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Multi-level distance labelings for the prism related graphs D_n^p

Saima Nazeer
Lahore college for Women University,
Lahore, PAKISTAN.
E-mail: saimanazeer123@yahoo.com

Imrana Kousar
Lahore college for Women University,
Lahore, PAKISTAN.
E-mail: imrana.kousar@hotmail.com

Abstract

Let G be a connected graph with diameter diam(G) and $d(x, y)$ denotes the distance between any two distinct vertices x, y in G. A radio labeling f of G is an assignment of non negative integer to the vertices of G satisfying $|f(x) - f(y)| \geq$diam$(G) - d(x, y) + 1$. The span of a radio labeling is the maximum integer assigned to a vertex. The radio number of G denoted by rn(G), is the minimum possible span. In this paper, we determine the radio number for the prism related graphs, D_n^p when $n = 4k + 2$.

Keywords: Multi- level distance labeling, radio number, prism graph.
AMS Subject Classification(2010): 05C78; 05C15.

1 Introduction

Radio labeling (multi-level distance labeling) is regarded as an extension of distance two labeling which is motivated by the channel assignment problem introduced by Hale [2]. It is a non negative integer with the condition that level of interference should be minimized. It means, for the two cities or two stations which are geographically very close, there is stronger chance of interference between the receiving waves. This effect is avoided by allocating such channels in which separation is large enough.

To model this problem, we construct a graph such that each station is represented by a vertex and there is an edge between two vertices when the geographical locations of the corresponding stations are very close. Most of the research papers available in this area discuss either how to determine upper bound of the radio number or the exact value of the radio number for a simple connected graph G .

Let G be a connected graph. The distance $d(x, y)$ between any pair of distinct vertices in G is the length of the shortest path between them. The diameter of G, denoted by diam$(G) = d$, is the maximum distance between any two vertices in G.

A radio labeling is a one-to-one mapping $f : V(G) \rightarrow Z^+ \bigcup\{0\}$ satisfying the condition $|f(x) - f(y)| \geq$ diam$(G) + 1 - d(x, y)$ for any pair of vertices x, y in G. The largest number in $f(V)$ is known as span of f. The minimum span taken over all radio labelings of G is called the radio number of G and it is denoted by rn(G).

In this paper, we determine the radio number of the graphs D_n^p which is an extension of the prism graph defined in [1]. For each vertex b_i, $i = 1, 2, 3, ..., n$ of the outer cycle introducing a new vertex

a_i and join a_i to b_i, $i = 1, 2, 3, ..., n$. Thus $V(D_n^p) = \bigcup_{i=1}^{n}\{a_i, b_i, c_i\}$. Here $\{c_i\}$ are the inner cycle vertices and $\{b_i\}$ are the outer cycle vertices and $\{a_i\}$, $i = 1, 2, 3, ..., n$ are the vertices pendant to outer cycle.
The main theorem of this paper is:

Theorem 1.1. For the prism related graphs, D_n^p, $n = 4k + 2$, $k \geq 2$,

$$\text{rn}(D_n^p) = \begin{cases} 6k^2 + 22k + 8, & \text{when } k \text{ is odd;} \\ \\ 6k^2 + 22k + 9, & \text{when } k \text{ is even.} \end{cases}$$

Remark 1.2. Note that, the diameter of D_n^p is:

$$\text{diam}(D_n^p) = 2k + 3, \ \ when \ \ n = 4k + 2.$$

2 Lower bound for the radio number of prism related graph D_n^p, $n = 4k + 2$

In this work, we determine the lower bound for the radio number of the prism related graph D_n^p, $n = 4k + 2$.

Lemma 2.1. Let D_n^p be the prism related graph, $n = 4k + 2$. Then the vertices of D_n^p satisfies the following conditions.

(a) For each vertex $\{a_i : 1 \leq i \leq n\}$, which is a pendant vertex there is exactly one pendant vertex at a distance d diameter of D_n^p.

