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$g^{\#}p^{\#}$-closed sets in topological spaces

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Abstract

In this paper, we introduce $g^{\#}p^{\#}$- closed sets of a topological space and study their properties.

Keywords: $g^{\#}$- closed set, $g^{\#}p^{\#}$-closed set.

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1 Introduction

N. Levine [10] introduced the class of g-closed sets. M.K.R.S. Veera kumar [19, 20, 21] introduced several generalized closed sets namely, $g^{\#}$-closed sets , g^{*}-closed sets , g^{*} p- closed sets and their properties. The authors [18] have already introduced $g^{\#}$p-closed sets and study their properties. In this paper we have introduce $g^{\#}p^{\#}$-closed sets and study their properties.

2 Preliminaries

Throughout this paper (X, τ) (or X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1. A subset A of a space (X,τ) is called

(i) a preclosed [14] set if $cl(int(A)) \subseteq A$.

(ii) a semi-open set [11] if $A \subseteq cl(int(A))$ and a semi-closed [11] set if $int(cl(A)) \subseteq A$.

(iii) an α-open set [16] if $A \subseteq int(cl(int(A)))$ and an α-closed set [15] if $cl(int(cl(A))) \subseteq A$.

(iv) a semi-preclosed set [2] (=β-closed [1]) if $int(cl(int(A))) \subseteq A$.

(v) a regular open set [9] if $A=int(cl(A))$ and a regular closed [9] set if $cl(int(A))=A$.

The semi-closure (α-closure) of a subset A of (X,τ) is denoted by scl(A) (αcl(A)) and is the intersection of all semi-closed (α-closed) sets containing A.

Definition 2.2. A subset A of a space (X, τ) is called

(i) a g^*-closed set [19] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is g-open in (X,τ).

(ii) semi-generalized closed (sg-closed) set [4] if $scl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi-open in (X,τ).

(iii) a generalized semi-closed (gs-closed) set [3] if $scl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ).

(iv) an α-generalized closed (αg-closed) set [12] if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ).

(v) a generalized α-closed (gα-closed) set [13] if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is α-open in (X,τ).

(vi) a generalized semi-preclosed (gsp-closed) set [6] if $spcl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ).

(vii) a generalized preregular closed (gpr-closed) set [8] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is regular open in (X,τ).

(viii) a regular generalized closed (rg-closed) set [17] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is regular open in (X,τ).

(ix) a $g^\#$-closed set [20] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is αg-open in (X,τ).

(x) a $g^\#$- pre closed [18] ($g^\#$p-closed) set if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is $g^\#$-open in (X,τ).

(xi) a g*- pre closed [21] (g*p-closed) set if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is g-open in (X,τ).

(xii) locally closed set [5] if it is the intersection of an open set and a closed set in (X,τ).

3 Basic properties of $g^\# p^\#$-closed sets

Now we introduce the following definitions.

Definition 3.1. A subset A of a space (X, τ) is called a **$g^\# p^\#$-closed** set if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is $g^\#$-open in (X,τ) **.**

Theorem 3.2. In a topological space (X, τ),

(i) Every closed set is $g^\# p^\#$-closed.

(ii) Every $g^\# p^\#$-closed set is αg-closed.

(iii) Every $g^\# p^\#$-closed set is gs-closed.

(iv) Every $g^\# p^\#$-closed set is gsp-closed.

(v) Every $g^\# p^\#$-closed set is $g^\#$p-closed.

(vi) Every $g^\# p^\#$-closed set is gpr-closed.

(vii) Every g*-closed set is $g^\# p^\#$-closed.

(viii) Every $g^\#$-closed set is $g^\# p^\#$-closed.

Proof. (i) It follows from the fact that $cl(A)=A$ for any closed set A of (X, τ) **.**

(ii) Since every open set is $g^\#$-open and $\alpha cl(A)\subseteq cl(A)$ for any subset A of (X, τ), (ii) follows.

(iii) is the consequence of the fact that every open set is $g^{\#}$-open and scl(A) $\subseteq$cl(A) for any subset A of (X, τ).

(iv) follows from the fact that every open set is $g^{\#}$-open and spcl(A) $\subseteq$cl(A) for any subset A of (X, τ).

(v) Since pcl(A) $\subseteq$cl(A) for any subset A of (X,τ), (v) follows.

(vi) Every regular open set is $g^{\#}$-open and pcl(A)$\subseteq$cl(A) for any subset A of (X,τ). Hence (vi) follows.

