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On felicitous labelings of $P_{r,2m+1}$, P_r^{2m+1} and $C_n \times P_m$

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Abstract

A simple graph G is called felicitous if there exists a one-to-one function $f : V(G) \to \{0,1,2, . . . , q\}$ such that the set of induced edge labels $f^*(uv) = (f(u) + f(v)) \pmod q$ are all distinct. In this paper we show that $P_{r,2m+1}$, P_r^{2m+1} and $C_n \times P_m$ are felicitous graphs.

Keywords: Labeling, felicitous labeling.

AMS Subject Classification(2010): 05C78.

1 Introduction

In this paper we consider only simple graphs. For notation and terminology, we refer to [2]. Lee, Schmeichel and Shee [7] introduced the concept of a felicitous graph as a generalization of a harmonious graph. A graph G with q edges is called *harmonious* if there is an injection $f : V(G) \to Z_q$, the additive group of integers modulo q such that when each edge xy of G is assigned the label $(f(x) + f(y)) \pmod q$, the resulting edge labels are all distinct. A *felicitous* labeling of a graph G, with q edges is an injection $f : V(G) \to \{0,1,2, . . . ,q\}$ so that the induced edge labels $f^*(xy) = (f(x) + f(y)) \pmod q$ are distinct. Clearly, a harmonious graph is felicitous. An example of a felicitous graph which is not harmonious is the graph $K_{m,n}$, where $m, n > 1$.

Throughout this paper, f denotes a 1-1 function from $V(G)$ to a subset of the set of non-negative integers and for any edge $e = xy \in E(G)$, $f^*(e) = f(x) + f(y)$.

In [4, 5, 6], Kathiresan introduced new classes of graphs denoted by $P_{a,b}$ and P_a^b and discussed the magic labeling of $P_{a,b}$ [5]. In [8], the gracefulness of $P_{a,b}$ was discussed. It motivates us to discuss the felicitousness of the graphs $P_{a,b}$ and P_a^b.

2 Definitions and basic results

Definition 2.1. *Let u and v be two fixed vertices. We connect u and v by means of $b \geq 2$ internally disjoint paths of length $a \geq 2$ each. The resulting graph embedded in a plane is denoted by $P_{a,b}$. Let $v_o^i, v_1^i, v_2^i, . . . , v_a^i$ be the vertices of the i^{th} copy of the path of length a, where $i = 1, 2, 3, . . . ,b$. $v_o^i = u$ and $v_a^i = v$ for all i.*

We observe that the graph $P_{a,b}$ has $(a - 1)b + 2$ vertices and ab edges.

Definition 2.2. *Let a and b be integers such that $a \geq 2$ and $b \geq 2$. Let $y_1, y_2, . . . , y_a$ be the fixed vertices. We connect the vertices y_i and y_{i+1} by means of b internally disjoint paths P_i^j of length i+1* *each,* $1 \le i \le a-1$ *and* $1 \le j \le b$. *Let* $y_i, x_{i,j,1}, x_{i,j,2}, \ldots, x_{i,j,i}, y_{i+1}$ *be the vertices of the path* P_i^j, $1 \le i \le a-1$ *and* $1 \le j \le b$. *The resulting graph embedded in a plane is denoted by* P_a^b, *where* $V(P_a^b) = \{y_i : 1 \le i \le a\} \cup \bigcup_{i=1}^{a-1} \bigcup_{j=1}^{b} \{x_{i,j,k} : 1 \le k \le i\}$ *and* $E(P_a^b) = \bigcup_{i=1}^{a-1} \{y_i x_{i,j,1} : 1 \le j \le b\} \cup \bigcup_{i=1}^{a-1} \bigcup_{j=1}^{b} \{x_{i,j,k} x_{i,j,k+1} : 1 \le k \le i-1\} \cup \bigcup_{i=1}^{a-1} \{x_{i,j,i} y_{i+1} : 1 \le j \le b\}$.

We observe that the number of vertices of the graph P_a^b is $\frac{ba(a-1)}{2} + a$ and the number of edges is $\frac{b(a-1)(a+2)}{2}$.

