International Journal of Mathematics and Soft Computing
Vol.3, No.3. (2013), 71 - 78.

ISSN Print : 2249 – 3328
ISSN Online: 2319 – 5215

On Topological #rg-quotient mappings

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Abstract

The authors introduced #rg-closed sets and #rg-open sets in topological spaces and established their relationships with some generalized sets in topological spaces. The aim of this paper is to introduce #rg-quotient maps using #rg-closed sets and characterize their basic properties.

Keywords: #rg-closed set, #rg-open set, #rg-continuous, #rg-irresolute, #rg-homeomorphism.

AMS Subject Classification (2010): 54C10.

1 Introduction

N. Levine [10] introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers like Balachandran, Sundaram and Maki[3], Bhattacharyya and Lahiri[5], Arockiarani[1], Arya and Gupta[2], Devi[7] and Wali[20] have worked on generalized closed sets, their generalizations and related concepts in general topology. In this paper, we introduce #rg-quotient functions. Several characterizations and its properties have been obtained for this function. Also the relationship between strong and weak form of #rg-quotient mappings have been established.

2 Preliminaries

Throughout the paper X and Y denote the topological spaces (X, τ) and (Y, σ) respectively and on which no separation axioms are assumed unless otherwise explicitly stated. For any subset A of a space (X, τ), the closure of A, interior of A and the complement of A are denoted by $cl\,(A)$, $int\,(A)$ and A^c or $X\backslash A$ respectively. (X, τ) may be replaced by X if there is no chance of confusion. Let us recall the following definitions.

Definition 2.1. A subset A of a space X is called

1) a preopen set[11] if $A \subseteq intcl(A)$ and a preclosed set if $clint\,(A) \subseteq A$.
2) a semiopen set[9] if $A \subseteq clint(A)$ and a semi closed set if $intcl\,(A) \subseteq A$.

3) a regular open set[15]if $A = intcl(A)$ and a regular closed set if $A = clint(A)$.
4) a π- open set[21] if A is a finite union of regular open sets.
5) regular semi open[6] if there is a regular open U such $U \subseteq A \subseteq cl(U)$.

Definition 2.2. A subset A of (X,τ) is called

1) a generalized closed set (briefly, g-closed)[10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
2) a regular generalized closed set (briefly, rg-closed)[14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
3) a generalized preregular closed set (briefly, gpr-closed)[8] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
4) a regular weakly generalized closed set (briefly, rwg-closed)[13] if $clint(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
5) rw-closed [4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open.

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3. A subset A of a space X is called #rg-closed[16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is rw-open. The complement of #rg-closed set is called a #rg-open set. The family of #rg-closed sets and #rg-open sets are denoted by $\#RGC(X)$ and $\#RGO(X)$.

Definition 2.4. A map $f:(X,\tau) \rightarrow (Y,\sigma)$ is said to be

(i) #rg-continuous [17] if $f^{-1}(V)$ is #rg-closed in (X,τ) for every closed set V in (Y,σ).
(ii) strongly #rg-continuous[19] if $f^{-1}(V)$ is a closed set of (X,τ) for each #rg-closed set V of (Y,σ).
(iii) #rg-irresolute [17] if $f^{-1}(V)$ is #rg-closed in (X,τ) for each #rg-closed set V of (Y,σ).
(iv) #rg-homeomorphism [18] if f is bijection and f and f^{-1} are #rg-continuous.
(v) #rg-closed [17] if $f(F)$ is #rg-closed in (Y,σ) for every #rg-closed set F of (X,τ).
(vi) #rg-open[17] if $f(F)$ is #rg-open in (Y,σ) for every #rg-open set F of (X,τ).
(vii) contra #rg continuous[19] if $f^{-1}(V)$ is #rg-closed in (X,τ) for each open set V in (Y,σ) .
(viii) a quotient map [12], provided a subset V of (Y,σ) is open if and only if $f^{-1}(V)$ is open in (X,τ).

Definition 2.5. A space X is called a $T_{\#rg}$-space[16] if every #rg-closed set in it is closed.

3 #rg-quotient maps

Definition 3.1. A surjective function $f:(X,\tau) \rightarrow (Y,\sigma)$ is said to be a #rg-quotient map if f is #rg-continuous and $f^{-1}(V)$ is open in (X,τ) implies V is a #rg-open set in (Y,σ).