(b) For each vertex $\{a_i : 1 \leq i \leq n\}$, which is a pendant vertex there is exactly one vertex on the inner cycle $\{c_i : 1 \leq i \leq n\}$ at a distance d diameter of D_n^p.

(c) For each vertex $\{a_i : 1 \leq i \leq n\}$, which is a pendant vertex there is exactly one vertex on the outer cycle $\{b_i : 1 \leq i \leq n\}$ at a distance $d - 1$ of D_n^p.

(d) For each vertex $\{b_i : 1 \leq i \leq n\}$ on the outer cycle there is exactly one vertex on the inner cycle at a distance $d - 1$ of D_n^p.

(e) For each vertex $\{c_i : 1 \leq i \leq n\}$ on the inner cycle there is exactly one vertex on the inner cycle at a distance $d - 2$ of D_n^p.

(f) For each vertex $\{b_i : 1 \leq i \leq n\}$ on the outer cycle there is exactly one vertex on the outer cycle at a distance $d - 2$ of D_n^p.

Proof: Since $n = 4k + 2$, there are equal number of vertices on the left and right half of the pendant vertices. The path from a_1 to a_{2k+2} is of length $2k + 3$ as
$a_1 \to b_1 \to b_2 \to b_3 \to ... \to b_{2k+1} \to b_{2k+2} \to a_{2k+2}$. This proves (a).
By representing the path in each case $(b) - (f)$ the result follows:
$d(a_1, c_{2k+2}) = 2k + 3 = d$: $\quad a_1 \to b_1 \to c_1 \to c_2 \to ... \to c_{2k+2}$.
$d(a_1, b_{2k+2}) = 2k + 2 = d - 1$: $\quad a_1 \to b_1 \to b_2 \to b_3 \to ... \to b_{2k+2}$.

$d(b_1, c_{2k+2}) = 2k+2 = d-1$: $a_1 \to b_1 \to b_2 \to b_3 \to \ldots \to b_{2k+2}$.
$d(b_1, b_{2k+2}) = 2k+1 = d-2$: $b_1 \to b_2 \to b_3 \to \ldots \to b_{2k+2}$.
$d(c_1, c_{2k+2}) = 2k+1 = d-2$: $c_1 \to c_2 \to c_3 \to c_4 \to \ldots \to c_{2k+2}$. ■

The following lemma determines the maximum distance between any three vertices of D_n^p.

Lemma 2.2. Let u, v, w be three vertices of D_n^p.

(a) If all the three vertices are pendant, then $d(u,v) + d(v,w) + d(w,u) \leq 2d + 2$.

(b) If two of the vertices are pendant and one vertex lies on the inner cycle, then $d(u,v) + d(v,w) + d(w,u) \leq 2d + 2$.

(c) If two of the vertices are on the inner cycle and one vertex lies on the outer cycle, then $d(u,v) + d(v,w) + d(w,u) \leq 2d - 2$.

(d) If one vertex is a pendant vertex, second vertex lies on the outer cycle and third vertex lies on the inner cycle then $d(u,v) + d(v,w) + d(w,u) \leq 2d$.

(e) If two of the vertices are on the outer cycle and one vertex lies on the inner cycle, then $d(u,v) + d(v,w) + d(w,u) \leq 2d - 2$.

Proof: By Lemma 2.1(a), $d(a_1, a_{2k+2}) = 2k+3 = d$.
Now $d(a_{2k+2}, a_{4k+1}) = 2k+1 = d-2$ and a path of length $2k+1$ between a_{2k+2} and a_{4k+1} is
$a_{2k+2} \to b_{(2k+1)+1} \to b_{(2k+1)+2} \to \ldots \to b_{(2k+1)+2k} = b_{4k+1} \to a_{4k+1}$,
and $d(a_{4k+1}, a_1) = 4$ as $a_{4k+1} \to b_{4k+1} \to b_{4k+2} \to b_{4k+3} = b_0 \to a_0$. Therefore, if u, v, w are three pendant vertices of D_n^p then $d(u,v) + d(v,w) + d(w,u) \leq 2d + 2$. This completes the proof of (a).

