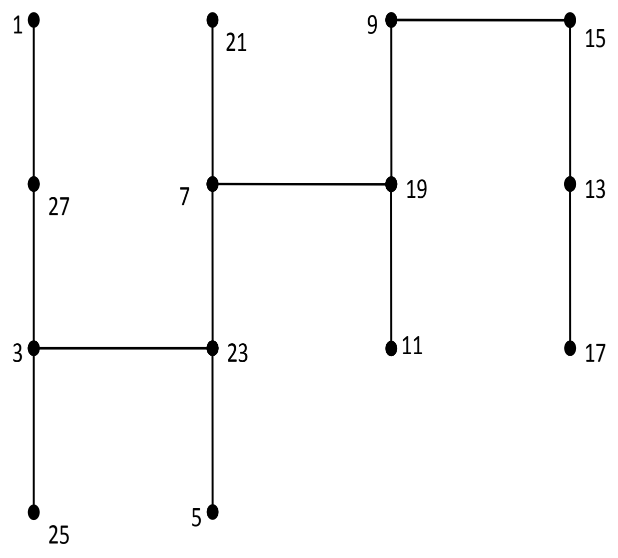

Figure 2: Extra skolem difference mean labeling of a T_P-tree with 14 vertices.

Theorem 2.2. Let T be a T_P – tree. Then the graph $T \odot \overline{K_n}$ is an extra skolem difference mean graph for all $n \geq 1$.

Proof: Let T be a T_P - tree with m vertices with the vertex set $V(T) = \{v_1, v_2, v_3, ..., v_m\}$. Let $u_1^j, u_2^j, ..., u_n^j$ be the pendant vertices joined with v_j $(1 \leq j \leq m)$ by an edge. Then, $V(T \odot \overline{K_n}) = \{v_j, u_i^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$. By the definition of a T_P - tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ and (ii) $E(P(T)) = (E(T) \setminus E_d) \cup E_P$, where E_d is the set of edges deleted from T and E_P is the set of edges newly added through the sequence $P = (P_1, P_2, ..., P_k)$ of the epts P used to arrive at the path $P(T)$. Clearly E_d and E_P have the same number of edges. Now denote the vertices of $P(T)$ successively as $v_1, v_2, v_3, ..., v_m$ starting from one pendant vertex of $P(T)$ right up to the other.

We define $f : V(T \odot \overline{K_n}) \rightarrow \{1,2,3,..., p+q = 2m(n+1)-1\}$ as follows:

$f(v_j) = (n+1)(2m+1-j)-1$ for j is odd, $1 \leq j \leq m$,

$f(v_j) = (n+1)j-1$ for j is even, $1 \leq j \leq m$,

$f(u_i^j) = (n+1)j-n-2+2i$ for j is odd, $1 \leq j \leq m$, $1 \leq i \leq n$,

$f(u_i^j) = (n+1)(2m+2-j)-1-2i$ for j is even, $1 \leq j \leq m$, $1 \leq i \leq n$.

Let v_iv_j be an edge of T for some indices i and j, $1 \leq i < j \leq n$ and let P_1 be the ept that deletes this edge and adds the edge $v_{i+t}\, v_{j-t}$ where t is the distance from v_i to v_{i+t} and also the distance from v_j to v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts. Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, it follows that $i + t + 1 = j - t$ which implies $j = i + 2t + 1$. The induced label of the edge v_iv_j is given by,

$$f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = \left\lceil \frac{|f(v_i) - f(v_{i+2t+1})|}{2} \right\rceil = |(n+1)(m-i-t)| \quad ...(3)$$

and $$f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = \left\lceil \frac{|f(v_{i+t}) - f(v_{i+t+1})|}{2} \right\rceil = |(n+1)(m-i-t)| \quad ...(4)$$

Therefore, from (3) and (4), $f^*(v_iv_j) = f^*(v_{i+t}v_{j-t})$.

Let $e_i^j = v_ju_i^j\ (1 \leq j \leq m,\ 1 \leq i \leq n),\ e_j = v_jv_{j+1}\ (1 \leq j \leq m-1)$ be the edges of $T \odot \overline{K_n}$.

For each vertex label f the induced edge label f^* is defined as follows:

$f^*(e_i^j) = (n+1)(m-j+1)-i$ for $1 \leq j \leq m$, $1 \leq i \leq n$,

$f^*(e_j) = (n+1)(m-j)$ for $1 \leq j \leq m-1$.