Example 3.2. Let $X = Y = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\}, \{b\},\{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be the identity mapping. Then f is #rg-continuous and $f^{-1}(V)$ is open in (X,τ) implies V is #rg-open in (Y,σ).Hence f is #rg-quotient map.

Theorem 3.3. If a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is surjective #rg-continuous and #rg-open then f is a #rg-quotient map.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be surjective #rg-continuous and #rg-open. Let $f^{-1}(V)$ be open in (X,τ). Then $f^{-1}(V)$ is #rg open in (X,τ). Since f is #rg-open, $f(f^{-1}(V))$ is #rg-open. Thus V is #rg-open. Hence f is a #rg-quotient map. ■

Corollary 3.4. If a map $f:(X,\tau)\rightarrow(Y,\sigma)$ is surjective #rg-continuous and f^{-1} is irresolute then f is a #rg-quotient map.

Proof: The proof follows from the fact that f^{-1} is irresolute and f is #rg-open is equivalent. ■

Theorem 3.5. If a map $f:(X,\tau)\rightarrow(Y,\sigma)$ is #rg-homeomorphism then f is #rg-quotient map.

Proof: Since f is #rg-homeomorphism, f is bijective and f is #rg-continuous. Let $f^{-1}(V)$ be open in X. Since f^{-1} is #rg-continuous, $f(f^{-1}(V))=V$ is #rg-open in Y. Hence f is a #rg-quotient map. ■

Theorem 3.6. Every quotient map is a #rg-quotient map.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a quotient map. Since every continuous map is #rg-continuous and every open set is #rg-open, the proof follows from the definition. ■

The converse of the above theorem is not true as seen from the following example.

Example 3.7. Let $X=Y=\{a,b,c,d\}$, $\tau=\{\phi,X,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$ and $\sigma=\{\phi,Y,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity mapping. Here f is #rg-quotient but f is not a quotient map since $f^{-1}(\{c\})=\{c\}$ which is open in X but $\{c\}$ is not open in Y.

Definition 3.8. A surjective function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be a #rg*-quotient map if f is #rg-irresolute and $f^{-1}(V)$ is #rg-open in (X,τ) implies V is open in (Y,σ).

Example 3.9. Let $X=\{a,b,c,d\}, Y=\{1,2,3\}$, $\tau=\{\phi,X,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$ and $\sigma=\{\phi,Y,\{1\},\{2\},\{1,2\}\}$. Define a map $f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(a)=f(b)=1$, $f(c)=2$ and $f(d)=3$. Then clearly f is irresolute and $f^{-1}(V)$ is #rg- open in (X,τ) implies V is open in (Y,σ).

Theorem 3.10. Every #rg*- quotient map is #rg-open.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a #rg*- quotient map. Let V be a #rg-open set in X. Then $f^{-1}((f(V))$ is #rg-open in X. Since f is #rg*-quotient map, $f(V)$ is open in Y. Hence $f(V)$ is #rg-open in Y and hence f is #rg-open. ■

The converse of the theorem need not be true as seen from the following example.

Example 3.11. Let $X=\{a,b,c,d\}$, $Y=\{1,2,3\}$, $\tau=\{\phi,X,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$ and $\sigma=\{\phi,Y,\{1\},\{2\},\{1,2\},\{1,3\}\}$. Define a map $f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(a)=1$, $f(b)=2$, $f(c)=f(d)=3$. Here f is #rg-open but f is not #rg*-quotient map since $f^{-1}(\{1,3\})=\{a,c,d\}$ is not #rg-open in X but $\{1,3\}$ is #rg-open in Y.

4 Strong form of #rg-quotient map

Definition 4.1. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a surjective map. Then f is called strongly #rg-quotient if a set U is open in Y if and only if $f^{-1}(U)$ is #rg-open set in X.

Example 4.2. Let $X=\{p,q,r,s\}$, $\tau=\{\phi, X, \{r\}, \{p,q\}, \{p,q,r\}\}$, $Y=\{1,2,3\}$ and $\sigma=\{\phi,Y,\{1\},\{2\},\{1,2\}\}$. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(p)=f(q)=1$, $f(r)=2$ and $f(s)=3$. Then clearly $f^{-1}(U)$ is #rg-open iff U is open. Hence f is strongly #rg-quotient map.