By Lemma 2.1(a), $d(a_1, a_{2k+2}) = 2k+3 = d$.
For each vertex pendant there is only one vertex on the inner cycle at a distance d. That is, $d(a_{2k+2}, c_1) = d$, as $a_{2k+2} \to b_{2k+2} \to c_{2k+2} \to c_{(2k+2)+1} \to \ldots \to c_{(2k+2)+2k} \to c_{(2k+2)+2k+1} = c_1$ and $d(c_1, a_1) = 2$. Therefore, $d(a_1, a_{2k+2}) + d(a_{2k+2}, c_1) + d(c_1, a_1) = d + d + 2 = 2d + 2$. Thus, if u, v, w are three vertices with two pendant vertices and one vertex on the inner cycle of D_n^p, then $d(u,v) + d(v,w) + d(w,u) \leq 2d + 2$. This completes the proof of (b).

By Lemma 2.1(e), $d(c_1, c_{2k+2}) = 2k+1 = d-2$.
For each vertex b_1 on the outer cycle there is only one vertex c_{2k+2} on the inner cycle at a distance $d-1$. That is, $d(b_1, c_{2k+2}) = d-1$. Therefore, $d(c_1, c_{2k+2}) + d(c_{2k+2}, b_1) + d(b_1, c_1) = d - 2 + d - 1 + 1 = 2d - 2$. Thus, if u, v, w are three vertices with two vertices on the inner and one vertex on the outer cycles of D_n^p, then $d(u,v) + d(v,w) + d(w,u) \leq 2d - 2$. This completes the proof of (c).

By Lemma 2.1(c), $d(a_1, b_{2k+2}) = 2k+2 = d-1$. For each vertex on the outer cycle there is exactly one vertex on the inner cycle at a distance $d-1$, that is, $d(b_{2k+2}, c_1) = d-1$. Also, $d(c_1, a_1) = 2$. Therefore, $d(a_1, b_{2k+2}) + d(b_{2k+2}, c_1) + d(c_1, a_1) = d - 1 + d - 1 + 2 = 2d$. Thus, if u, v, w are three vertices with one pendant vertex, second vertex on the outer cycle and third vertex on the inner cycle of D_n^p, then $d(u,v) + d(v,w) + d(w,u) \leq 2d$. This completes the proof of (d).
The proof of (e) is similar as (c). ■

Lemma 2.3. Let f be a radio labeling of D_n^p, $n = 4k+2$ and $k \geq 2$.
(a) Suppose $\{x_i : 1 \leq i \leq n\}$ is the set of pendant vertices and $f(x_i) < f(x_j)$ whenever $i < j$. Then, $|f(x_{i+2}) - f(x_i)| \geq \phi(n)$ where $\phi(n) = k+2$.
(b) Suppose $\{y_i : 1 \leq i \leq n\}$ is the set of vertices on the outer and inner cycles and $f(y_i) < f(y_j)$ whenever $i < j$. Then, $|f(y_{i+2}) - f(y_i)| \geq \psi(n)$ where $\psi(n) = k+4$.

Proof: Let $\{x_i, x_{i+1}, x_{i+2}\}$ be any set of three pendant vertices of D_n^p. Applying the radio condition to each pair in the vertex set $\{x_i, x_{i+1}, x_{i+2}\}$ and take the sum of these inequalities.

$$\begin{aligned} |f(x_{i+1}) - f(x_i)| &\geq \text{diam}(G) - d(x_{i+1}, x_i) + 1 \\ |f(x_{i+2}) - f(x_{i+1})| &\geq \text{diam}(G) - d(x_{i+2}, x_{i+1}) + 1 \\ |f(x_{i+2}) - f(x_i)| &\geq \text{diam}(G) - d(x_{i+2}, x_i) + 1 \end{aligned}$$