It can be verified that f is a mean labeling of $T \odot \overline{K_n}$. Hence, $T \odot \overline{K_n}$ is an extra skolem difference mean graph. ■

The example for an extra skolem difference mean labeling of $T \odot \overline{K_n}$, where *T* is a Tp-tree with 10 vertices, is given in Figure 3.

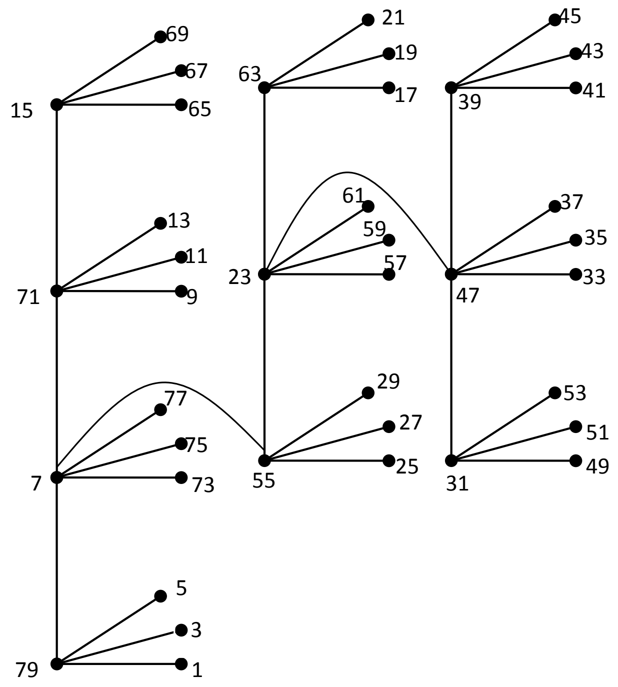

Figure 3: Extra skolem difference mean labeling of $T \odot \overline{K_n}$.

Theorem 2.3. Let T be a Tp- tree on m vertices. Then the graph $T @ P_n$ is an extra skolem difference mean graph.

Proof: Let *T* be a Tp - tree with *m* vertices. By the definition of a Tp - tree there exists a parallel transformation *P* of *T* such that for the path *P*(*T*) we have (i) $V(P(T)) = V(T)$ and (ii)$E(P(T)) = (E(T) \setminus E_d) \cup E_P$, where E_d is the set of edges deleted from *T* and E_P is the set of edges newly added through the sequence $P = (P_1, P_2, ..., P_k)$ of the epts *P* used to arrive at the path *P*(*T*). Clearly E_d and E_P have the same number of edges.

Now denote the vertices of *P*(*T*) successively as $v_1, v_2, v_3, ..., v_m$ starting from one pendant vertex of *P*(*T*) right up to other. Let $u_1^j, u_2^j, u_3^j, ..., u_n^j$ $(1 \le j \le m)$ be the vertices of j^{th} copy of P_n. Then $V(T @ P_n) = \{u_i^j : 1 \le i \le n, 1 \le j \le m$ with $u_n^j = v_j\}$.

Define $f : V(T @ P_n) \to \{1,2,3,..., p+q = 2mn-1\}$ as follows:

$f(u_i^j) = n(j-1) + i$ for i is odd, j is odd, $1 \le i \le n, 1 \le j \le m$,

$f(u_i^j) = (2m+1-j)n + 1 - i$ for i is even, j is odd, $1 \le i \le n, 1 \le j \le m$,

$f(u_i^j) = (2m-j)n + i$ for i is odd, j is even, $1 \le i \le n, 1 \le j \le m$,

$f(u_i^j) = nj + 1 - i$ for i is even, j is even, $1 \le i \le n, 1 \le j \le m$.

Let v_iv_j be a transformed edge in T for some indices i and j, $1 \le i < j \le m$ and let P_1 be the ept that deletes the edge v_iv_j and adds the edge $v_{i+t}\, v_{j-t}$ where t is the distance of v_i from v_{i+t} and also the distance of v_j from v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts. Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, $i + t + 1 = j - t$ which implies $j = i + 2t + 1$. The induced label of the edge v_iv_j is given by,

$$f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = \left\lceil \frac{|f(v_i) - f(v_{i+2t+1})|}{2} \right\rceil = |m - i - t|n \qquad \ldots(5)$$

$$f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = \left\lceil \frac{|f(v_{i+t}) - f(v_{i+t+1})|}{2} \right\rceil = |m - i - t|n \qquad \ldots(6)$$

Therefore from (5) and (6), $f^*(v_iv_j) = f^*(v_{i+t}\, v_{j-t})$.