Theorem 4.3. Every strongly #rg-quotient map is #rg-quotient.

Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a strongly #rg-quotient map. Then clearly f is continuous. Let $f^{-1}(V)$ be an open set in X. Then, $f^{-1}(V)$ is #rg-open set in X. Since f is strongly #rg-quotient map, V is open. Hence, V is #rg-open. Thus f is #rg-quotient map. ■

The converse of the above theorem is not true as seen from the following example.

Example 4.4. Let $X= Y= \{a,b,c,d\}$, $\tau=\{\phi,X, \{a\}, \{b\},\{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. The identity mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ is a #rg-quotient map but not strongly #rg-quotient map, since $f^{-1}\{a,c\}= \{a,c\}$ which is #rg-open in X but $\{a,c\}$ is not open in Y.

Theorem 4.5. Every #rg*-quotient map is strongly #rg-quotient.

Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be #rg*-quotient map. Let U be an open set in Y. Then U is #rg-open in Y. Since f is #rg-irresolute $f^{-1}(U)$ is #rg-open. Thus U is an open set in Y implies $f^{-1}(U)$ is #rg-open in X. If $f^{-1}(V)$ is a #rg-open set in X then f is #rg*-quotient map implies that V is an open set in Y. Hence f is strongly #rg-quotient. ■

Corollary 4.6. Every #rg*- quotient map is #rg-quotient.

Theorem 4.7. Every strongly #rg-quotient map is #rg-open.

Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a strongly #rg-quotient map. Let V be an open set in X. That is, $f^{-1}(f(V))$ is #rg-open in X. Since f is strongly #rg-quoitent, $f(V)$ is open. Hence $f(V)$ is #rg-open and f is a #rg-open set. ■

The converse of the above theorem is not true as seen from the following example.

Example 4.8. Let $X=\{p,q,r,s\}$, $\tau=\{\phi, X, \{p\}, \{q\}, \{p,q\}, \{p,q,r\}\}$, $Y=\{1,2,3\}$ and $\sigma=\{\phi,Y,\{1\},\{2\},\{1,2\}, \{2,3\}\}$. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(p)=f(q)=1$, $f(r)=2$ and $f(s)=3$. Then f is #rg-open but not strongly #rg-quotient, since $f^{-1}(\{2,3\}) = \{r,s\}$ which is not #rg-open in X.

Definition 4.9. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a surjective map. Then f is called strongly #rg*-quotient if a set U is #rg-open in Y if and only if $f^{-1}(U)$ is #rg-open in X.

Example 4.10. Let $X=\{p,q,r,s\}$, $\tau=\{\phi, X, \{p\}, \{q\}, \{p,q\}, \{p,q,r\}\}$, $Y=\{1,2,3\}$ and $\sigma=\{\phi,Y,\{1\},\{2\},\{1,2\}\}$. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(p)=f(q)=1$, $f(r)=2$ and $f(s)=3$. Then clearly $f^{-1}(U)$ is #rg-open iff U is #rg- open. Hence f is a strongly #rg*-quotient map.

Theorem 4.11. Every #rg*-quotient map is strongly #rg*- quotient.

Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be #rg*- quotient map. Let U bea #rg-open set in Y. Since f is #rg-irresolute, $f^{-1}(U)$ is #rg-open. Let $f^{-1}(U)$ be #rg-open in X. Since f is #rg*-quotient, it follows that U is open. Hence U is #rg-open and $f\,a$ is strongly #rg*- quotient map. ■

Theorem 4.12. Every strongly #rg*-quotient map is #rg-quotient.

Proof: Let f:$(X,\tau) \to (Y,\sigma)$ be strongly #rg*-quotient map. Let V be an open set in Y. Then V is a #rg-open set. Since f is strongly #rg*- quotient, $f^{-1}(V)$ is #rg-open. Hence f is #rg-continuous. Let $f^{-1}(V)$ be an open set in X. Then $f^{-1}(V)$ is #rg-open in X. Hence V is #rg-open and f is a #rg-quotient map. ■