$|f(x_{i+1}) - f(x_i)| + |f(x_{i+2}) - f(x_{i+1})| + |f(x_{i+2}) - f(x_i)| \geq 3 \cdot diam(G) + 3 - d(x_{i+1}, x_i) - d(x_{i+2}, x_{i+1}) - d(x_{i+2}, x_i)$.
We drop the absolute sign because $f(x_i) < f(x_{i+1}) < f(x_{i+2})$ and by using Lemma 2.2, we have $2[f(x_{i+2}) - f(x_i)] \geq 3 + 3d - (2d+2) = d+1$ $[f(x_{i+2}) - f(x_i)] \geq \frac{d+1}{2} = \frac{2k+4}{2} = k+2$.
Thus, $\phi(n) = k+2$. This completes the proof of (a).

Let $\{y_i, y_{i+1}, y_{i+2}\}$ be any set of three vertices on the outer and inner cycles of D_n^p. Applying radio condition to each pair in the above manner and by using Lemma 2.2(c) and (e), we get
$2[f(y_{i+2}) - f(y_i)] \geq 3 + 3d - 2d + 2 = d + 5$
$[f(x_{i+2}) - f(x_i)] \geq \frac{d+5}{2} = \frac{2k+3+5}{2} = k+4$. Thus, $\psi(n) = k+4$. ∎

Theorem 2.4. For the prism related graph, D_n^p, $n = 4k+2$ and $k \geq 2$,

$$\text{rn}(D_n^p) \geq \begin{cases} 6k^2 + 22k + 8, & \text{when } k \text{ is odd;} \\ 6k^2 + 22k + 9, & \text{when } k \text{ is even.} \end{cases}$$

Proof: A prism related graph D_n^p has $3n$ vertices. First we divide set of all vertices into three subsets $\{a_1, a_2, a_3, ..., a_n\}$, $\{b_1, b_2, b_3, ..., b_n\}$ and $\{c_1, c_2, c_3, ..., c_n\}$. Let f be a distance labeling for D_n^p. We order the vertices of D_n^p which are pendant to the outer cycle by $x_1, x_2, x_3, ..., x_n$ with $f(x_i) < f(x_{i+1})$ and the vertices on the outer and inner cycles by $y_1, y_2, y_3, ..., y_{2n}$ with $f(y_i) < f(y_{i+1})$, $\text{diam}(D_n^p) = d = 2k+3$.
For $i = 1, 2, 3, ...n-1$, set $d_i = d(x_i, x_{i+1})$ and $f_i = f(x_{i+1}) - f(x_i)$.
Then, $f_i \geq d - d_i + 1$ for all i.
By Lemma 2.3(a), the span of a distance labeling f of D_n^p for the pendant vertices is,

$$\begin{aligned} f(x_n) &= \sum_{i=1}^{n-1} f_i = f_1 + f_2 + f_3 + ... + f_{n-2} + f_{n-1} \\ &= [f(x_2) - f(x_1)] + [f(x_3) - f(x_2)] + ... + [f(x_{n-1}) - f(x_{n-2})] + [f(x_n) - f(x_{n-1})] \\ &= (f_1 + f_2) + (f_3 + f_4)) + (f_5 + f_6) + ... + (f_{n-3} + f_{n-2}) + f_{n-1} \\ &= \sum_{i=1}^{\frac{n-2}{2}} (f_{2i-1} + f_{2i}) + f_{n-1} \end{aligned}$$

$$\geq \frac{n-2}{2}\phi(n)+1.$$

Thus, $f(x_n) \geq 2k^2+4k+1$.

Applying Lemma 2.1 and Lemma 2.3(b) to the vertices x_{n-1}, x_n, y_1 such that $f(x_{n-1}) < f(x_n) < f(y_1)$, we obtain $|f(y_1) - f(x_{n-1})| \geq k+2$. That is, $f(y_1) \geq 2k^2+5k+2$.