(vii) is the consequence of the fact that every $g^{\#}$- open set is g-open.

(viii) follows from the fact that every $g^{\#}$- open set is αg-open. ■

The converses of Theorem 3.2 need not be true as seen from the following examples.

Example 3.3. Consider the space (X, τ) where X= {a, b, c} and τ={X,Φ,{a, b}}. Let A = {b, c}, then A is $g^{\#}p^{\#}$-closed but not a closed set.

Example 3.4. Consider the space (X, τ) where X={a, b, c} and τ={X,Φ,{c},{b, c}}. Let A = {b}, then A is αg-closed, gs-closed, gsp-closed and $g^{\#}$p-closed sets but not a $g^{\#}p^{\#}$-closed set.

Example 3.5. Consider the space (X, τ) where X={a, b, c} and τ={X,Φ,{c}}. Let A={c}, then A is gpr-closed but not a $g^{\#}p^{\#}$-closed set.

Example 3.6. Consider the space (X,τ) given in Example 3.5. Let A={a}, then A is $g^{\#}p^{\#}$-closed but not a g^{*}- closed set and $g^{\#}$-closed set.

Thus the class of $g^{\#}p^{\#}$-closed sets properly contains the class of closed sets, g^{*}-closed sets, $g^{\#}$-closed sets and is properly contained in the classes of αg-closed set, gs-closed sets, gsp-closed sets, gpr-closed sets and $g^{\#}$p-closed sets.

Remark 3.7. $g^{\#}p^{\#}$-closed sets are independent of semiclosed set, α-closed set, semipre-closed set, sg-closed set, gα-closed set, preclosed set, rg-closed set, and g^{*}p-closed sets as it can be seen in the following examples.

Example 3.8. Consider the space (X,τ) given in Example 3.4. Let A={a, c}. Then A is $g^{\#}p^{\#}$-closed but not a semi-closed , α-closed, semi-preclosed, sg-closed, gα-closed and preclosed sets. Also the set B={b} is semi-closed, α-closed, semi-preclosed, sg-closed, gα-closed and preclosed sets but not a $g^{\#}p^{\#}$-closed.

Example 3.9. Consider the space (X,τ) where X={a, b, c} and τ={X,Φ,{a},{b},{a, b}}. Let A={b, c}, then A is $g^{\#}p^{\#}$-closed set but not rg-closed set. Also the set B={a, b} is rg-closed but not a $g^{\#}p^{\#}$-closed set.

Example 3.10. Consider the space (X,τ) given in Example 3.4. Let A= {b}, then A is g^{*}p-closed but not a $g^{\#}p^{\#}$- closed set.

Example 3.11. Consider the space (X,τ) given in Example 3.5. Let A={b, c}, then A is $g^{\#}p^{\#}$-closed but not a g^{*}p- closed set.

Remark 3.12. Union of two $g^{\#}p^{\#}$-closed set is $g^{\#}p^{\#}$-closed.

Remark 3.13. Intersection of two $g^{\#}p^{\#}$-closed sets need not be $g^{\#}p^{\#}$-closed set as seen in the following example.

Example 3.14. Consider the space (X,τ) in Example 3.5. Let $A=\{b, c\}$ and $B=\{a, c\}$, then A and B are $g^{\#}p^{\#}$-closed sets but $A\cap B$ is not a $g^{\#}p^{\#}$-closed set of (X,τ).

Theorem 3.15. If A is $g^{\#}$-open and $g^{\#}p^{\#}$-closed, then A is a closed set.

The proof is obvious from Definition 2.2.(xi) and 3.1.

Theorem 3.16. If A is a $g^{\#}p^{\#}$-closed set of (X,τ), then cl(A)-A does not contain any non-empty $g^{\#}$-closed set.

Proof. Let F be a $g^{\#}$-closed set contained in cl(A)-A. Then $A\subseteq X-F$ and $X-F$ is a $g^{\#}$-open set of (X,τ). Since A is $g^{\#}p^{\#}$-closed, cl(A) $\subseteq X-F$. This implies $F\subseteq X$-cl(A). Then $F\subseteq(X\text{-cl}(A))\cap(\text{cl}(A)\text{-}A)\subseteq(X\text{-cl}(A))\cap\text{cl}(A)=\Phi$. Therefore, $F=\Phi$. ■

Theorem 3.17. If A is a $g^{\#}p^{\#}$-closed set of (X,τ) and $A\subseteq B\subseteq$cl(A),then B is also a $g^{\#}p^{\#}$-closed set of (X,τ).