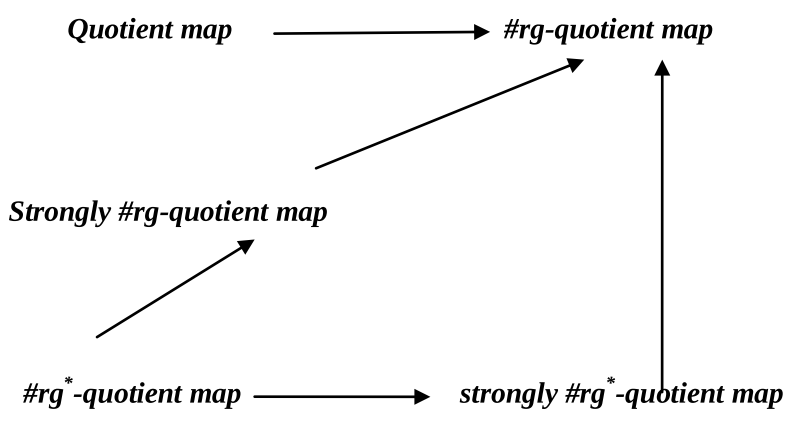
Corollary 4.13. Every #rg*-quotient map is #rg-quotient.

Proof: The proof is the consequence of the fact that #rg*-quotient map is strongly #rg*-quotient and Theorem 4.11.

Theorem 4.14. Let f:$(X,\tau) \to (Y,\sigma)$ be strongly #rg*- quotient map and Y is $T_{\#rg}$- Space. Then f is strongly #rg-quotient map.

Proof: Let U be an open set in Y. Then U is #rg-open in Y. Since f is strongly #rg*-quotient, $f^{-1}(U)$ is #rg-open. Let $f^{-1}(U)$ be #rg-open in X. Then U is #rg-open in Y. Since Y is $T_{\#rg}$- Space, U is open in Y and hence f is a strongly #rg-quotient map. ■

Remark 4.15. From the above results we have the following diagram where $A \to B$ represent A implies B but not conversely.



5 Properties of #rg-quotient maps

Theorem 5.1. Let f:$(X,\tau) \to (Y,\sigma)$ be an open surjective #rg-irresolute map and g: $(Y,\sigma) \to (Z,\eta)$ be a #rg-quotient map. Then their composition $g \circ f = (X,\tau) \to (Z,\eta)$ is a #rg-quotient map.

Proof: Let U be any open set in (Z,η). Since g is #rg-quotient, $g^{-1}(U)$ is #rg-open in Y. Since f is #rg-irresolute, $f^{-1}(g^{-1}(U))$ is a #rg-open set in X. This implies that $(g \circ f)^{-1}(U)$ is #rg-open. Thus $g \circ f$ is #rg-continuous. Let $(g \circ f)^{-1}(U)$ be open in X. That is, $f^{-1}(g^{-1}(U))$ is open in X. Since f is an open map, $f(f^{-1}(g^{-1}(U)))$ is an open set in Y. Hence $g^{-1}(U)$ is open in Y. Since g is #rg-quotient map, U is #rg-open in Z and hence $g \circ f$ is #rg-quotient. ■

Theorem 5.2. Let $f:(X,\tau) \to (Y,\sigma)$ be a #rg-open, surjective and #rg-irresolute map and $g:(Y,\sigma) \to (Z,\eta)$ be a strongly #rg-quotient map. Then $g \circ f$ is strongly #rg-quotient.

Proof: Let U be an open set in Z. Then U is #rg-open. Since g is a strongly quotient map, $g^{-1}(U)$ is #rg-open in Y. Then $f^{-1}(g^{-1}(U))$ is #rg-open in X (Since f is #rg-irresolute). Hence $(g \circ f)^{-1}(U)$ is #rg-open in Y. Let $(g \circ f)^{-1}(U)$ be #rg-open in X. That is, $f^{-1}(g^{-1}(U))$ is #rg- open in X. Since f is #rg-open map, $f(f^{-1}(g^{-1}(U)))$ is #rg-open in Y and hence $g^{-1}(U)$ is #rg-open in Y. Since g is a strongly #rg-quotient map, U is open in Z and hence $g \circ f$ is strongly #rg-quotient. ■

Theorem 5.3. If $h : (X,\tau) \to (Y,\sigma)$ is a #rg-quotient map and $g: (X,\tau) \to (Z,\eta)$ is a continuous map that is constant on each set $h^{-1}(y)$, for $y \in Y$, then g induces a #rg-continuous map $f:(Y,\sigma) \to (Z,\eta)$ such that $f \circ h = g$.