Case 1: k is odd.

By Lemma 2.3(b), the span of a distance labeling f of D_n^p for the vertices on the outer and inner cycles is

$$\begin{aligned}
f(y_{2n}) - f(y_1) &= \sum_{i=1}^{2n-1} f_i' = (f_1'+f_2') + (f_3'+f_4') + ... + (f_{2n-3}'+f_{2n-2}') + f_{2n-1}' \\
&= \sum_{i=1}^{\frac{2n-2}{2}} (f_{2i-1}'+f_{2i}') + f_{2n-1}' \\
&\geq \frac{2n-2}{2}\psi(n)+2 \\
f(y_{2n}) - f(y_1) &\geq \frac{2n-2}{2}(k+4)+2 \\
f(y_{2n}) &\geq 4k^2+17k+6+f(y_1) \\
f(y_{2n}) &\geq 6k^2+22k+8.
\end{aligned}$$

Case 2: When k is even.

By Lemma 2.3(b), the span of a distance labeling f of D_n^p for the vertices on the outer and inner cycles is

$$\begin{aligned}
f(y_n) - f(y_1) &= \sum_{i=1}^{n-1} f_i' = (f_1'+f_2') + (f_3'+f_4') + ... + (f_{n-3}'+f_{n-2}') + f_{n-1}' \\
&= \sum_{i=1}^{\frac{n-2}{2}} (f_{2i-1}'+f_{2i}') + f_{2n-1}' \\
&\geq \frac{n-2}{2}\psi(n)+2 \\
f(y_n) - f(y_1) &\geq \frac{n-2}{2}(k+4)+2 \\
f(y_n) &\geq 2k^2+8k+2+f(y_1) \\
f(y_n) &\geq 4k^2+13k+4.
\end{aligned}$$

Now, $f(y_{n+1}) - f(y_{n-1}) \geq k+5$ and hence we have

$f(y_{n+1}) \geq k+5+4k^2+13k+2 = 4k^2+14k+7$.

$$\begin{aligned}
f(y_{2n}) - f(y_{n+1}) &= \sum_{i=n+1}^{2n-1} f_i' \\
&= (f_{n+1}'+f_{n+2}') + (f_{n+3}'+f_{n+4}') + ... + (f_{2n-3}'+f_{2n-2}') + f_{2n-1}'
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=n+1}^{\frac{2n-2}{2}} (f'_{2i-1} + f'_{2i}) + f'_{2n-1} \\
&\geq \frac{n-2}{2}\psi(n) + 2 \\
&= 2k^2 + 8k + 2 \\
&\geq 6k^2 + 22k + 9.
\end{aligned}
$$

■

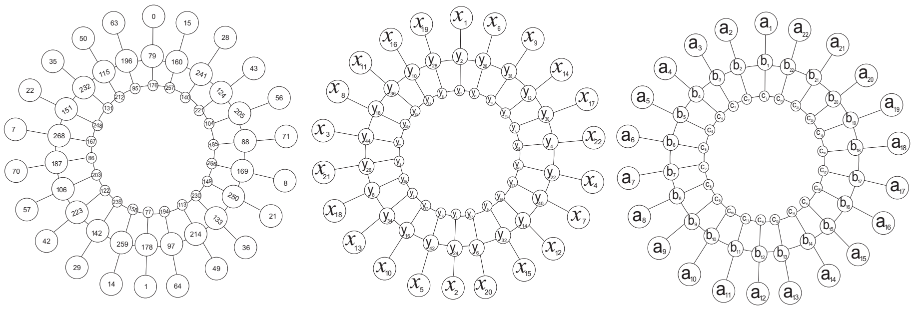

Figure 1: *Ordinary labeling and Radio labeling for* D_{22}^p.

3 An upper bound for the radio number of the prism related graph D_n^p, $n = 4k + 2$

To complete the proof of Theorem 1.1, it remains to find the distance labeling f for D_n^p with span equal to the desired numbers.