Definition 2.3. *A subgraph H of a graph G is said to be an even subgraph of G, if the degree of every vertex of H is even in H.*

Result 2.4. [1] *An even subgraph of a felicitous graph with an even number of edges contains an even number of odd labelled edges.*

Result 2.5. [1] *No even graph with 4n + 2 edges is felicitous.*

Lemma 2.6. [1] *Let G be a graph with an odd number of edges and let* $f : V(G) \to \{0, 1, 2, \ldots, q\}$ *be an odd edge labeling of G. Then, f is a felicitous labeling for G.*

Proof. As f is an odd edge labeling of a graph G with odd number of edges, $f(E(G)) = \{1, 3, 5, \ldots, 2q-1\}$. After taking mod q, $f(E(G)) = \{1, 2, 3, \ldots, q\}$. So, f becomes felicitous labeling of G.

Remark 2.7. *It is observed that as in Result 2.5, most of the even graphs are not felicitous. So, finding felicitous graphs with even number of edges are very difficult.*

3 Main Results

Theorem 3.1. $P_{r,\,2m+1}$ *is a felicitous graph for all values of m and for odd values of r.*

Proof : Let u and v be the origin and the terminal vertices of the $(2m + 1)$ internally disjoint paths of length r in $P_{r,\,2m+1}$. Let $v_0^i, v_1^i, v_2^i, \ldots, v_r^i$ be the vertices of the i^{th} copy of the path, where $i = 1, 2, 3, \ldots, 2m + 1$, $v_0^i = u$ and $v_r^i = v$ for all i. The number of vertices of the graph $P_{r,\,2m+1}$ is $(r-1)(2m+1)+2$ and the number of edges is $(2m + 1)r$.

It is enough to show that $P_{r,\,2m+1}$ admits odd edge labeling.

Define f on the vertex set of $P_{r,\,2m+1}$ as follows:

$f(u) = 0 \quad f(v) = (2m+1)r$

For $1 \le j \le \frac{r-1}{2}$,

$f\left(v_{2j-1}^i\right) = (4m+2)(j-1) + (2i-1)$, $1 \le i \le 2m+1$,

$$f\left(v_{2j}^i\right) = \begin{cases} (6m+2) + (4m+2)(j-1) - 4(i-1), & 1 \le i \le m+1 \\ 6m + (4m+2)(j-1) - 4(i-(m+2)), & m+2 \le i \le 2m+1 \end{cases}$$

Let $E_1 = \{ v_o^i\, v_1^i : 1 \le i \le 2m + 1\}$.

$E_2 = \{v_j^{m+1}\, v_{j+1}^{m+1},\ v_j^{m}\, v_{j+1}^{m}, \ldots, v_j^{1}\, v_{j+1}^{1}, v_j^{2m+1}\, v_{j+1}^{2m+1}, v_j^{2m}\, v_{j+1}^{2m}, \ldots, v_j^{m+2}\, v_{j+1}^{m+2} : 1 \le j \le r - 2\}$.

$E_3 = \{ v_{r-1}^{m+1}\, v_r^{m+1}, v_{r-1}^{2m+1}\, v_r^{2m+1}, v_{r-1}^{m}\, v_r^{m}, v_{r-1}^{2m}\, v_r^{2m}, \ldots, v_{r-1}^{2}\, v_r^{2},\ v_{r-1}^{m+2}\, v_r^{m+2}, v_{r-1}^{1}\, v_r^{1} \}$.

The labels of the edges in E_1 are $2i - 1$, $1 \le i \le 2m + 1$.

For $1 \le j \le r - 2$, the labels of the edges in E_2 are $2j\,(2m + 1) + 1$, $2j\,(2m + 1) + 3, \ldots, 2(j + 1)(2m + 1) - (2m - 1)$, $2(j + 1)(2m + 1) - (2m - 3), \ldots, 2(j + 1)(2m + 1) - 1$.