Proof: The set $g(h^{-1}(y))$ is a one point set in Z, since g is constant on $h^{-1}(y)$,for each $y \in Y$. If $f(y)$ denotes this point, then it is clear that f is well defined and for each $x \in X$, $f(h(x)) = g(x)$. We claim that f is #rg-continuous. Let U be any open set in Z. Since g is continuous, $g^{-1}(U)$ is open in X. That is $h^{-1}(f^{-1}(U))$ is open in X. Since h is #rg-quotient map, $f^{-1}(U)$ is #rg-open and hence f is #rg-continuous.

■

Theorem 5.4. Let $f:(X,\tau) \to (Y,\sigma)$ be a #rg-quotient map where X and Y are $T_{\#rg}$-space. Then $g:(Y,\sigma) \to (Z,\eta)$ is strongly #rg-continuous if and only if $g \circ f = (X,\tau) \to (Z,\eta)$ is strongly #rg-continuous.

Proof: Let $g:(Y,\sigma) \to (Z,\eta)$ be strongly #rg-continuous and U be a #rg-open set in Z. Since g is strongly #rg-continuous, $g^{-1}(U)$ is open in Y. Then $f^{-1}(g^{-1}(U))$ is #rg-open in X (since f is #rg-quotient). Thus $(g \circ f)^{-1}(U)$ is #rg-open in X. Since X is $T_{\#rg}$ -Space, $(g \circ f)^{-1}(U)$ is open in X and hence $g \circ f$ is strongly #rg-continuous.

Conversely, suppose that $g \circ f$ is strongly #rg-continuous. Let V be a #rg-oopen set in Z. Since $g \circ f$ is strongly #rg-continuous, $(g \circ f)^{-1}$ is open in X. That is, $f^{-1}(g^{-1}(U))$ is open in X. Since f is #rg-quotient, $g^{-1}(V)$ is #rg-open in Y . Y is $T_{\#rg}$ -Space, implies that $g^{-1}(V)$ is open in Y. Hence, g is strongly #rg-continuous. ■

Theorem 5.5. Let $f: (X,\tau) \to (Y,\sigma)$ be surjective #rg-open and #rg-irresolute and $g:(Y,\sigma) \to (Z,\eta)$ be a #rg*-quotient map then $g \circ f = (X,\tau) \to (Z,\eta)$ is #rg^{*-}quotient.

Proof: Let V be a #rg-open set in Z. Then $g^{-1}(V)$ is #rg-open in Y (since g is #rg*-quotient map). Since f is #rg-irresolute, $f^{-1}(g^{-1}(V))$ is #rg-open in X. That is, $(g \circ f)^{-1}(V)$ is #rg-open in X. Hence $g \circ f = (X,\tau) \to (Z,\eta)$ is #rg-irresolute. Let $(g \circ f)^{-1}(V)$ be #rg-open in X. That is, $f^{-1}(g^{-1}(V))$ is #rg-open in X. Since f is #rg-open map, $f(f^{-1}(g^{-1}(V)))= g^{-1}(V)$ is #rg-open in Y. Since g is a #rg*-quotient map, V is open and hence $g \circ f$ is #rg*-quotient. ■

Theorem 5.6. Let $f:(X,\tau) \to (Y,\sigma)$ be a strongly #rg-quotient map and $g: (Y,\sigma) \to (Z,\eta)$ be a #rg*-quotient map and Y be a $T_{\#rg}$ Space. Then $g \circ f = (X,\tau) \to (Z,\eta)$ is #rg^{*-}quotient.

Proof: Let V be a #rg-open set in Z. Then $g^{-1}(V)$ is #rg-open in Y (since g is #rg*-quotient map). Since Y is a $T_{\#rg}$ Space, $g^{-1}(V)$ is an open set in Y. Since f is strongly #rg-quotient, $f^{-1}(g^{-1}(V))$ is #rg-open in X. That is, $(g \circ f)^{-1}(V)$ is #rg-open in X and hence $g \circ f = (X,\tau) \to (Z,\eta)$ is #rg-irresolute. Let $(g \circ f)^{-1}(V)$ be a #rg-open set in X. That is, $f^{-1}(g^{-1}(V))$ is #rg-open in X. This implies $g^{-1}(V)$ is open in Y. Hence $g^{-1}(V)$ is #rg-open set. Since g is #rg*-quotient, V is open and hence $g \circ f$ is #rg*-quotient map. ■