Case 1: When k is odd.

The labeling is generated by the pair of sequences namely,

the distance gap sequence

$$D = (d_1, d_2, d_3, ..., d_{n-1})$$

$$D' = (d'_1, d'_2, d'_3, ..., d'_{2n-1})$$

and the color gap sequence

$$F = (f_1, f_2, f_3, ..., f_{n-1})$$

$$F' = (f'_1, f'_2, f'_3, ..., f'_{2n-1}).$$

The distance gap sequence for the pendant vertices D is given by:

$$d_i = \begin{cases} 2k+3, & \text{if } i \text{ is odd;} \\ \\ k+3, & \text{if } i \text{ is even.} \end{cases}$$

The distance gap sequence for the outer and inner cycles vertices D' is given by:

$$d'_i = \begin{cases} 2k+2, & \text{if } i \text{ is odd;} \\ \\ k+2, & \text{if } i \text{ is even.} \end{cases}$$

For each i, we have $d(x_i, x_{i+1}) = d_i$, $d(y_i, y_{i+1}) = d'_i$ and

$$d' = d(x_n, y_1) = k+3.$$

The color gap sequence F for the pendant vertices is given by:

$$f_i = \begin{cases} 1, & \text{if } i \text{ is odd;} \\ \\ k+1, & \text{if } i \text{ is even.} \end{cases}$$

The color gap sequence F' for the outer and inner cycles vertices is given by:

$$f'_i = \begin{cases} 2, & \text{if } i \text{ is odd;} \\ \\ k+2, & \text{if } i \text{ is even.} \end{cases}$$

$$f' = f(y_1) - f(x_n) = k+1.$$

The position function $p : V(D_n^P) = \{a_i, b_i, c_i : 1 \leq i \leq n\} \to \{x_1, x_2, ..., x_n, y_1, y_2, ..., y_{2n}\}$ is defined as follows.
For $i = 1, 2, ..., 2k+1$, $p(a_{3i-2}) = x_{2i-1}$, $p(a_{3i+5}) = x_{2i}$.
where as, for $i = 1, 2, 3, ..., n$, $p(c_{11i-3}) = y_{2i-1}$, $p(b_{11i-10}) = y_{2i}$.

$$\begin{aligned} \text{span of} f &= f_1 + f_2 + f_3 +, ..., f_{n-2} + f_{n-1} + f' + f'_1 + f'_2 + f'_3 +, ..., f'_{2n-2} + f'_{2n-1} \\ &= [(f_1 + f_3 + f_5 +, ..., + f_{n-1})] + [(f_2 + f_4 + f_6 +, ..., + f_{n-2})] + f' \\ &\quad + [(f'_1 + f'_3 + f'_5 +, ..., + f'_{2n-1})] + [(f'_2 + f'_4 + f'_6 +, ..., + f'_{2n-2})] \\ &= \frac{n}{2}(1) + \frac{n-2}{2}(k+1) + k + 1 + \frac{2n}{2}(2) + \frac{2n-2}{2}(k+2) \\ &= 6k^2 + 22k + 8. \end{aligned}$$

Case 2: When k is even.
The labeling is generated by the two sequences namely,
the distance gap sequences

$$D = (d_1, d_2, d_3, ..., d_{n-1})$$

$$D' = (d'_1, d'_2, d'_3, ..., d'_{n-1}, d'_{n+1}, ..., d'_{2n-1})$$

and the color gap sequences

$$F = (f_1, f_2, f_3, ..., f_{n-1})$$

$$F' = (f'_1, f'_2, f'_3, ..., f'_{n-1}, f'_{n+1}, ..., f'_{2n-1}).$$

The distance gap sequences D for the pendant vertices is given by:

$$d_i = \begin{cases} 2k+3, & \text{if } i \text{ is odd;} \\ \\ k+3, & \text{if } i \text{ is even.} \end{cases}$$

The distance gap sequences D' for the outer and inner cycles vertices is given by:

$$d'_i = \begin{cases} 2k+2, & \text{if } i \text{ is odd;} \\ \\ k+2, & \text{if } i \text{ is even.} \end{cases}$$

For each i, we have $d(x_i, x_{i+1}) = d_i$, $d(y_i, y_{i+1}) = d'_i$ and $d' = d(x_n, y_1) = k+3$, $d'' = d(y_n, y_{n+1}) = k+3$.

