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Star related subset cordial graphs

D. K. Nathan
Department of Mathematics,
Sri S.R.N.M.College, Sattur - 626 203,
Tamil Nadu, INDIA.
E-mail: dknathansrnmcstr@gmail.com

K. Nagarajan
Department of Mathematics,
Sri S.R.N.M.College, Sattur - 626 203,
Tamil Nadu, INDIA.
E-mail: k_nagarajan_srnmc@yahoo.co.in

Abstract

A subset cordial labeling of a graph G with vertex set V is an injection f from V to the power set of$\{1, 2, \dots, n\}$ such that an edge uv is assigned the label 1 if either $f(u)$ is a subset of $f(v)$ or $f(v)$ is a subset of $f(u)$ and 0 otherwise. Then the number of edges labeled 0 and 1 differ by at most 1. If a graph has a subset cordial labeling then it is called a subset cordial graph. In this paper, we prove that some star related graphs such as splitting graph of star, identification of star with a graph are subset cordial.

Keywords: Cordial labeling, subset cordial labeling, subset cordial graphs, splitting graph.
AMS Subject Classification(2010): 05C78.

1 Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges and for terms not defined here, we refer to Harary [6]. We recall some well known definitions and results which are useful for the present studies. [8].

Let $X = \{1, 2, \dots, n\}$ be a set and $\wp(X)$ be the collection of all subsets of X, called the power set of X. If A is a subset of B, we denote it by $A \subset B$, otherwise by $A \not\subset B$. Note that $\wp(X)$ contains 2^n subsets. Let $e_f(0)$ and $e_f(1)$ denote the number of edges labeled wih 0 and 1 respectively.

Graph labeling [5] is a strong communication between Algebra [8] and structure of graphs [6]. By combining the set theory concept in algebra and cordial labeling concept in graph labeling, we introduced a new concept called subset cordial labeling [9]. In this paper, we prove some star related graphs such as splitting graph of star, identification of star with a graph and the like are subset cordial[11].

A vertex labeling [5] of a graph G is an assignment f of labels to the vertices of G that induces each edge uv a label depending on the vertex label $f(u)$ and $f(v)$. Graceful and harmonious labeling are two well known labelings. Cordial labeling is a variation of both graceful and harmonious labeling [3].

Definition 1.1. Let $G = (V, E)$ be a graph. A mapping $f : V(G) \rightarrow \{0, 1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f. Let $e_f(0)$ and $e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*.

Definition 1.2. A binary vertex labeling of a graph G is called a cordial labeling, if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial, if it admits cordial labeling.

2 Main Results

Sundaram, Ponraj and Somasundaram [10] have introduced the notion of prime cordial labeling and proved that some graphs are prime cordial. R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan [12] introduced divisor cordial labeling and they have proved that some graphs are divisor cordial[13].

Definition 2.1. [10] A prime cordial labeling of a graph G with vertex set V is a bijection $f : V \longrightarrow \{1, 2, \ldots, |V|\}$ such that if each edge uv is assigned the label 1 if gcd$(f(u), f(v)) = 1$ and 0 if gcd$(f(u), f(v)) > 1$; Then the number of edges labeled 1 and the number of edges labeled 0 differ by at most 1.

Definition 2.2. Let G=(V, E) be a simple graph and $f : V \rightarrow \{1, 2, \ldots |V|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u) \mid f(v)$ or $f(v) \mid f(u)$ and the label 0 if $f(u) \nmid f(v)$. f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$.

A graph with a divisor cordial labeling is called a divisor cordial graph.

Using set theory concepts, some authors have established set sequential graphs[2] and set graceful labeling[7]. Motivated by the concepts of prime cordial labeling and divisor cordial labeling, we introduced a new cordial labeling called **subset cordial labeling** [9].

Definition 2.3. [9] Let $X = \{1, 2, \ldots, n\}$ be a set. Let $G = (V, E)$ be a simple (p, q)-graph and $f : V \rightarrow \wp(X)$ be an injection. Let $2^{n-1} < p \leq 2^n$. For each edge uv, assign label 1 if either $f(u) \subset f(v)$ or $f(v) \subset f(u)$ and assign 0 if either $f(u) \not\subset f(v)$ or $f(v) \not\subset f(u)$. f is called a subset cordial labeling if $|e_f(0) - e_f(1)| \leq 1$.

