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On (Super) edge-antimagic total labeling of subdivided stars

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Abstract

Labeling is an interesting technique to assign labels to vertices and edges of a graph under certain conditions. There are many types of labeling, for example magic, antimagic, graceful, odd graceful, cordial, radio, sum and mean labeling. This paper deals with different results related to (a, d)-edge-antimagic total and super (a, d)-edge-antimagic total labelings of a subclass of subdivided stars denoted by $T(n, n+2, n+5, 2n+7, n_5, ..., n_r)$, where $n \equiv 1 \pmod 2$, $n_m = 2^{m-3}(n+3)+1$ and $5 \leq m \leq r$.

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1 Introduction

Let G be a graph with $V(G)$ and $E(G)$ as set of vertices and edges, respectively. Moreover, suppose that $|V(G)| = v$ and $|E(G)| = e$. All graphs in this paper are finite, simple and undirected. A general reference for graph-theoretic ideas can be seen in [27]. A *labeling* (or *valuation*) of a graph is a map that carries graph elements to numbers (usually to positive or non-negative integers). In this paper, the domain is the set of all vertices and edges and such a labeling is called *a total labeling*. Some labelings use only the vertex-set or the edge-set and we shall call them *vertex-labelings* or *edge-labelings*, respectively.

Definition 1.1. An (s,d)-edge-antimagic vertex ((s,d)-EAV) labeling of a (v,e)-graph G is a bijective function $\lambda : V(G) \rightarrow \{1,2,\ldots,v\}$ such that the set of edge-weights of all edges in G, $\{w(xy) = \lambda(x)+\lambda(y) : xy \in E(G)\}$, forms an arithmetic progression $\{s, s+d, s+2d, \ldots, s+(e-1)d\}$, where $s > 0$ and $d \geq 0$ are two fixed integers.

Definition 1.2. A bijection $\lambda : V(G) \cup E(G) \rightarrow \{1,2,...,v+e\}$ is called an (a,d)-edge-antimagic total ((a,d)-EAT) labeling of a (v,e)-graph G if the set of edge-weights $\{\lambda(x)+\lambda(xy)+\lambda(y) : xy \in V(G)\}$ forms an arithmetic progression starting at a and having common difference d, where $a > 0$ and $d \geq 0$ are two fixed integers. A graph that admits an (a,d)-EAT labeling is called an (a,d)-EAT graph.

Definition 1.3. *If λ is an (a,d)-EAT labeling such that $\lambda(V(G)) = \{1,2,...,v\}$ then λ is called a super (a,d)-EAT labeling and G is known as a super (a,d)-EAT graph.*

In the above definition, if $d = 0$ then a super $(a,0)$-EAT labeling is called a super edge-magic total labeling. Moreover, a is called minimum edge-weight for $d \geq 0$ and magic constant for $d = 0$. The definition of an (a,d)-EAT labeling was introduced by Simanjuntak, Bertault and Miller in [24] as a natural extension of *magic valuation* defined by Kotzig and Rosa [18, 19]. A super (a,d)-EAT labeling is a natural extension of the notion of *super edge-magic labeling* defined by Enomoto, Llado, Nakamigawa and Ringel [5]. They also proposed the conjecture that every tree admits a super $(a,0)$-EAT labeling. Lee and Shah [20] verified this conjecture for trees with at most 17 vertices with the help of computer. However, in general, this conjecture is still an open problem. The results related to (a,d)-EAT and super (a,d)-EAT labelings can be found for some particular families of trees, for example banana trees [7], w-trees [8], extended w-trees [9–11], generalized extended w-trees [12], stars [21], subdivided stars [13–16, 22, 23, 28, 29], path-like trees [2], caterpillars [18, 25] and subdivided caterpillars [17]. More details on antimagic labeling can be found in [1, 3, 4, 6, 26].

The notion of a dual labeling has been introduced by Kotzig and Rosa [18]. According to him, if λ is an $(a,0)$-EAT labeling with magic constant a then λ' is also an $(a',0)$-EAT labeling with magic constant $a' = 3(v+e+1)-a$. The labeling is defined as $\lambda'(x) = v+e+1-\lambda(x)$ for ever $x \in V(G) \cup E(G)$.

The following lemma of duality has been studied by Baskoro [4].