The labels of the edges in E_3 are $2(r - 1)(2m + 1) + 1$, $2(r - 1)(2m + 1) + 3, \ldots, 2r(2m + 1) - 1$.

$f(E_1) = \{1, 3, 5, \ldots, 2(2m + 1) - 1\} = \{1, 3, 5, \ldots, 4m + 1\}$.

$f(E_2) = \{2(2m + 1) + 1, 2(2m + 1) + 3, \ldots, 4(2m + 1) - (2m - 1), 4(2m + 1) - (2m - 3), \ldots, 4(2m + 1) - 1, \ldots, 2(r - 2)(2m + 1) + 1, 2(r - 2)(2m + 1) + 3, \ldots, (2(r - 2) + 2)(2m + 1) - 1\} = \{4m + 3, 4m + 5, \ldots, 6m + 5, 6m + 7, \ldots, 8m + 3, \ldots, 2(r - 2)(2m + 1) + 1, 2(r - 2)(2m + 1) + 3, \ldots, 2(r - 1)(2m + 1) - 1\}$.

$f(E_3) = \{2(r - 1)(2m + 1) + 1, 2(r - 1)(2m + 1) + 3, \ldots, 2r(2m + 1) - 1\}$.

Now, $f(E(G)) = f(E_1) \cup f(E_2) \cup f(E_3)$.

$f(E(G)) = \{1, 3, 5, \ldots, 4m + 1, 4m + 3, 4m + 5, \ldots, 6m + 5, 6m + 7, \ldots, 8m + 3, \ldots, 2(r - 2)(2m + 1) + 1, 2(r - 2)(2m + 1) + 3, \ldots, 2(r - 1)(2m + 1) - 1, 2(r - 1)(2m + 1) + 1, 2(r - 1)(2m + 1) + 3, \ldots, 2r(2m + 1) - 1\} = \{1, 3, 5 \ldots, 2q - 1\}$.

Clearly, the above edge labelings are distinct and odd and hence G admits odd edge labeling. Therefore, by Lemma 2.6, $P_{r,\,2m+1}$ is a felicitous graph for all the values of m and for odd values of r. ■

Example 3.2. A felicitous labeling of $P_{7,5}$ is shown in Figure 1.

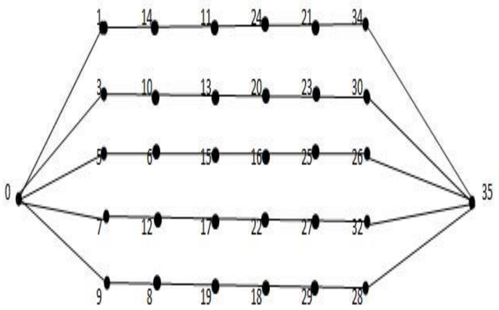

Figure 1: *A felicitous labeling of $P_{7,5}$.*

Corollary 3.3. *$P_{a,\,b}$ is not a felicitous graph when $a \equiv 1 (\mathrm{mod}\ 2)$ and $b \equiv 2 (\mathrm{mod}\ 4)$.*

Proof. The number of edges of $P_{a,b} = ab = (2k+1)(4m+2) = (8km+4k+4m+2) = 4(2km+k+m) + 2 = 4l+2$ where $l = 2km+k+m$ and $l \in Z^+$. Further, the graph $P_{a,b}$ is even. Hence, $P_{a,b}$ is not a felicitous graph when $a \equiv 1 (\text{mod } 4)$ and $b \equiv 2 (\text{mod } 4)$. ■

Theorem 3.4. *P_r^{2m+1} is a felicitous graph for all values of m and $r \equiv 0, 3 (mod\ 4)$.*

Proof. Let $y_1, y_2, \ldots, y_r$ be the fixed vertices. We connect the vertices y_i and y_{i+1} by means of $2m+1$ internally disjoint paths P_i^j of length $i+1$ each, $1 \le i \le r-1$ and $1 \le j \le 2m+1$. Let $y_i, x_{i,j,1}, x_{i,j,2}, \ldots, x_{i,j,i}, y_{i+1}$ be the vertices of the path P_i^j, $1 \le i \le r-1$ and $1 \le j \le 2m+1$. We observe that the number of vertices of the graph P_r^{2m+1} is $\frac{(2m+1)r(r-1)}{2} + r$ and the number of edges is $\frac{(2m+1)(r-1)(r+2)}{2}$.

