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Order six block integrator for the solution of first-order ordinary differential equations

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Abstract

In this research work, we present the derivation and implementation of an order six block integrator for the solution of first-order ordinary differential equations using interpolation and collocation procedures. The approximate solution used in this work is a combination of power series and exponential function. We further investigate the properties of the block integrator and found it to be zero-stable, consistent and convergent. The block integrator is further tested on some real-life numerical problems and found to be computationally reliable.

Keywords: Block integrator, exponential function, order, power series.

AMS Subject Classification(2010): 65L05, 65L06, 65D30.

1 Introduction

In recent times, the integration of Ordinary Differential Equations (ODEs) is carried out using some kinds of block methods. In this paper, we propose an order six block integrator for the solution of first-order ODEs of the form:

$$y' = f(x, y),\ y(a) = \eta \quad \forall a \leq x \leq b \tag{1}$$

where f is continuous within the interval of integration $[a, b]$. We assume that f satisfies Lipchitz condition which guarantees the existence and uniqueness of solution of (1). The problem (1) occurs mainly in the study of dynamical systems and electrical networks. According to [7] and [14], equation (1) is used in simulating the growth of population, trajectory of a particle, simple harmonic motion, deflection of a beam and the like. It is also important to note that mixture models, SIR model and other similar models can be written in the form of equation (1).

Development of Linear Multistep Methods (LMMs) for solving ODEs can be generated using methods such as Taylor's series, numerical integration and collocation methods, which are restricted by an assumed order of convergence [10]. In this work, we follow suite from [13] by deriving an order six block integrator in a multistep collocation technique introduced in [15].

Block integrators for solving ODEs have been proposed by W. E. Milne[18] who used them as starting values for predictor-corrector algorithm, [8] developed Milne's method in the form of implicit integrators and [12] also contributed greatly to the development and application of block integrators. Various authors [1], [3], [4], [5], [9], [11], [17] and [19] proposed LMMs to generate numerical solution to (1).

They proposed integrators in which the approximate solution ranges from power series, Chebychev's, Lagrange's and Laguerre's polynomials. The advantages of LMMs over single step methods have been extensively discussed in [2].

In this paper, we propose an order six block integrator, in which the approximate solution is the combination of power series and exponential function. This work is an improvement on [13].

2 Main Results

To derive this integrator, interpolation and collocation procedures are used by choosing interpolation point $s$at a grid point and collocation points r at all points giving rise to $\xi = s + r - 1$ system of equations whose coefficients are determined by using appropriate procedures.

The approximate solution to (1) is taken to be a combination of power series and exponential function given by:

$$y(x) = \sum_{j=0}^{5} a_j x^j + a_6 \sum_{j=0}^{6} \frac{\alpha^j x^j}{j!} \tag{2}$$

with the first derivative given by:

$$y'(x) = \sum_{j=0}^{5} j a_j x^{j-1} + a_6 \sum_{j=1}^{6} \frac{\alpha^j x^{j-1}}{(j-1)!} \tag{3}$$

where $a_j, \alpha^j \epsilon \mathbb{R}\ for\ j = 0(1)6$ and $y(x)$ is continuously differentiable.

Let the solution of (1) be sought on the partition $\pi_n: a = x_0 < x_1 < x_2 < \cdots < x_n < x_{n+1} < \cdots < x_N = b$, of the integration interval $[a, b]$with a constant step-size h, given by,

$$h = x_{n+1} - x_n, n = 0,1, \ldots, N.$$

Then, substituting (3) in (1) gives:

$$f(x, y) = \sum_{j=0}^{5} j a_j x^{j-1} + a_6 \sum_{j=1}^{6} \frac{\alpha^j x^{j-1}}{(j-1)!} \tag{4}$$

Now, interpolating (2) at point $x_{n+s}, s = 0$ and collocating (4) at points $x_{n+r}, r = 0(1)5$, leads to the following system of equations:

$$AX = U \tag{5}$$

where $A = [a_0, a_1, a_2, a_3, a_4, a_5, a_6]^T$;

$U = [y_n, f_n, f_{n+1}, f_{n+2}, f_{n+3}, f_{n+4}, f_{n+5}]^T$

and

$$X=\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & \left(1+\alpha x_n+\frac{\alpha^2x_n^2}{2!}+\frac{\alpha^3x_n^3}{3!}+\frac{\alpha^4x_n^4}{4!}+\frac{\alpha^5x_n^5}{5!}+\frac{\alpha^6x_n^6}{6!}\right) \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & \left(\alpha+\alpha^2x_n+\frac{\alpha^3x_n^2}{2!}+\frac{\alpha^4x_n^3}{3!}+\frac{\alpha^5x_n^4}{4!}+\frac{\alpha^6x_n^5}{5!}\right) \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & \left(\alpha+\alpha^2x_{n+1}+\frac{\alpha^3x_{n+1}^2}{2!}+\frac{\alpha^4x_{n+1}^3}{3!}+\frac{\alpha^5x_{n+1}^4}{4!}+\frac{\alpha^6x_{n+1}^5}{5!}\right) \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & \left(\alpha+\alpha^2x_{n+2}+\frac{\alpha^3x_{n+2}^2}{2!}+\frac{\alpha^4x_{n+2}^3}{3!}+\frac{\alpha^5x_{n+2}^4}{4!}+\frac{\alpha^6x_{n+2}^5}{5!}\right) \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & \left(\alpha+\alpha^2x_{n+3}+\frac{\alpha^3x_{n+3}^2}{2!}+\frac{\alpha^4x_{n+3}^3}{3!}+\frac{\alpha^5x_{n+3}^4}{4!}+\frac{\alpha^6x_{n+3}^5}{5!}\right) \\ 0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & \left(\alpha+\alpha^2x_{n+4}+\frac{\alpha^3x_{n+4}^2}{2!}+\frac{\alpha^4x_{n+4}^3}{3!}+\frac{\alpha^5x_{n+4}^4}{4!}+\frac{\alpha^6x_{n+4}^5}{5!}\right) \\ 0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & \left(\alpha+\alpha^2x_{n+5}+\frac{\alpha^3x_{n+5}^2}{2!}+\frac{\alpha^4x_{n+5}^3}{3!}+\frac{\alpha^5x_{n+5}^4}{4!}+\frac{\alpha^6x_{n+5}^5}{5!}\right) \end{bmatrix}$$