Proof. Let U be an $g^{\#}$-open set of (X,τ) such that $B\subseteq U$. Then $A\subseteq U$. Since A is $g^{\#}p^{\#}$-closed, cl(A) $\subseteq U$. Since $B\subseteq$cl(A), cl(B) $\subseteq$cl(cl(A))=cl(A) $\subseteq U$. Therefore, B is also a $g^{\#}p^{\#}$-closed set. ■

Theorem 3.18. Let A be a locally closed set of (X,τ). Then A is closed iff A is $g^{\#}p^{\#}$-closed.

Proof. Let U be a $g^{\#}$-open set of (X,τ) such that $A\subseteq U$. Since A is closed, cl(A)=A, cl(A) $\subseteq U$. Hence A is $g^{\#}p^{\#}$-closed.

Conversely, suppose A is $g^{\#}p^{\#}$-closed. By Proposition 5.1.3.3 of Bouraki [5], AU(X-cl(A)) is open in (X,τ), since A is locally closed. Now AU(X-cl(A)) is a $g^{\#}$- open set of (X,τ) such that $A\subseteq$ AU(X-cl(A)). Since A is $g^{\#}p^{\#}$-closed, then cl(A)$\subseteq$ AU(X-cl(A)). But cl(A)$\cap$(X-cl(A))=Φ. Thus we have cl(A)$\subseteq A$. Trivially A $\subseteq$cl(A). Hence A is a closed set. ■

Corollary 3.19. In a submaximal space (X,τ), every $g^{\#}p^{\#}$-closed set is closed.

Proof. Ganster and Reilly [7] proved that (X,τ) is submaximal iff every subset of X is locally closed. By Theorem 3.18 every $g^{\#}p^{\#}$-closed set is closed. ■

Theorem 3.20. Let A be a $g^{\#}p^{\#}$-closed set of a topological space (X,τ). Then

(i) pcl(A) is $g^{\#}p^{\#}$-closed set.

(ii) If A is regular open, then scl(A) is $g^{\#}p^{\#}$-closed set.

Proof. First we note that for a subset A of (X,τ), scl(A)=$A\cup$int(cl(A)) and pcl(A)=$A\cup$cl(int(A)).
(i) Since cl(int(A)) is a closed set,then A and cl(int(A)) are $g^{\#}p^{\#}$-closed sets. By Theorem 3.12, $A\cup$ cl(int(A)) is also a $g^{\#}p^{\#}$-closed set.

(ii) Since A is regular open, A = int(cl(A)). Then, scl(A) = A $\cup$ int(cl(A)) = A. Thus scl(A) is $g^{\#}p^{\#}$-closed set. ■

The converse of Theorem 3.20 need not be true as seen from the following examples.

Example 3.21. Consider the space (X,τ) given in Example 3.4. Let $A=\{c\}$, then A is not a $g^{\#}p^{\#}$-closed but pcl(A)=X is $g^{\#}p^{\#}$- closed set.

Example 3.22. Consider the space (X,τ) given in Example 3.3. Let $A=\{b\ ,c\}$. Cleary A is not regular open but A is $g^{\#}p^{\#}$-closed and scl(A) = X is $g^{\#}p^{\#}$- closed set.

The following diagram shows the relationships of $g^{\#}p^{\#}$-closed sets with other sets.

$A \rightarrow B(A \not\leftrightarrow B)$ represents A implies B but not conversely (A and B are independent).