Let $e_i^j = u_i^j u_{i+1}^j$ for $1 \le i \le n-1, 1 \le j \le m$ and $e_j = v_jv_{j+1}$ for $1 \le i \le m-1$.

For each vertex label f, the induced edge label f^* is defined as follows:

$f^*(e_i^j) = n(m - j + 1) - i$ for j is odd, $1 \le i \le n-1, 1 \le j \le m,$

$f^*(e_i^j) = n(m - j) + i$ for j is even, $1 \le i \le n-1, 1 \le j \le m,$

$f^*(e_j) = (m - j)n$ for $1 \le j \le m-1.$

It can be verified that f is a skolem difference mean labeling of $T @ P_n$ .Hence $T @ P_n$ is an extra skolem difference mean graph. ■

The example for an extra skolem difference mean labeling of $T @ P_4$, where T is a Tp-tree with 8 vertices, is given in Figure 4.

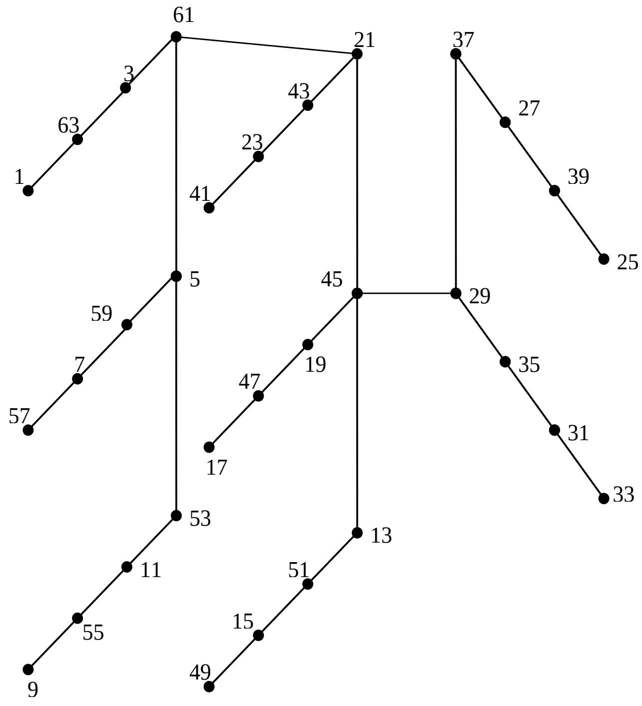

Figure 4: extra skolem difference mean labeling of $T @ P_4$.

Theorem 2.4: Let T be a Tp - tree on m vertices. Then the graph $T@2P_n$ is an extra skolem difference mean graph.

Proof: Let T be a Tp- tree with m vertices. By the definition of Tp – tree, there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ and (ii)$E(P(T)) = (E(T) \setminus E_d) \cup E_P$, where E_d is the set of edges deleted from T and E_P is the set of edges newly added through the sequence $P = (P_1, P_2, ..., P_k)$ of the epts P used to arrive at the path $P(T)$. Clearly E_d and E_P have the same number of edges.

Now denote the vertices of $P(T)$ successively as $v_1, v_2, v_3, ..., v_m$ starting from one pendant vertex of $P(T)$ right up to other. Let $u_{1,1}^j, u_{1,2}^j, u_{1,3}^j, ..., u_{1,n}^j$ and $u_{2,1}^j, u_{2,2}^j, u_{2,3}^j, ..., u_{2,n}^j$ $(1 \le j \le m)$ be the vertices of the two vertex disjoint paths joined with j^{th} vertex of T such that $v_j = u_{1,n}^j = u_{2,n}^j$.Then $V(T@2P_n) = \{v_j, u_{1,i}^j, u_{2,i}^j : 1 \le i \le n, 1 \le j \le m$ with $v_j = u_{1,n}^j = u_{2,n}^j\}$.