A graph is called a **subset cordial graph** if it has a subset cordial labeling. Here $\wp(X)$ denotes the power set of X.

Example 2.4. Consider the following graph G. Take $X = \{1, 2, 3\}$. Here $e_f(0) = 5$ and $e_f(1) = 5$ and so $|e_f(0) - e_f(1)|$ = 0. Thus G is subset cordial.

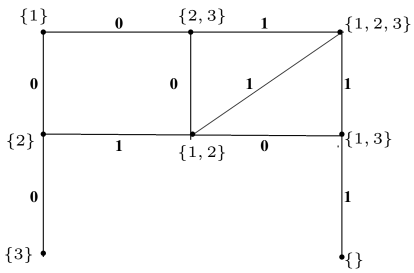

Figure 1: *A subset cordial labeling of a given graph G.*

Remark 2.5. If $2^{n-1} < p < 2^n$ and $X = \{1, 2, \ldots, n\}$, we have more number of subsets than vertices. We can easily label the vertices with $|e_f(0) - e_f(1)| \leq 1$.

Example 2.6. In Example 2.4, by removing the vertices labeled $\{3\}$ and $\{\}$, we get the following subset cordial graph.

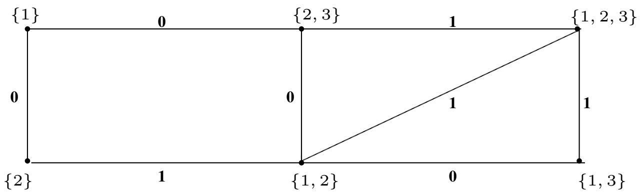

Figure 2: *A subset cordial labeling of the graph given in Figure 1 by removing the vertices labeled* $\{3\}$ *and* $\{\}$.

In this paper we prove that some star related graphs such as splitting graph of star, identification of star with a graph and the like are subset cordial.

Theorem 2.7. The star graph $K_{1,q}$ is subset cordial.

Proof: Let $X = \{1, 2, \ldots, n\}$. Assume that $p = 2^n$. Then $q = 2^n - 1$. Let v_p be the center of the star and $v_1, v_2, v_3, \ldots, v_{p-1}$ be the end vertices. Assign the subset $\{1\}$ to v_p and the remaining 2^{n-1} subsets to $v_1, v_2, v_3, \ldots, v_{p-1}$. All the subsets of $\{2, 3, \ldots, n\}$ excluding $\{\}$ contribute 0 to $e_f(0)$ and so $e_f(0) = 2^{n-1} - 1$. The remaining subsets of $\{1, 2, 3, \ldots, n\}$ containing 1 are labeled in the remaining vertices of $v_1, v_2, v_3 \ldots, v_{p-1}$ contribute 1 to $e_f(1)$ and so $e_f(1) = 2^n - 1 - 2^{n-1} + 1 = 2^{n-1}$. Thus $|e_f(0) - e_f(1)| = 1$.

Suppose $2^{n-1} < p \leq 2^n$. If $2^n - p$ is even, we remove $(2^n - p)/2$ edges labeled 0 and $(2^n - p)/2$ edges labeled 1. If $2^n - p$ is odd, we remove $(2^n - p - 1)/2$ edges labeled 1 and $(2^n - p - 1)/2$ edges labeled 0. In both cases, we see that $|e_f(0) - e_f(1)| \leq 1$. Thus the star graph $K_{1,q}$ is subset cordial. ■

The labeling pattern given in Theorem 2.7 is illustrated below.

Example 2.8. Consider the following star graph $K_{1,7}$. Take $X = \{1, 2, 3\}$. Here $e_f(0) = 3$ and $e_f(1) = 4$ and so $|e_f(0) - e_f(1)| = 1$. Thus the star graph $K_{1,7}$ is subset cordial.

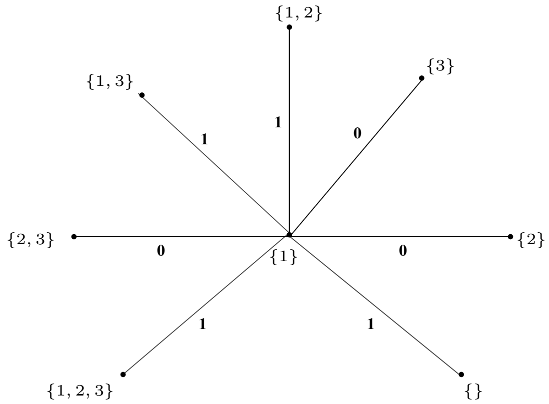

Figure 3: *A subset cordial labeling of* $K_{1,7}$.