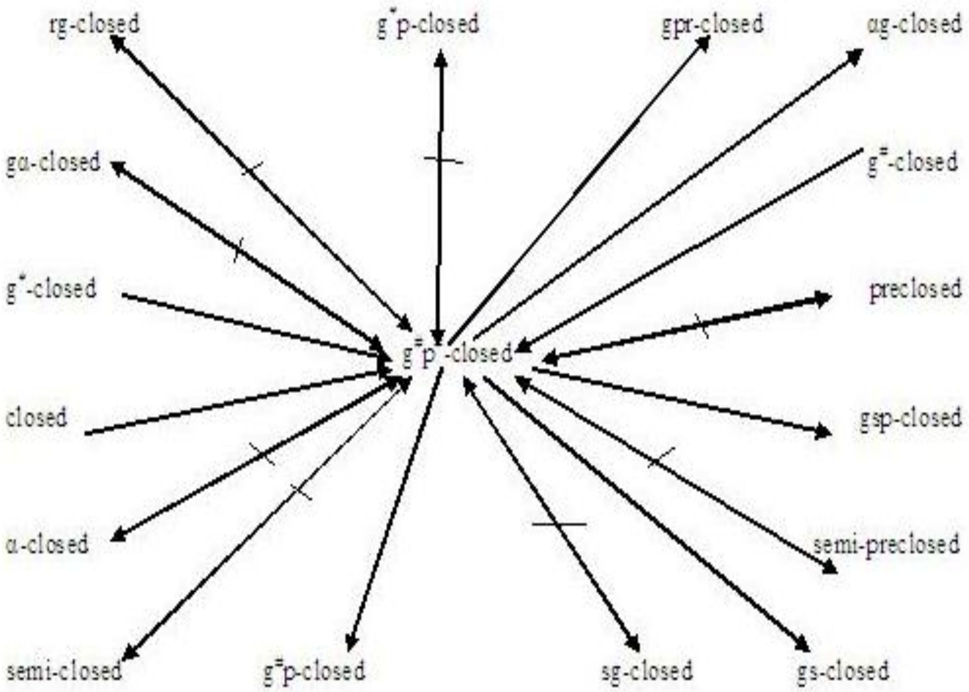

Figure 1: *Relationships of $g^{\#}p^{\#}$-closed sets with other sets.*

References

[1] M.E. Abd EI-Monsef, S.N. EI-Deeb and R.A. Mahmoud, *β-open sets and β-continuous mappings*, Bull. Assiut Univ., 12(1983), 77-90.

[2] D. Andrijevic, *Semi-preopen sets*, Mat. Vesnik, 38(1) (1986), 24-32.

[3] S.P. Arya and T. Nour, *Characterizations of s-normal spaces*, Indian J. Pure Appl. Math., 21(8) (1990), 717-719.

[4] P. Bhattacharya and B.K. Lahiri, *Semi-generalized closed sets in topology*, Indian J. Math., 29(3) (1987), 375-382.

[5] N. Bourbaki, *General Topology Part I*, Addision-Wesley, Reading, Mass., 1966.

[6] J. Dontchev, *On generalizing semi-preopen sets*, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 16(1995), 35-48.

[7] M. Ganster and I.L. Reilly, *Locally closed sets and LC-continuous functions*, Internat. J. Math., & Math. Sci., 12(3)(1989),417-424.

[8] Y. Gnanambal, *On generalized pre regular closed sets in topological spaces*, Indian J. Pure. Appl. Math., 28(3) (1997), 351-360.

[9] C. Kuratowski, *Topology I*, Hafner, New York,1958.

[10] N. Levine, *Generalized closed sets in topology*, Rend. Circ. Math. Palermo, 19(2)(1970), 89-96.

[11] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70(1963), 36-41.

[12] H. Maki, R. Devi and K. Balachandran, *Associated topologies of generalized α- closed sets and α-generalized closed sets*, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 15(1994), 51-63.

[13] H. Maki, R. Devi and K. Balachandran, *Generalized α-closed sets in topology*, Bull. Fukuoka Univ. Ed. Part III, 42(1993), 13-21.

[14] A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deeb, *On pre-continuous and weak pre-continuous mappings*, Proc. Math. and Phys. Soc. Egypt, 53(1982),47-53.

[15] A.S. Mashhour, I.A. Hasanein and S.N. El-Deeb, *α-continuous and α-open mappings*, Acta Math Hung., 41(3-4)(1983), 213-218.

[16] O. Njastad, *On some classes of nearly open sets*, Pacific J. Math., 15 (1965), 961-970.

[17] N. Palaniappan and K.C. Rao, *Regular generalized closed sets*, Kyungpook Math.J., 33(1993), 211-219.

[18] S. Pious Missier, K. Alli and A. Subramanian, $g^{\#}$p-*closed set in topological spaces*, International Journal of Mathematical Archive, 4(1) (2013), 176-181

[19] M.K.R.S.Veera kumar, *Between closed sets and g-closed sets*, Mem. Fac. Sci., Kochi Univ. Ser. A. Math., 21(2000), 1-19.

[20] M.K.R.S. Veera kumar, $g^{\#}$-*closed sets in topological spaces*, Mem. Fac. Sci. Kochi Univ. Ser. A. Math 24(2003), 1-13.

[21] M.K.R.S. Veera kumar, g*-*preclosed sets*, Acta ciencia Indica (Maths) Meerut XXVIII(M)(1) 2002, 51-60.