Define $f : V(T@2P_n) \to \{1,2,3,..., p+q = 2m(2n-1)-1\}$ as follows:

$f(u_{1,i}^j) = (j-1)(2n-1)+i$ for i is odd, j is odd, $1 \le i \le n,\ 1 \le j \le m$,

$f(u_{2,i}^j) = (j-1)(2n-1)+2n-i$ for i is odd, j is odd, $1 \le i \le n,\ 1 \le j \le m$,

$f(u_{1,i}^j) = (2m-j+1)(2n-1)-(i-1)$ for i is even, j is odd, $1 \le i \le n,\ 1 \le j \le m$,

$f(u_{2,i}^j) = (2m-j)(2n-1)+i$ for i is even, j is odd, $1 \le i \le n,\ 1 \le j \le m$,

$f(u_{1,i}^j) = (j-1)(2n-1)+i$ for i is even, j is even, $1 \le i \le n,\ 1 \le j \le m$,

$f(u_{2,i}^j) = j(2n-1)-(i-1)$ for i is even, j is even, $1 \le i \le n,\ 1 \le j \le m$,

$f(u_{1,i}^j) = (2m+1-j)(2n-1)-(i-1)$ for i is odd, j is even, $1 \le i \le n,\ 1 \le j \le m$,

$f(u_{2,i}^j) = (2m-j)(2n-1)+i$ for i is odd, j is even, $1 \le i \le n,\ 1 \le j \le m$.

Let v_iv_j be a transformed edge in T for some indices i and j, $1 \le i < j \le m$ and let P_1 be the ept that deletes the edge v_iv_j and adds the edge $v_{i+t}\, v_{j-t}$ where t is the distance of v_i from v_{i+t} and also the distance of v_j from v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts. Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, $i + t + 1 = j - t$ which implies $j = i + 2t + 1$. The induced label of the edge v_iv_j is given by,

$$f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = \left\lceil \frac{|f(v_i) - f(v_{i+2t+1})|}{2} \right\rceil = (2n-1)|m-i-t| \quad \text{…(7)}$$

and $$f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = \left\lceil \frac{|f(v_{i+t}) - f(v_{i+t+1})|}{2} \right\rceil = (2n-1)|m-i-t| \quad \text{…(8)}$$

Therefore, from (7) and (8), $f^*(v_i v_j) = f^*(v_{i+t} v_{j-t})$.

$e_{1,i}^{j} = u_{1,i}^{j} u_{1,i+1}^{j}$ for $1 \le i \le n-1,\ 1 \le j \le m$,

$e_{2,i}^{j} = u_{2,i}^{j} u_{2,i+1}^{j}$ for $1 \le i \le n-1,\ 1 \le j \le m$ and $e_j = v_j v_{j+1}$ for $1 \le j \le m-1$.

For each vertex label f, the induced edge label f^* is defined as follows:

$f^*(v_i v_{i+1}) = (2n-1)(m-i)$ for $1 \le i \le m-1$,

$f^*(e_{1,i}^{j}) = (m-j+1)(2n-1)-i$ for $1 \le i \le n-1,\ 1 \le j \le m$,

$f^*(e_{2,i}^{j}) = (m-j)(2n-1)+i$ for $1 \le i \le n-1,\ 1 \le j \le m$.

It can be verified that f is an extra skolem difference mean labeling of $T\,@2\,P_n$. Hence, $T\,@\,2\,P_n$ is an extra skolem difference mean graph. ∎

The example for an extra skolem difference mean labeling of $T\,@2\,P_3$, where T is a Tp-tree with 9 vertices is given in Figure 5.

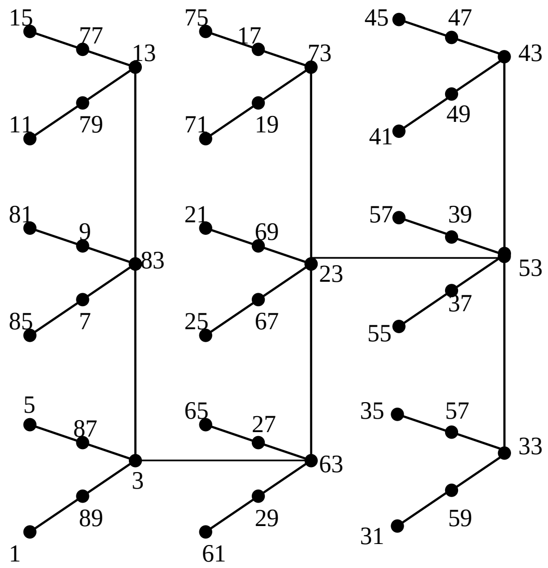

Figure 5: Extra skolem difference mean labeling of $T\,@2\,P_3$.

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