The color gap sequences F and F' are given by

$$f_i = \begin{cases} 1, & \text{if } i \text{ is odd;} \\ \\ k+1, & \text{if } i \text{ is even.} \end{cases}$$

$$f'_i = \begin{cases} 2, & \text{if } i \text{ is odd;} \\ \\ k+2, & \text{if } i \text{ is even.} \end{cases}$$

and

$$f' = f(y_1) - f(x_n) = k+1,\ f'' = f(y_{n+1}) - f(y_n) = k+3.$$

The position function $p : V(D_n^P) = \{a_i, b_i, c_i : 1 \leq i \leq n\} \rightarrow \{x_1, x_2, ..., x_n, y_1, y_2, ..., y_{2n}\}$ is defined as follows.
For $i = 1, 2, ..., 2k+1$,

$$\begin{aligned} p(a_{4i-3}) &= x_{2i-1}, \\ p(a_{4i+6}) &= x_{2i}, \\ p(c_{14i-3}) &= y_{2i-1}, \\ p(b_{14i-12}) &= y_{2i}. \end{aligned}$$

However, for $i = 2k+1, 2k+2, ..., n$, $p(c_{14i-2}) = y_{2i-1}$, $p(b_{14i-11}) = y_{2i}$.

$$\begin{aligned} \text{span of} f &= f_1 + f_2 +, ..., f_{n-2} + f_{n-1} + f' + f'_1 + f'_2 +, ..., f'_{n-1} + f'' + f'_{n+1} +, ..., f'_{2n-1} \\ &= [(f_1 + f_3 + f_5 +, ..., + f_{n-1})] + [(f_2 + f_4 + f_6 +, ..., + f_{n-2})] + f' \\ &\quad + [(f'_1 + f'_3 + f'_5 +, ..., + f'_{n-1})] + f'' + [(f'_2 + f'_4 + f'_6 +, ..., + f'_{n-2})] \\ &\quad [(f'_{n+1} + f'_{n+3} + f'_5 +, ..., + f'_{2n-1})] + [(f'_{n+2} + f'_{n+4} + f'_{n+6} +, ..., + f'_{2n-2})] \\ &= \frac{n}{2}(1) + \frac{n-2}{2}(k+1) + k + 1 + \frac{n}{2}(2) + \frac{n-2}{2}(k+2) + k + 3 + \frac{n}{2}(2) + \frac{n-2}{2}(k+2) \\ &= 6k^2 + 22k + 9. \end{aligned}$$

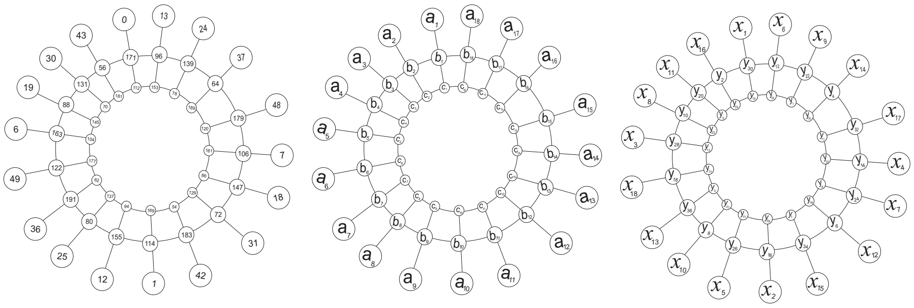

Figure 2: *Ordinary labeling and Radio labeling for* D_{18}^p.

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