Lemma 1.4. [4] If λ is a super $(a,0)$-EAT labeling of G with the magic constant a, then the bijection $\lambda_1 : V(G) \cup E(G) \rightarrow \{1,2,...,v+e\}$ defined by

$$\lambda_1(x) = \begin{cases} v+1-\lambda(x), & \text{for } x \in V(G), \\ \\ 2v+e+1-\lambda(x), & \text{for } x \in E(G). \end{cases}$$

is also a super $(4v+e+3-a, 0)$-EAT labeling of G.

Now, we consider the following proposition which establishes the relation between (s,d)-EAV and (a,d)-EAT labelings:

Proposition 1.5. [2] If a (v, e)-graph G has an (s, d)-EAV labeling then
(i) G has a super $(s + v + 1, d + 1)$-EAT labeling,
(ii) G has a super $(s + v + e, d - 1)$-EAT labeling.

2 Important Results

In this section, we define the concept of a subdivided star and present the values for the bounds of the antimagic labeling parameters a and d. At the end, we state some known results related to a super (a, d)-EAT labeling of different subclasses of subdivided stars.

Definition 2.1. Let $n_i \geq 1$, $1 \leq i \leq r$, and $r \geq 2$. A subdivided star $T(n_1, n_2, ..., n_r)$ is a tree obtained by inserting $n_i - 1$ vertices to each of the ith edge of the star $K_{1,r}$. Moreover suppose that $V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \leq i \leq r \; ; \; 1 \leq l_i \leq n_i\}$ is the vertex-set and $E(G) = \{c\ x_i^1 \mid 1 \leq i \leq r\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \leq i \leq r \; ; \; 1 \leq l_i \leq n_i - 1\}$ is the edge-set of the subdivided star $G \cong T(n_1, n_2, ..., n_r)$ then $v = |V(G)| = \sum\limits_{i=1}^{r} n_i + 1$ and $e = |E(G)| = \sum\limits_{i=1}^{r} n_i$.

Bača and Miller [3] stated a necessary condition for a graph to be super (a, d)-EAT, which provides an upper bound on the parameter d. Let a (v, e)-graph G be a super (a, d)-EAT. The minimum possible edge-weight is at least $v + 4$. The maximum possible edge-weight is no more than $3v + e - 1$. Thus $a+(e-1)d \leq 3v+e-1$ or $d \leq \frac{2v+e-5}{e-1}$. For any subdivided star, where $v = e+1$, it follows that $d \leq 3$.

Ngurah et al. [22] found lower and upper bounds of the magic constant a for a subclass of the subdivided stars, which is stated as below.

Lemma 2.2. If $T(n_1, n_2, n_3)$ is a super $(a, 0)$-EAT graph, then $\frac{1}{2l}(5l^2+3l+6) \leq a \leq \frac{1}{2l}(5l^2+11l-6)$, where $l = \sum\limits_{i=1}^{3} n_i$.

The lower and upper bounds of the magic constant a for a subclass of subdivided stats established by Salman et al. [23] are given below.

Lemma 2.3. If $T\underbrace{(n, n, ..., n)}_{n-times}$ is a super $(a, 0)$-EAT graph, then $\frac{1}{2l}(5l^2 + (9 - 2n)l + n^2 - n) \leq a \leq \frac{1}{2l}(5l^2 + (2n + 5)l + n - n^2)$, where $l = n^2$.

For $d = 0$, Javaid [16] proved the lower and upper bounds of the magic constant a for the most extended subclasses of the subdivided stars denoted by $T(n_1, n_2, n_3, ..., n_r)$ with any $n_i \geq 1$ for $1 \leq i \leq r$, which are presented in the following lemma:

Lemma 2.4. If $T(n_1, n_2, n_3, ..., n_r)$ is a super $(a, 0)$-EAT graph, then $\frac{1}{2l}(5l^2 + (9-2r)l + (r^2 - r)) \leq a \leq \frac{1}{2l}(5l^2 + (5 + 2r)l - (r^2 - r))$, where $l = \sum\limits_{i=1}^{r} n_i$.