Definition 2.9. Consider the m stars $K_{1,q}^{(1)}$, $K_{1,q}^{(2)}$, $K_{1,q}^{(3)}$,...,$K_{1,q}^{(m)}$. Then $G = \left(K_{1,q}^{(1)}, K_{1,q}^{(2)}, \ldots, K_{1,q}^{(m)}\right)$ is the graph obtained by joining one of the pendant vertices $K_{1,q}^{(i)}$ to any one of the pendant vertices of $K_{1,q}^{(i+1)}$ by an edge for $i = 1, 2, \ldots, m-1$.

Note that G has $m(n+1)$ vertices and $m(n+1) - 1$ edges.

Theorem 2.10. The graph $G = (K_{1,q}^{(1)}, K_{1,q}^{(2)}, \ldots, K_{1,q}^{(2^m)})$ is subset cordial.

Proof: Let $v_1^{(i)}$, $v_2^{(i)}, \ldots, v_n^{(i)}$ be the pendant vertices and $v^{(i)}$ be the central vertex of $K_{1,q}^{(i)}$ for $i = 1, 2, \ldots, 2^m$. Let $X = \{1, 2, \ldots, n, n+1, \ldots, n+m-1\}$. Assume that each $K_{1,q}^{(i)}$ contains 2^n vertices. Then G has 2^{n+m} vertices and $2^{n+m} - 1$ edges.

Now assign 2^n subsets of $\{1, 2, \ldots, n\}$ to the vertices of $K_{1,q}^{(1)}$ as in Theorem 2.7 . The number of 1's contributed to $e_f(1)$ is 2^{n-1} and the number of 0's contributed to $e_f(0)$ is $2^{n-1} - 1$. Next, we construct 2^n subsets by adding the element $n+1$ to each subset of $\{1, 2, \ldots, n\}$ and replace 0 by $n+1$.

Then assign these 2^n subsets to the star $K_{1,q}^{(2)}$ as in $K_{1,q}^{(1)}$. We also join one pendant vertex of $K_{1,q}^{(1)}$ to a pendant vertex of $K_{1,q}^{(2)}$ such that their labels themselves are not subsets . Such vertices exist because singleton sets and disjoint sets are available in $K_{1,q}^{(1)}$ and $K_{1,q}^{(2)}$. Now the number of 1's contributed to $e_f(1)$ is $2^{n-1} + 2^{n-1} = 2^n$ and 0's contributed to $e_f(0)$ is $2^{n-1} - 1 + 2^{n-1} - 1 + 1 = 2^n - 1$.

Now we construct 2^{n+1} subsets by adding the element $n+2$ to each subset of $\{1, 2, \ldots, n, n+1\}$ and replace 0 by $n+2$. Of these 2^{n+1} subsets, we assign 2^n subsets to $K_{1,q}^{(3)}$ and 2^n subsets to $K_{1,q}^{(4)}$ as in $K_{1,q}^{(1)}$ and $K_{1,q}^{(2)}$ respectively. As above, we join one pendant vertex of $K_{1,q}^{(2)}$ to a pendant vertex of $K_{1,q}^{(3)}$ and another pendant vertex $K_{1,q}^{(3)}$ to a pendant vertex $K_{1,q}^{(4)}$ such that their labels are not subsets themselves. Then the number of 1's contributed to $e_f(1)$ is 2^{n+1} and 0's contributed to $e_f(0)$ is $2^{n+1}-1$.

Continuing in this way, we label all the vertices of G, using the subsets of $\{1, 2, 3, \dots, n, n+1, \dots, n+m\}$ to 2^{n+m} vertices and also join one pendant vertex of $K_{1,q}^{(i)}$ to the pendant vertex of $K_{1,q}^{(i+1)}$, where $i = 1, 2, \dots, m-1$.

Thus, $e_{f(1)} = 2^{n+m-1}$ and $e_{f(0)} = 2^{n+m-1} - 1$ and so $|e_f(0) - e_f(1)| = 1$. Hence, G is subset cordial. ■

Remark 2.11. We can prove that the subset cordiality remains unchanged for $G = (K_{1,q}^{(1)}, K_{1,q}^{(2)}, \dots, K_{1,q}^{(2^m)})$ for all m.