It is enough to show that P_r^{2m+1} admits odd edge labeling.

Define f on $V(P_r^{2m+1})$ as follows :

$$f(y_i) = \left(\frac{i(i+1)}{2} - 1\right)(2m+1),\ 1 \le i \le r,$$

$$f(x_{1,j,1}) = 2j-1,\quad 1 \le j \le 2m+1,$$

$$f(x_{2,j,2}) = \begin{cases} 5(2m+1)-1-4(j-1), & 1 \le j \le m+1 \\ 5(2m+1)-3-4(j-(m+2)), & m+2 \le j \le 2m+1 \end{cases}$$

For $1 \le j \le 2m+1$,

$$f(x_{i,j,k}) = \begin{cases} f(x_{i-1,j,k}) + (2m+1)i & \text{if } k = 1,2 \text{ and } k+1 \le i \le r\text{-}1; \\ f(x_{i,j,1}) + (k-1)(2m+1) & \text{if } k \text{ is odd}, 3 \le k \le r\text{-}1 \text{ and } k \le i \le r\text{-}1; \\ f(x_{i,j,2}) + (k-2)(2m+1) & \text{if } k \text{ is even}, 4 \le k \le r\text{-}1 \text{ and } k \le i \le r\text{-}1. \end{cases}$$

Let $E_1 = \{y_i x_{i,1,1},\ y_i x_{i,2,1},\ y_i x_{i,3,1}, \ldots, y_i x_{i,2m+1,1} : 1 \le i \le r-1\}$, $E_2 = \{x_{i,m+1,k} x_{i,m+1,k+1},\ x_{i,m,k} x_{i,m,k+1}, \ldots, x_{i,1,k} x_{i,1,k+1},\ x_{i,2m+1,k} x_{i,2m+1,k+1},\ x_{i,2m,k} x_{i,2m,k+1}, \ldots, x_{i,m+2,k}\ x_{i,m+2,k+1} : 2 \le i \le r-1$ and $1 \le k \le i-1\}$, $E_3 = \{x_{i,1,i} y_{i+1},\ x_{i,2,i} y_{i+1},\ x_{i,3,i} y_{i+1}, \ldots, x_{i,2m+1,i} y_{i+1} : 1 \le i \le r-1\}$ and $E_4 = \{x_{i,m+1,i} y_{i+1},\ x_{i,2m+1,i} y_{i+1},\ x_{i,m,i} y_{i+1},\ x_{i,2m,i} y_{i+1}, \ldots, x_{i,2,i} y_{i+1},\ x_{i,m+2,i} y_{i+1},\ x_{i,1,i} y_{i+1} : 1 \le i \le r-1\}$.

The edge labels of P_r^{2m+1} are as follows:

For $1 \le i \le r-1$, the labels of the edges in E_1 are $(i(i+1)-2)(2m+1)+1$, $(i(i+1)-2)(2m+1)+3$, $\ldots$, $(i(i+1)-2)(2m+1)+2(2m+1)-1$.

For $2 \le i \le r-1$ and $1 \le k \le i-1$, the labels of the edges in E_2 are $(i(i+1)-2+2k)(2m+1)+1$, $(i(i+1)-2+2k)(2m+1)+3$, $\ldots$, $(i(i+1)+2k-1)(2m+1)$, $(i(i+1)+2k-1)(2m+1)+2$, $\ldots$, $(i(i+1)+2k)(2m+1)-1$.

For $1 \le i \le r-1$ and $i \equiv 1 (\text{mod } 2)$, the labels of the edges in E_3 are $(i(i+3)-2)(2m+1)+1$, $(i(i+3)-2)(2m+1)+3$, $\ldots$, $(i(i+3)-2)(2m+1)+2(2m+1)-1$.