For $d \in \{0, 1, 2, 3\}$, Javaid and Akhlaq [15] proved the following lower and upper bounds of the minimum edge-weight a on the same class of subdivided stars:

Lemma 2.5. If $T(n_1, n_2, n_3, ..., n_r)$ has a super (a, d)-EAT labeling, then $\frac{1}{2l}(5l^2 + r^2 - 2lr + 9l - r - (l-1)ld) \le a \le \frac{1}{2l}(5l^2 - r^2 + 2lr + 5l + r - (l-1)ld)$, where $l = \sum_{i=1}^{r} n_i$ and $d \in \{0, 1, 2, 3\}$.

Many authors have proved the antimagicness for subdivided stars. Lu [28, 29] called the subdivided star $T(n_1, n_2, n_3)$ as a three-path tree and proved that it is a super $(a, 0)$-EAT graph if n_1 and n_2 are odd with $n_3 = n_2 + 1$ or $n_3 = n_2 + 2$. Ngurah et al. [22] proved that the subdivided star $T(n_1, n_2, n_3)$ is also a super $(a, 0)$-EAT graph if $n_3 = n_2 + 3$ or $n_3 = n_2 + 4$. Salman et al. [23] found a super $(a, 0)$-EAT labeling on the subdivided stars $T(n_1, n_2, n_3, ..., n_r)$, where $n_1 = n_2 = n_3 = ... = n_r = 2$ or 3. Javaid et al. [14] proved the existence of a super $(a, 0)$-EAT labeling on the subdivided stars $T(n, n, n, n)$, $T(n, n, n-1, n)$ and $T(n, n, m, m)$, where n, m are odd. Moreover, Javaid et al. [15] proved the following results:

- For $r \ge 5$ and even $n \ge 4$, $T(n, n+2, n+5, 2n+7, n_5, ..., n_r)$ admits a super (a, d)-EAT labeling, where $d \in \{0, 2\}$, $n_m = 2^{m-3}(n+3) + 1$ and $5 \le m \le r$ [15].
- For any $r \ge 5$ and even $n \ge 4$, $T(n, n+2, n+5, 2n+7, n_5, ..., n_r)$ admits a super $(a, 1)$-EAT labeling, where $v = |V(G)|$ is even $n_m = 2^{m-3}(n+3) + 1$ and $5 \le m \le r$ [15].

In the present paper, we prove the existence of (a, d)-EAT and super (a, d)-EAT labelings on the same class of subdivided stars $T(n, n+2, n+5, 2n+7, n_5, ..., n_r)$ but under the certain condition of $n \equiv 1 \pmod 2$.

3 Super (a, d)-EAT labeling of subdivided stars

In this section, we prove some results related to (a, d)-EAT and super (a, d)-EAT labelings of the subdivided stars for different values of d.

Theorem 3.1. For $r \ge 5$ and $n \equiv 1 \pmod 2$, $G \cong T(n_1, n_2, n_3, n_4, n_5, ..., n_r)$ admits a super $(a, 0)$-EAT labeling with $a = v + e + s$ and a super $(a, 2)$-EAT labeling with $a = v + s + 1$, where $v = |V(G)|$, $s = \frac{5n+21}{2} + \sum_{m=5}^{r}[2^{m-4}(n+3)+1]$, $n_1 = n, n_2 = n+2, n_3 = n+5, n_4 = 2n+7$ and $n_m = 2^{m-3}(n+3) + 1$ for $5 \le m \le r$.

Proof: If $v = |V(G)|$ and $e = |E(G)|$ then $v = (5n+15) + \sum_{m=5}^{r}[2^{m-3}(n+3)+1]$ and $e = v - 1$. Throughout the labeling take $1 \le l_i \le n_i$ and $1 \le i \le r$. Define $\lambda : V(G) \to \{1, 2, ..., v\}$ as follows:

$$\lambda(c) = (3n+9) + \sum_{m=5}^{r}[2^{m-4}(n+3)+1].$$

Case (i): $l_i \equiv 1 \pmod 2$.
For $1 \le i \le 2$,

$$\lambda(u) = \begin{cases} \frac{n+2-l_1}{2}, & \text{for } u = x_1^{l_1}, \\ \\ \frac{n+4+l_2}{2}, & \text{for } u = x_2^{l_2}, \end{cases}$$

For $i=3$,

$$\lambda(u)=\begin{cases}\frac{n+3}{2}, & \text{for } u=x_3^1,\\ \\ \frac{3n+12-l_3}{2}, & \text{for } u=x_3^{l_3},\end{cases}$$