Remark 2.12. We can also prove Theorem 2.10, for any number of edges in G, by removing or adding the edges in the stars of G. This can be done as follows. If the number of edges to be removed is even, say k, then we remove $\frac{k}{2}$ edges labeled with 0 and $\frac{k}{2}$ edges labeled with 1. If the number of edges to be removed is odd, then we remove $\frac{k-1}{2}$ edges labeled with 0 and $\frac{k+1}{2}$ edges labeled with 1. Then the subset cordiality remains unchanged.

Example 2.13. A subset cordial labeling of the graph $G = \left(K_{1,7}^{(1)}, K_{1,7}^{(2)}, K_{1,7}^{(3)}, K_{1,7}^{(4)}\right)$ is given in Figure 4. Take $X = \{1, 2, 3, 4, 5\}$, $m = 4$, $p = 2^5 = 32$ and $q = 31$. Here $e_f(0) = 16$ and $e_f(1) = 15$. Thus $|e_f(0) - e_f(1)| = 1$.

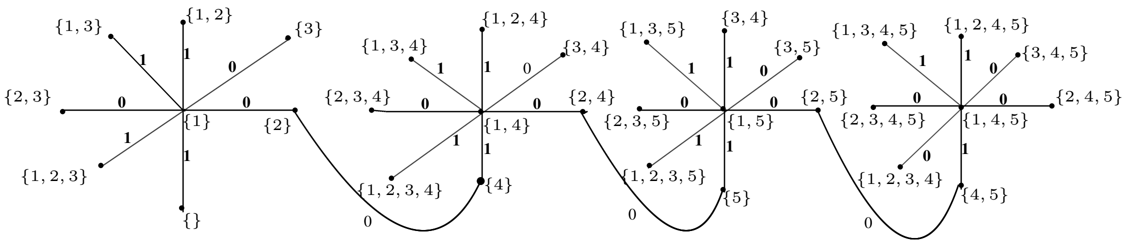

Figure 4: *A subset cordial labeling of* $\left(K_{1,7}^{(1)}, K_{1,7}^{(2)}, K_{1,7}^{(3)}, K_{1,7}^{(4)}\right)$.

Theorem 2.14. Let $X = \{1, 2, \dots, n\}$ and G be any subset cordial $(2^n, q)$-graph, where q is even. Then the graph $G * K_{1,2^n}$ obtained by identifying the central vertex of $K_{1,2^n}$ with that labeled $\{1\}$ in G is also subset cordial.

Proof: Let q be the even size of G. Since G is a subset cordial, $e_f(0) = e_f(1) = \frac{q}{2}$. Already 2^n subsets of $\{1, 2, \dots, n\}$ were labeled to the vertices of G. Let $v_1, v_2, \dots, v_{2^n}$ be the pendant vertices of $K_{1,2^n}$. Now, we construct 2^n subsets by adding a new element $n+1$ to each subset of X and by replacing 0 with $n+1$. Assign these 2^n subsets to the pendant vertices of $K_{1,2^n}$. From Theorem 2.7, it follows that $e_f(0) = \frac{q}{2} + 2^{n-1} = e_f(1)$. Hence G is a subset cordial. ■

Example 2.15. A subset labeling of $G * K_{1,8}$ where G is as in Example 2.4 is given in Figure 5. We see that $e_f(0) = 9 = e_f(1)$. Thus $|e_f(0) - e_f(1)| = 0$.

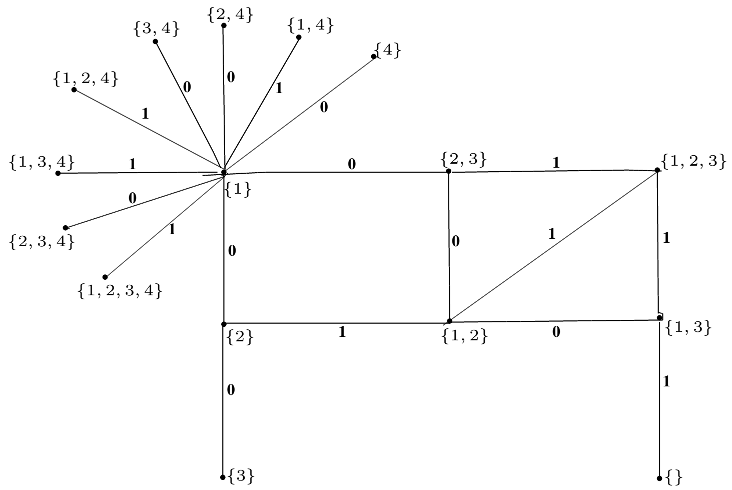

Figure 5: *A subset cordial labeling of $G * K_{1,8}$ for the graph G given in Example 2.4.*