For $1 \le i \le r-1$ and $i \equiv 0 \pmod 2$, the labels of the edges in E_4 are $(i(i+3)-2)(2m+1)+1$, $(i(i+3)-2)(2m+1)+3$, ... , $(i(i+3)-2)(2m+1)+2(2m+1)-1$ respectively.

Now, $f(E(G)) = f(E_1) \cup f(E_2) \cup f(E_3) \cup f(E_4) = \{1, 3, 5, \ldots, 2(2m+1)-1, \ 4(2m+1)+1, 4(2m+1)+3, \ldots, 4(2m+1)+2(2m+1)-1, \ldots, (r(r-1)-2)(2m+1)+1, (r(r-1)-2)(2m+1)+3, \ldots, (r(r-1)-2)(2m+1)+2(2m+1)-1\} \cup \{6(2m+1)+1, 6(2m+1)+3, \ldots, 7(2m+1), 7(2m+1)+2, \ldots, ((r(r-1)-2)+2(r-2))(2m+1)+1, ((r(r-1)-2)+2(r-2))(2m+1)+3, \ldots, (r(r-1)+2(r-2))(2m+1)-1\} \cup \{2(2m+1)+1, 2(2m+1)+3, \ldots, 2(2m+1)+2(2m+1)-1, \ldots, ((r-1)(r+2)-2)(2m+1)+1, ((r-1)(r+2)-2)(2m+1)+3, \ldots, ((r-1)(r+2)-2)(2m+1)+2(2m+1)-1\}$.

That is, $f(E(G)) = \{1, 3, 5, \ldots, 2(2m+1)-1, 2(2m+1)+1, 2(2m+1)+3, \ldots, 4((2m+1)-1, 4(2m+1)+1, 4(2m+1)+3, \ldots, 6(2m+1)-1, 6(2m+1)+1, 6(2m+1)+3, \ldots, 7(2m+1), 7(2m+1)+2, \ldots, ((r(r-1)-2)+2(r-2))(2m+1)+1, ((r(r-1)-2)+2(r-2))(2m+1)+3, \ldots, (r(r-1)+2(r-2))(2m+1)-1, \ldots, ((r-1)(r+2)-2)(2m+1)+1, ((r-1)(r+2)-2)(2m+1)+3, \ldots, (r-1)(r+2)(2m+1)-1\}$
$= \{1, 3, 5, \ldots, 2q-1\}$.

Clearly, the above edge labelings are distinct and odd and hence G admits odd edge labeling. Therefore, P_r^{2m+1} is a felicitous graph for all values of m and $r \equiv 0, 3 \pmod 4$. ■

Example 3.5. A felicitous labeling of P_4^5 is shown in Figure 2.

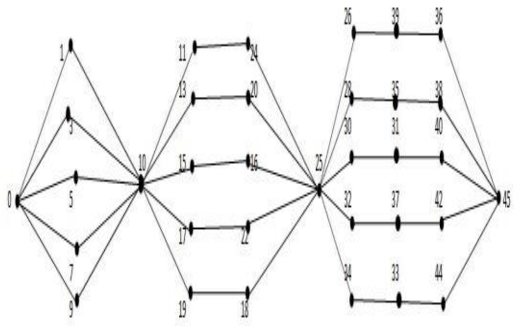

Figure 2: *A felicitous labeling of* P_4^5.

Corollary 3.6. P_a^b *is not felicitous when* $b \equiv 2(mod\ 4)$ *and* (i) $a \equiv 0(mod\ 4)$ *or* (ii) $a \equiv 3(mod\ 4)$.

Proof. (i) Let $a \equiv 0(\text{mod } 4)$ and $b \equiv 2(\text{mod } 4)$.