For $i=4$,

$$\lambda(u)=\begin{cases}\frac{5n+18-l_4}{2}, & \text{for } u=x_4^{l_4},\end{cases}$$

and for $5\leq i\leq r$,

$$\lambda(x_i^{l_i})=\frac{5n+17}{2}+\sum_{m=5}^{i}[2^{m-4}(n+3)+1]-\frac{l_i-1}{2}.$$

Case (ii): $l_i\equiv 0 \pmod 2$ and $\alpha=\frac{5n+17}{2}+\sum\limits_{m=5}^{r}[2^{m-4}(n+3)+1]$.
For $i=1,2,3,4$,

$$\lambda(u)=\begin{cases}(\alpha+\frac{n-1}{2})-\frac{l_1-2}{2}, & \text{for } u=x_1^{l_1},\\ \\ (\alpha+\frac{n+3}{2})+\frac{l_2-2}{2}, & \text{for } u=x_2^{l_2},\\ \\ (\alpha+\frac{3n+7}{2})-\frac{l_3-2}{2}, & \text{for } u=x_3^{l_3},\\ \\ (\alpha+\frac{5n+13}{2})-\frac{l_4-2}{2}, & \text{for } u=x_4^{l_4},\end{cases}$$

and for $5\leq i\leq r$,

$$\lambda(x_i^{l_i})=(\alpha+\frac{5n+13}{2})+\sum_{m=5}^{i}[2^{m-4}(n+3)]-\frac{l_i-2}{2}.$$

The set of all edge-sums generated by the above formulas forms a consecutive integer sequence $s=(\alpha+1)+1,(\alpha+1)+2,\cdots,(\alpha+1)+e$. It follows that λ is an $(s,1)$-EAV labeling. Therefore, by Proposition 1.1, λ can be extended to a super $(a,0)$-EAT labeling with $a=s+v+e=\frac{25n+79}{2}+\frac{1}{4}\sum\limits_{m=5}^{r}[2^{m-2}(5n+15)+12]$ and to a super $(a,2)$-EAT labeling with $a=s+v+1=\frac{15n+53}{2}+\frac{1}{2}\sum\limits_{m=5}^{r}[2^{m-3}(3n+9)+4]$. ∎

Theorem 3.2. For $r\geq 5$, $n\equiv 1 \pmod 2$ and $v=|V(G)|$ even, $G\cong T(n_1,n_2,n_3,n_4,n_5,...,n_r)$ admits a super $(a,1)$-EAT labeling with $a=s+\frac{3}{2}v$, where $s=\frac{5n+21}{2}+\sum\limits_{m=5}^{r}[2^{m-4}(n+3)+1]$, $n_1=n, n_2=n+2, n_3=n+5, n_4=2n+7$ and $n_m=2^{m-3}(n+3)+1$ for $5\leq m\leq r$.

Proof: Suppose that $V(G)$, $E(G)$ and λ are defined as in the proof of Theorem 3.1. It follows that the edge-sums of all the edges of G constitute an arithmetic sequence $s=\alpha+2,\alpha+3,\cdots,\alpha+1+e$ with common difference 1, where $\alpha=\frac{5n+17}{2}+\sum\limits_{m=5}^{r}[2^{m-4}(n+3)+1]$. We denote it by $A=\{a_i; 1\leq i\leq e\}$. Consequently, the set of edge-labels is $B=\{b_j\ ;\ 1\leq j\leq e\}$, where $b_j=v_j+1$. Define the set of edge-weights $C=\{a_{2i-1}+b_{e-i+1}\ ;\ 1\leq i\leq\frac{e+1}{2}\}\cup\{a_{2j}+b_{\frac{e-1}{2}-j+1}\ ;\ 1\leq j\leq\frac{e+1}{2}-1\}$. It is easy to see that C constitutes an arithmetic sequence with $d = 1$ and $a = s + \frac{3v}{2} = (10n + 33) + \frac{1}{2}\sum_{m=5}^{r}[2^{m-1}(n + 3) + 5]$. Since all the vertices receive the smallest labels, λ is a super $(a, 1)$-EAT labeling. ∎

As a consequence of Lemma 1.4, and Theorem 3.1, we have the following corollary:

Corollary 3.3. For $n \equiv 1 \pmod 2$ and $r \geq 5$, $G \cong T(n, n+2, n+5, 2n+7, n_5, ..., n_r)$ admits a super $(a, 0)$-EAT labeling with the magic constant $a = \frac{25n+76}{2} + \sum_{m=5}^{r}[2^{m-4}(5n + 15) + 2]$, where $n_m = 2^{m-3}(n + 3) + 1$ and $5 \leq m \leq r$.