Theorem 2.16. Let $X = \{1, 2, \ldots, n\}$. Let G be any subset cordial graph of size q and $K_{2,2^n}$ be a bipartite graph with the bipartition $V = V_1 \cup V_2$ with $V_1 = \{x_1, x_2\}$ and $V_2 = \{y_1, y_2, \ldots, y_{2^n}\}$. Then the graph $G * K_{2,2^n}$ obtained by identifying the vertices x_1 and x_2 of $K_{2,2^n}$ with that labeled $\{1\}$ and $\{2\}$ respectively in G is also a subset cordial.

Proof: Let G be any subset cordial graph of size q. Then $e_f(0) = e_f(1) = \frac{q}{2}$ if q is even. If q is odd, then $e_f(0) = \frac{q-1}{2}$ or $\frac{q+1}{2}$ and $e_f(1) = \frac{q+1}{2}$ or $\frac{q-1}{2}$. Let u and v be the vertices having the labels $\{1\}$ and $\{2\}$ respectively.

Suppose the vertices of G are labeled by the 2^n subsets of $\{1, 2, \ldots, n\}$.

To obtain $G * K_{1,2^n}$, we identify the vertices x_1 and x_2 of $K_{2,2^n}$ with that labeled $\{1\}$ and $\{2\}$ respectively in G. Then we label the remaining 2^n vertices of $G*K_{1,2^n}$ by the 2^n subsets of $\{1, 2, \ldots, n\}$ by adding $n+1$ to each subset and by replacing 0 with $n+1$.

We observe that $\{1\}$ is not a subset of the 2^{n-1} subsets of $\{2, 3, \ldots, n\}$ by adding $n+1$ to each subset and of $n+1$. So it contributes 2^{n-1}, 0's to $e_f(0)$ and 2^{n-1}, 1's to $e_f(1)$. The same is true for the element $\{2\}$ also. Thus we see that for $G * K_{1,2^n}$, $e_f(0) = e_f(1) = \frac{q}{2} + 2^{n-1} + 2^{n-1} = \frac{q}{2} + 2^n$, if q is even. If q is odd, then $e_f(0) = \frac{q-1}{2} + 2^n$ or $e_f(0) = \frac{q+1}{2} + 2^n$ and $e_f(1) = \frac{q+1}{2} + 2^n$ or $e_f(1) = \frac{q-1}{2} + 2^n$. Hence $|e_f(0) - e_f(1)| \leq 1$. So $G * K_{1,2^n}$ is a subset cordial. ■

Example 2.17. Consider the subset cordial graph G in Example 2.4 and $K_{2,2^3}$. Then we identify the vertices labeled $\{1\}$and $\{2\}$ in G to the vertices with label $\{1\}$and $\{2\}$ in $K_{2,2^3}$. The subset cordiality of $G * K_{2,2^3}$ is given below. Here, $e_f(0) = 13$ and $e_f(1) = 13$. Thus $|e_f(0) - e_f(1)| = 0$.

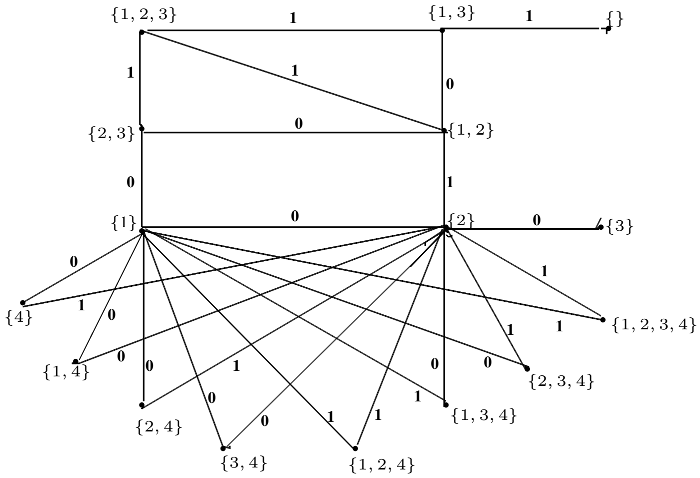

Figure 6: *A subset cordial labeling of $G * K_{2,2^3}$ for the graph G given in Example 2.4.*

Definition 2.18. [11] For a graph G, the splitting graph $S^{'}(G)$ of a graph G is obtained by adding a new vertex v corresponding to each vertex u of G such that $N(u) = N(v)$.