The number of edges of $P_a^b = \dfrac{b(a-1)(a+2)}{2} = \dfrac{(4k+2)(4m-1)(4m+2)}{2}$

$= \dfrac{2(2k+1)(4m-1)(4m+2)}{2}$

$= 2(8m^2 + 2m - 1)(2k+1)$

$= 2(16m^2k + 8m^2 + 4mk + 2m - 2k - 1)$

$= 4(8m^2k + 4m^2 + 2mk + m - k - 1) + 2 = 4l + 2$ where $l = 8m^2k + 4m^2 + 2mk + m - k - 1$ and $l \in Z^+$.

(ii) Let $a \equiv 3 (\text{mod } 4)$ and $b \equiv 2 (\text{mod } 4)$.

The number of edges of $P_a^{\,b} = \dfrac{b(a-1)(a+2)}{2}$

$= \dfrac{(4k+2)(4m+3-1)(4m+3+2)}{2}$

$= \dfrac{2(2k+1)(4m+2)(4m+5)}{2}$

$= 2(2k + 1)\,(2m + 1)\,(4m + 5)$

$= 2\,(2k + 1)\,(8m^2 + 14m + 5)$

$= 2(16m^2k + 28km + 10k + 8m^2 + 14m + 5)$

$= 4(8m^2k + 4m^2 + 14mk + 7m + 5k + 2) + 2 = 4l + 2$ where $l = 8m^2k + 4m^2 + 14mk + 7m + 5k + 2$ and $l \in Z^+$. ■

Remark 3.7. *Let G be a (p, q) graph. Let f be a felicitous labeling. Define $f_1(uv) = f(u) + f(v)$ for every $uv \in E(G)$. Then $f^*(uv) = f_1(uv)(\text{mod } q)$.*

Theorem 3.8. *$C_n \times P_m$ is felicitous for $m \geq 1$ and $n \equiv 1(\text{mod } 2)$.*

Proof. Case (i): when $n = 3$.

Let $V(C_3 \times P_m) = \{u_{ij} : 1 \leq i \leq 3 \text{ and } 1 \leq j \leq m\}$.

Define $f : V(C_3 \times P_m) \rightarrow \{0,1,2,\ldots,q = 6m - 3\}$ by

$f(u_{i1}) = i - 1,\ 1 \leq i \leq 3;$

$f(u_{2i}) = 3 + i,\ 1 \leq i \leq 2$ and $f(u_{23}) = 3;$

$f(u_{3i}) = 5 + i,\ 1 \leq i \leq 3.$

$$f(u_{ij}) = \begin{cases} f(u_{1(j-1)}) + i, & 5 \leq j \leq m \text{ and } j \equiv 1(\text{mod } 2); \\ f(u_{3(j-1)}) + \sigma_1(i), & 4 \leq j \leq m \text{ and } j \equiv 0(\text{mod } 2); \end{cases} \quad \text{where } \sigma_1 = (1\ 3\ 2).$$

Let $E_1 = \{(u_{2j}u_{1j}),\ (u_{1j}u_{3j}),\ (u_{3j}u_{2j}) : 1 \leq j \leq m \text{ and } j \equiv 1(\text{mod } 2)\}$,

$E_2 = \{(u_{12}u_{32}), (u_{32}u_{22}), (u_{22}u_{12}), (u_{3j}u_{2j}), (u_{2j}u_{1j}), (u_{1j}u_{3j}) : 4 \leq j \leq m \text{ and } j \equiv 0(\text{mod } 2)\}$ and

$E_3 = \{(u_{11}u_{12}), (u_{31}u_{32}), (u_{21}u_{22}), (u_{12}u_{13}), (u_{32}u_{33}), (u_{22}u_{23}), (u_{2j}u_{2(j+1)}), (u_{1j}u_{1(j+1)}), (u_{3j}u_{3(j+1)}) : 3 \leq j \leq m - 1\}$.

Now, $E = E1 \cup E_2 \cup E_3$.

The labels of the edges in E_1 and E_2 are $f_1(E_1) \cup f_1(E_2) = \{6j - 5, 6j - 4, 6j - 3 : 1 \leq j \leq m\}$. The labels of the edges in E_3 are $f_1(E_3) = \{6j - 2, 6j - 1, 6j : 1 \leq j \leq m - 1\}$.