Now by the concept of dual labeling of $(a, 0)$-EAT labeling stated by Kotzig and Rosa, we have the following corollaries from Theorem 3.3 and the Corollary 3.1:

Corollary 3.4. For $n \equiv 1 \pmod 2$ and $r \geq 5$, $G \cong T(n, n+2, n+5, 2n+7, n_5, ..., n_r)$ admits an $(a, 0)$-EAT labeling with the magic constant $a = \frac{35n+101}{2} + \sum_{m=5}^{r}[2^{m-4}(7n + 21) + 3]$, where $n_m = 2^{m-3}(n + 3) + 1$ and $5 \leq m \leq r$.

Corollary 3.5. For $n \equiv 1 \pmod 2$ and $r \geq 5$, $G \cong T(n, n+2, n+5, 2n+7, n_5, ..., n_r)$ admits an $(a, 0)$-EAT labeling with the magic constant $a = \frac{25n+75}{2} + \sum_{m=5}^{r}[2^{m-4}(5n + 15) + 2]$, where $n_m = 2^{m-3}(n + 3) + 1$ and $5 \leq m \leq r$.

In the following theorem, we construct another $(a, 0)$-EAT labeling:

Theorem 3.6. For $r \geq 5$ and $n \equiv 1 \pmod 2$, $G \cong T(n_1, n_2, n_3, n_4, n_5, ..., n_r)$ admits an $(a, 0)$-EAT labeling with $a = v + e + s - 1$, where $v = |V(G)|$ $s = \frac{5n+19}{2} + \sum_{m=5}^{r}[2^{m-3}(n + 3) + 2]$, $n_1 = n, n_2 = n + 2, n_3 = n + 5, n_4 = 2n + 7$ and $n_m = 2^{m-3}(n + 3) + 1$ for $5 \leq m \leq r$.

Proof: Suppose that $V(G)$, $E(G)$ and λ are defined as in the proof of Theorem 3.1. Now we define f as follows:

$$f(x) = 2\lambda(x) - 1 \text{ for all } x \in V(G).$$

Consequently, all the vertices receive the odd labels and the set of edge-sums $S = \{f(x) + f(y) : xy \in E(G)\}$ forms an arithmetic progression starting from $\frac{5n+19}{2} + \sum_{m=5}^{r}[2^{m-3}(n + 3) + 2]$ with common difference 2. If we define

$$f(xy) = (15n + 47) + \sum_{m=5}^{r}[2^{m-4}(3n + 9) + 4] - \lambda(x) - \lambda(y).$$

Thus, we have an $(a, 0)$-EAT labeling with magic constant $a = v + e + s - 1 = (15n + 47) + \sum_{m=5}^{r}[2^{m-4}(3n + 9) + 4]$. ∎

As a consequence of Theorem 3.6 and the concept of Kotzig and Rosa related to a dual labeling, we have the following corollary:

Corollary 3.7. For $n \equiv 1 \pmod 2$ and $r \geq 5$, $G \cong T(n, n+2, n+5, 2n+7, n_5, ..., n_r)$ admits a super $(a, 0)$-EAT labeling with the magic constant $a = (45n+43) + \sum_{m=5}^{r} [2^{m-4}(9n+27)+2]$, where $n_m = 2^{m-3}(n+3)+1$ and $5 \leq m \leq r$.

4 Conclusion

In this paper, we show that a subclass of trees, namely subdivided star denoted by $T(n, n+2, n+5, 2n+7, n_5, ..., n_r)$ admits (a, d)-EAT and super (a, d)-EAT labelings, where $n \equiv 1 \pmod 2$, and $n_m = 2^{m-3}(n+3)+1$ for $5 \leq m \leq r$. However, for different values of the minimum edge-weight a and n_i, problem is still open.

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