Theorem 2.19. $S^{'}(K_{1,2^n-1})$ is a subset cordial graph.

Proof: Let $u_1, u_2, \ldots, u_{2^n-1}$ be the pendant vertices and u be the central vertex of $K_{1,2^n-1}$ and $v, v_1, v_2, \ldots, v_{2^n-1}$ are added vertices corresponding to $u, u_1, u_2, \ldots, u_{2^n-1}$ to obtain $S^{'}(K_{1,2^n-1})$. Note that it has 2^n vertices and $3(2^n - 1)$ edges.

Let $X = \{1, 2, 3, \ldots, n, n+1\}$. Now consider the star $K_{1,2^n-1}$ and label its vertices as in Theorem 2.7 by the subsets of $\{1, 2, \ldots, n\}$. From Theorem 2.7, it is clear that the number of 1's contributed to $e_f(1)$ is 2^{n-1} and the number of 0's contributed to $e_f(0)$ is $2^{n-1} - 1$.

Next, we construct 2^n subsets by just adding the element $n+1$ to each subset of $\{1, 2, \ldots, n\}$ and by replacing 0 with $n + 1$. Then we label the vertices $v, v_1, v_2, \ldots, v_{2^n-1}$ by these 2^n subsets as in $K_{1,n}$. Note that the label of vertices $v, v_1, v_2, \ldots, v_{2^n-1}$ are the same as for the vertices $u, u_1, u_2, \ldots, u_{2^n-1}$, but the only difference is the element $n + 1$ has been added.

Thus, the label of vertices $v, v_1, v_2, \ldots, v_{2^n-1}$ are the copies of the labels of $u, u_1, u_2, \ldots, u_{2^n-1}$. Hence, the number of 1's contributed to $e_f(1)$ is 2^{n-1} and the number of 0's contributed to $e_f(0)$ is $2^{n-1} - 1$.

Now consider the labels of the vertices $v, v_1, v_2, \ldots, v_{2^n-1}$. Here there are 2^{n-1}, 0's contributing to $e_f(0)$ and $2^{n-1} - 1$, 1's to $e_f(1)$. Since we have replaced the subset $\{\}$ with $\{n + 1\}$ and so $\{1\}$ is not a subset of $\{n + 1\}$. Then the label of the edge uv_n is 0.

Thus, $e_f(0) = 2^{n-1} - 1 + 2^{n-1} - 1 + 2^{n-1} = 3.2^{n-1} - 2$ and $e_f(1) = 2^{n-1} + 2^{n-1} + 2^{n-1} - 1 = 3.2^{n-1} - 1$. Hence, $|e_f(0) - e_f(1)| = 1$ and so $S^{'}(K_{1,2^n-1})$ is a subset cordial graph. ∎

Example 2.20. Consider the following graph $S^{'}(K_{1,7})$. We have, $|V(S^{'}(K_{1,7}))| = 16$ and $|E(S^{'}(K_{1,7}))| = 21$. Here $e_f(0) = 10$ and $e_f(1) = 11$. Thus $|e_f(0) - e_f(1)| = 1$.