Clearly, $f^*(E(G)) = f_1(E_1) \cup f_1(E_2) \cup f_1(E_3) = \{1, 2, 3, \ldots, 6m - 6, 6m - 5, 6m - 4, 6m - 3\}$. After taking (mod q), $f^*(E(G)) = f_1(E(G))$ (mod q) $= \{0, 1, 2, 3, \ldots, 6m - 5, 6m - 4\}$.

Case (ii): when $n \geq 5$.

Let $V(C_n \times P_m) = \{ u_{ij} : 1 \le i \le n \text{ and } 1 \le j \le m\}$.

Define $f : V(C_n \times P_m) \rightarrow \{0,1,2,\ldots,q = (2m-1)n\}$ by

$f(u_{i1}) = i - 1,\ 1 \le i \le n$.

Throughout this proof, addition being taken modulo n with residues 1, 2, 3, . . . , n.

Let $\sigma_j = \begin{pmatrix} 1 & 2 & 3 & \ldots & n \\ n-j+2 & n-j+3 & n-j+4 & \ldots & n-j+1 \end{pmatrix}$

$f(u_{\sigma_j(i),j}) = n(j-2) + (n-1) + i,\ 1 \le i \le n$ and $2 \le j \le m$.

The labels of the edges are,

For $1 \le j \le m$,

$$f_1\left(u_{\sigma_j(i),j}u_{\sigma_j(i+1),j}\right) = \begin{cases} n(2j-2)+(2i-1), & 1 \le i \le n-1 \\ n(2j-2)+(n-1), & i = n \end{cases}$$

For $2 \le j \le m-1$,

$f_1\left(u_{\sigma_j(i),j}u_{\sigma_j(i),j+1}\right) = f\left(u_{\sigma_j(i),j}\right) f\left(u_{\sigma_j(i),j+1}\right) = f\left(u_{\sigma_j(i),j}\right) f\left(u_{\sigma_{j+1}(i+1),j+1}\right)$ by the definition of σ_j.

Therefore, $f_1\left(u_{\sigma_j(i),j}u_{\sigma_j(i),j+1}\right) = \begin{cases} n(2j-1)+(2i-1), & 1 \le i \le n-1 \\ n(2j-1)+(n-1), & i = n \end{cases}$

Let $E_1 = \left\{ f_1\left(u_{\sigma_j(i),j}u_{\sigma_j(i+1),j}\right) : 1 \le i \le n-1 \text{ and } 1 \le j \le m \right\} \cup \left\{ f_1\left(u_{\sigma_j(n),j}u_{\sigma_j(1),j}\right) : 1 \le j \le m \right\}$ and

$E_2 = \left\{ f_1\left(u_{i,1}u_{i,2}\right) : 1 \le i \le n \right\} \cup \left\{ f_1\left(u_{\sigma_j(i),j}u_{\sigma_j(i),j+1}\right) : 1 \le i \le n-1 \text{ and } 2 \le j \le m-1 \right\}$.

The labels of the edges in E_1 are,

$f_1(E_1) = \{1, 3, 5, 7, 9, \ldots, 2(n-1)-1, n-1, 2n+1, 2n+3, \ldots, 2n+2(n-1)-1, 2n+n-1, \ldots, 2n(m-1)+1, 2n(m-1)+3, \ldots, 2n(m-1)+2(n-2)-1, 2n(m-1)+2(n-1)-1, 2n(m-1)+(n-1)\}$.

That is, $f_1(E_1) = \{1, 3, 5, 7, 9, \ldots, 2n-3, n-1, 2n+1, 2n+3, \ldots, 3n-1, \ldots, 4n-3, \ldots, 2mn-2n+1, 2mn-2n+3, \ldots, 2mn-5, 2mn-3, 2mn-n-1\}$.