Theorem 5.7. The composition of two #rg*-quotient maps is #rg*-quotient.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ and $g: (Y,\sigma) \to (Z,\eta)$ be two #rg*-quotient maps. Let U be a #rg-open set in Z. Then $g^{-1}(U)$ is a #rg-open set in Y. Since f is a #rg*-quotient map, $f^{-1}(g^{-1}(U))$ is a #rg-open set in X. That is, $(g \circ f)^{-1}(U)$ is #rg-open in X. Hence $g \circ f = (X,\tau) \to (Z,\eta)$ is #rg-irresolute. Let $(g \circ f)^{-1}(V)$ be a #rg-open set in X. Then $f^{-1}(g^{-1}(V))$ is #rg-open in X. This implies $g^{-1}(V)$ is open in Y and hence $g^{-1}(V)$ is #rg-open. Since g is a #rg*-quotient map, V is open. Hence $g \circ f$ is #rg*-quotient map. ■

References

[1] I. Arockiarani, *Studies on generalizations of generalized closed sets and maps in topological spaces*, Ph.D Thesis, 1997, Bharathiar University, Coimbatore.

[2] S.P. Arya and R.Gupta,*On strongly continuous mappings*, Kyungpook Math., J. 14(1974), 131-143.

[3] K. Balachandran, P.Sundaram and H. Maki, *On generalized continuous maps in topological spaces*, Mem. I ac Sci. Kochi Univ. Math., 12(1991), 5-13.

[4] S.S. Benchalli and R.S. Wali, *On RW-Closed sets in topological spaces*, Bull. Malays. Math. Sci. Soc(2) 30(2)(2007)- 99 – 110.

[5] P. Bhattacharyya and B.K. Lahiri, *Semi-generalized closed sets in topology*, Indian J. Math., 29(1987), 376-382.

[6] D.E. Cameron, *Properties of S-closed spaces*, Proc. Amer Math. Soc. 72(1978), 581–586.

[7] R. Devi, K.Balachandran and H.Maki, *On generalized α-continuous maps*, Far. East J. Math. Sci. Special Volume, part 1(1997), 1-15.

[8] Y. Gnanambal, *On generalized preregular closed sets in topological spaces,* Indian J. Pure App. Math. 28(1997), 351–360.

[9] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly,70(1963), 36–41.

[10] N. Levine, *Generalized closed sets in topology,* Rend. Circ. Mat. Palermo 19(1970), 89–96.

[11] A.S. Mashhour, M.E. Abd. El-Monsef and S.N. El-Deeb , *On pre continuous mappings and weak pre-continuous mappings*, Proc Math, Phys. Soc. Egypt, 53(1982), 47–53.

[12] J.R. Munkres, *Topology,* A first Course, Fourteenth Indian Reprint.

[13] N. Nagaveni, *Studies on Generalizations of Homeomorphisms in Topological Spaces*, Ph.D. Thesis 1999, Bharathiar University, Coimbatore.

[14] N. Palaniappan, and K.C. Rao, *Regular generalized closed sets* , Kyungpook Math. J. 33(1993), 211–219.

[15] M. Stone, *Application of the theory of Boolean rings to general topology,* Trans. Amer. Math. Soc. 41(1937), 374–481.

[16] S. Syed Ali Fathima.and M. Mariasingam, *On #regular generalized closed sets in topological spaces*, International journal of mathematical archive-2(11)(2011), 2497 – 2502.

[17] S. Syed Ali Fathima and M. Mariasingam, *On #RG-Continuous and #RG-irresolute functions*,Journal of Advanced Studies in Topology, 3(4)(2012), 28-33.

[18] S. Syed Ali Fathima and M. Mariasingam, *#RG- Homeomorphisms in topological spaces*, International Journal of Engineering Research and Technology, 1(5)(2012).

[19] S. Syed Ali Fathima and M. Mariasingam, *On contra #RG-Continuous functions*, International journal of Modern Engineering Research, 3(2)(2013), 939-943.

[20] R.S. Wali, *Some topics in general and fuzzy topological spaces*, Ph.D., Thesis, 2006, Karnatak University, Karnataka.

[21] V. Zaitsav , *On certain classes of topological spaces and their bicompactifications*, Dokl. Akad. Nauk SSSR178(1968): 778-779.