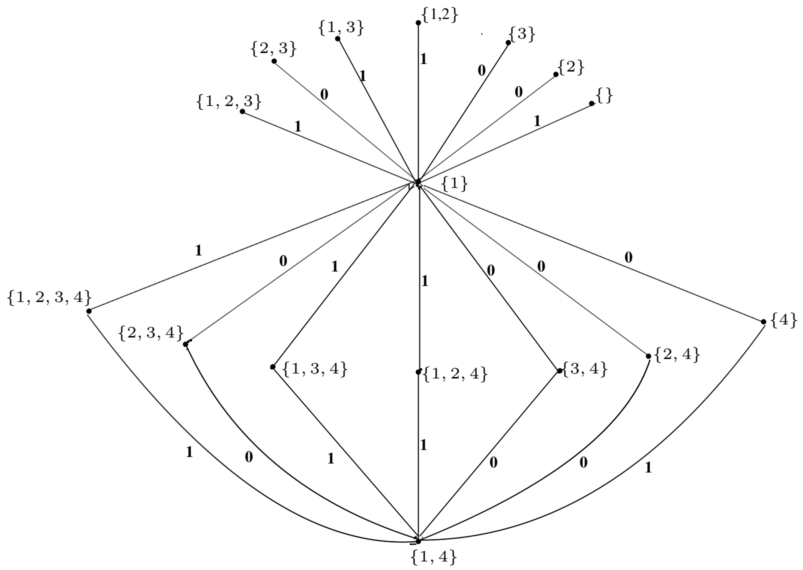

Figure 7: *A subset cordial labeling of* $S^{'}(K_{1,7})$.

Definition 2.21. The bistar $B(m, n)$ is the graph obtained by joining the centres of the stars $K_{1,m}$ and $K_{1,n}$. Note that $B(m, n)$ has $m + n + 2$ vertices and $m + n + 1$ edges.

Theorem 2.22. $B(2^{n-1} - 1, 2^{n-1} - 1)$is subset cordial.

Proof: Take $X = \{1, 2, \dots, n\}$. Let $V(B(2^{n-1} - 1, 2^{n-1} - 1)) = \{u_i, v_j : 1 \leq i, j \leq 2^{n-1}\}$ where $u_{2^{n-1}}$ and $v_{2^{n-1}}$ are central vertices.

Let $E(B(2^{n-1} - 1, 2^{n-1} - 1)) = \{u_{2^{n-1}}v_{2^{n-1}}, u_{2^{n-1}}u_i, v_{2^{n-1}}v_j : 1 \leq i, j \leq 2^{n-1} - 1\}$. Note that $B(2^{n-1} - 1, 2^{n-1} - 1)$ has 2^n vertices and $2^n - 1$ edges. Now we label the 2^n subsets of X to the vertices of $B(2^{n-1} - 1, 2^{n-1} - 1)$. First label the central vertex $u_{2^{n-1}}$ of the first star by $\{n\}$ and label the remaining pendent vertices of the star by the subsets of $\{1, 2, \dots, n\}$.

Next label the central vertex of $v_{2^{n-1}}$ of the second star by the empty set $\{\}$ and label the remaining pendent vertices of the second star by the subsets adding the element n to each subset of $\{1, 2, \dots, n - 1\}$. We observe that $\{n\}$ is not a subset of any set labeled in the vertices $u_i (1 \leq i \leq 2^{n-1} - 1)$ and $\{\}$ is a subset of every set labeled in the vertices of $v_j (1 \leq j \leq 2^{n-1} - 1)$ and $u_{2^{n-1}}$.

Hence, $e_f(0) = 2^{n-1} - 1$ and $e_f(1) = 2^{n-1}$ and $|e_f(0) - e_f(1)| = 1$. Thus $B(2^{n-1} - 1, 2^{n-1} - 1)$ is subset cordial. ∎

Example 2.23. Let $X = \{1, 2, 3, 4\}$. The subset cordial labeling of $B(7, 7)$ is given below. We see that $e_f(0) = 7$ and $e_f(1) = 8$.

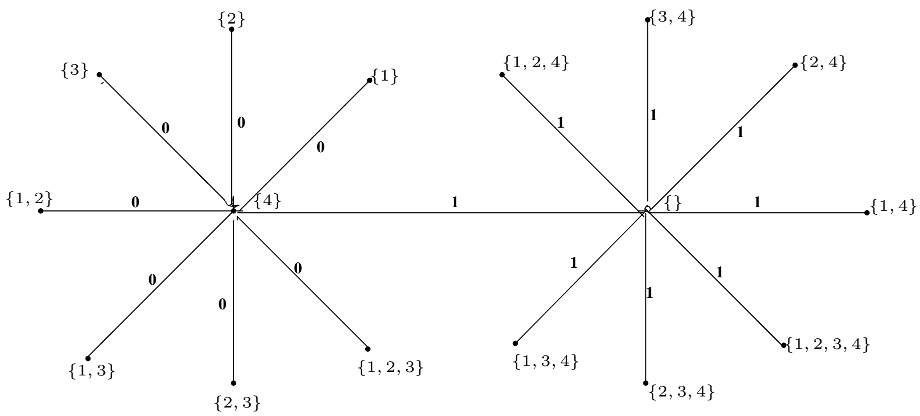

Figure 8: *A subset cordial labeling of* $B(7, 7)$.