The labels of the edges in E_2 are,

$f_1(E_2) = \{8, 10, 12, 14, 16, \ldots, 2(n-1)+6, 2n-1, 3n+1, 3n+3, \ldots, 3n+2(n-1)-1, 3n+n-1, \ldots, n(2(m-1)-1)+1, n(2(m-1)-1)+2(n-2)-1, n(2(m-1)-1)+2(n-1)-1, n(2(m-1)-1)+(n-1)\} = \{8, 10, 12, 14, 16, \ldots, 2n-1, 2n+4, 3n+1, 3n+3, \ldots, 4n-1, \ldots, 5n-3, 2mn-3n+1, 2mn-3n+3, \ldots, 2mn-n-5, 2mn-n-3, 2mn-2n-1\}$.

$f_1(E_1) \cup f_1(E_2) = \{1, 3, 7, 8, 9, 10, \ldots, 2n-3, 2n-2, 2n-1, 2n, 2n+1, 2n+2, 2n+3, 2n+4, \ldots, 3n-1, 3n, 3n+1, 3n+2, \ldots, 4n-3, 4n-2, 4n-1, \ldots, 2mn-3n+1, 2mn-3n+2, 2mn-3n+3,$

. . . , $2mn - 2n - 1$, $2mn - 2n$, $2mn - 2n + 1$. . . , $2mn - n - 3$, $2mn - n - 2$, $2mn - n - 1$, $2mn - n$, $2mn - 5$, $2mn - 3$}.

After taking (mod q), $f^*(E(G)) = f_1(E(G))$ (mod q) = {1, 2, 3, . . . , $n - 3$, $n - 2$, $n - 1$, n, $n + 1$, $n + 2$, . . . , $2n - 1$, $2n$, $2n + 1$, $2n + 3$, . . . , $3n - 1$, $3n$, $3n + 1$, . . . , $4n - 2$, $4n - 1$, $n(2m - 3) + 1$, $n(2m - 3) + 2$, . . . , $2n(m - 1)$, $2n(m - 1) + 1$, . . . , $n(2m - 1) - 1$, $n(2m - 1)$}.

Hence, $C_n \times P_m$ is a felicitous graph for $m \geq 1$ $n \geq 5$ and $n \equiv 1 (\text{mod } 2)$. ■

Example 3.9. *A felicitous labeling of $C_3 \times P_4$ is shown in Figure 3.*

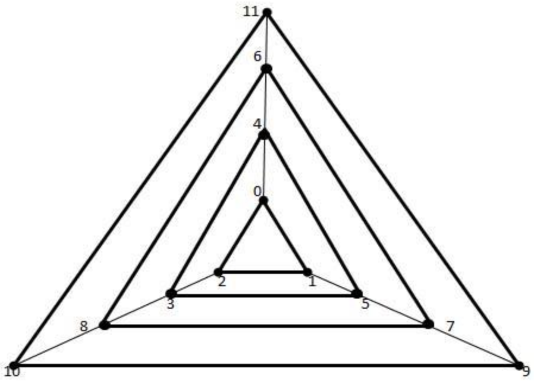

Figure 3: *A felicitous labeling of $C_3 \times P_4$.*

Example 3.10. *A felicitous labeling of $C_7 \times P_4$ is shown in Figure 4.*

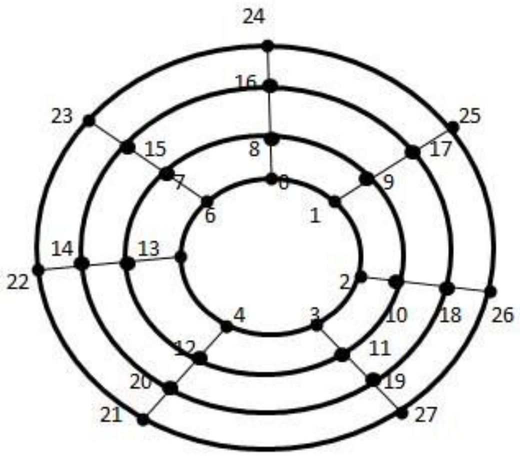

Figure 4: *A felicitous labeling of $C_7 \times P_4$.*

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