Next we prove that the splitting graph of bistar $B(2^{n-1} - 1, 2^{n-1} - 1)$ is subset cordial.

Theorem 2.24. $S^{'}(B(2^{n-1} - 1, 2^{n-1} - 1))$ is subset cordial.

Proof: Consider $B(2^{n-1} - 1, 2^{n-1} - 1)$ with vertex set $\{u_i v_j : 1 \leq i, j \leq 2^{n-1}\}$ where $u_{2^{n-1}}$ and $v_{2^{n-1}}$ are central vertices and other vertices are pendant. In order to obtain $S^{'}(B(2^{n-1} - 1, 2^{n-1} - 1))$, add the vertices $u_i^{'}$,$v_j^{'}$ where $1 \leq i, j \leq 2^{n-1}$.

Then $|V(S^{'}(B(2^{n-1} - 1, 2^{n-1} - 1)))| = 2^{n+1}$ and $|E(S^{'}(B(2^{n-1}, 2^{n-1})))| = 3(2^n - 1)$. Take $X = \{1, 2, \ldots, n, n+1\}$. First label the vertices of Bistar $B(2^{n-1} - 1, 2^{n-1} - 1)$ as in Theorem 2.22.

Next we label the vertex $u_{2^n}^{'}$ by $n(n+1)$ and $u_i^{'}(1 \leq i \leq 2^n - 1)$ by the labels as in $u_i(1 \leq i \leq 2^n - 1)$ and adding the subset $\{n + 1\}$ to each label. So the corresponding edges get the label 0. Then we label the v_{2^n}' by $n + 1$ and $v_j^{'}(1 \leq i \leq 2^n - 1)$ by the labels as in $v_j(1 \leq i \leq 2^n - 1)$ and adding the subset $\{n + 1\}$ to each label. So the corresponding edges get the label 1. We also note that the edge $u_{2^n} v_{2^n}^{'}$ gets the label 0 and the edge $v_{2^n} u_{2^n}^{'}$ gets the label 1.

Thus we have $e_f(0) = 2^{n-1} - 1 + 2^{n-1} - 1 + 2^{n-1} - 1 + 1 = 3.2^{n-1} - 2$ and $e_f(1) = 2^{n-1} - 1 + 2^{n-1} - 1 + 2^{n-1} - 1 + 1 = 3.2^{n-1} - 1$.

Hence $|e_f(0) - e_f(1)| = 1$ and so $S^{'}(B(2^{n-1} - 1, 2^{n-1} - 1))$ is subset cordial. ■

Example 2.25. Consider the graph $S^{'}(B(7, 7))$. Take $X = \{1, 2, 3, 4, 5\}$. Here $|V(S^{'}(B(7, 7)))| = 32$ and $|E(S^{'}(B(7, 7)))| = 45$. The subset cordiality is given in Figure 9.

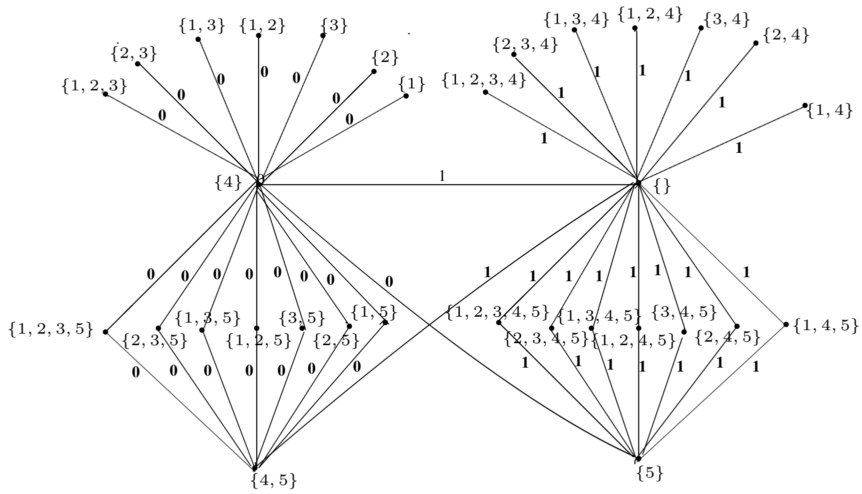

Figure 8: *A subset cordial labeling of* $S^{'}(B(7,7))$.

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