Solving (5), for $a_j's, j=0(1)6$ and substituting back into (2) gives a continuous linear multistep method of the form:

$$y(x)=\alpha_0(x)y_n+h\sum_{j=0}^{5}\beta_j(x)\,f_{n+j} \tag{6}$$

where the coefficients of $y_n and\ f_{n+j}$ are given by:

$$\left.\begin{aligned} \alpha_0&=1\\ \beta_0&=-\frac{1}{1440}(2t^6-36t^5+255t^4-900t^3+1644t^2-1440t)\\ \beta_1&=\frac{1}{1440}(10t^6-168t^5+1065t^4-3080t^3+3600t^2)\\ \beta_2&=-\frac{1}{720}(10t^6-156t^5+885t^4-2140t^3+1800t^2)\\ \beta_3&=\frac{1}{720}(10t^6-144t^5+735t^4-1560t^3+1200t^2)\\ \beta_4&=-\frac{1}{1440}(10t^6-132t^5+615t^4-1220t^3+900t^2)\\ \beta_5&=\frac{1}{1440}(2t^6-24t^5+105t^4-200t^3+144t^2) \end{aligned}\right\} \tag{7}$$

where $t=(x-x_n)/h$. Evaluating (6) at $t=1(1)5$ gives a block scheme of the form:

$$A^{(0)}\boldsymbol{Y}_m=\boldsymbol{E}\boldsymbol{y}_n+hd\boldsymbol{f}(\boldsymbol{y}_n)+hb\boldsymbol{F}(\boldsymbol{Y}_m) \tag{8}$$

where

$\boldsymbol{Y}_m=[y_{n+1},y_{n+2},y_{n+3},y_{n+4},y_{n+5}]^T$,

$\boldsymbol{y}_n=[y_{n-4},y_{n-3},y_{n-2},y_{n-1},y_n]^T$.

$\boldsymbol{F}(\boldsymbol{Y}_m)=[f_{n+1},f_{n+2},f_{n+3},f_{n+4},f_{n+5}]^T$,

$\boldsymbol{f}(\boldsymbol{y}_n)=[f_{n-4},f_{n-3},f_{n-2},f_{n-1},f_n]^T$

$$A^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{E} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \ d = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{95}{288} \\ 0 & 0 & 0 & 0 & \frac{14}{45} \\ 0 & 0 & 0 & 0 & \frac{51}{160} \\ 0 & 0 & 0 & 0 & \frac{14}{45} \\ 0 & 0 & 0 & 0 & \frac{95}{288} \end{bmatrix}$$

$$b = \begin{bmatrix} \frac{1427}{1440} & \frac{-133}{240} & \frac{241}{720} & \frac{-173}{1440} & \frac{3}{160} \\ \frac{43}{30} & \frac{7}{45} & \frac{7}{45} & \frac{-1}{15} & \frac{1}{90} \\ \frac{219}{160} & \frac{57}{80} & \frac{57}{80} & \frac{-21}{160} & \frac{3}{160} \\ \frac{64}{45} & \frac{8}{15} & \frac{64}{45} & \frac{14}{45} & 0 \\ \frac{125}{96} & \frac{125}{144} & \frac{125}{144} & \frac{125}{96} & \frac{95}{288} \end{bmatrix}$$

The algorithm for implementing the block integrator (8) using Matlab is given by,

function output $= f(x, y)$

output=?;

functionexactsol$= fr(x)$

exact sol=?

$x0 = ?; y0 = ?; h = ?;$

disp(‘x-value Exact Solution Computed Solution Error ’)

$for\ j = 1:1:4;$

$i = 1:5$

$f0 = g(x0, y0);$

$x(i) = x0 + i * h;$

$y(i) = y0 + (i * h) * f0 + (((i * h)^2)/2)) * dx(x0, y0)$

$+(((i * h)^3)/6) * ddx(x0, y0) + \cdots$

$yi(x) = gi_L$

$mi = toc;$

$erri = abs\big(fr\big(x(i)\big) - yri\big);$

$fprintf('\%2.4f\ \%3.16f\ \%3.16f\ \%1.6e\ \%2.4f\ \backslash n', x(i), fr\big(x(i)\big), yri, erri, mi)$

$x0 = x(5); y0 = yr(5);$

end

Note that: $y(x) = g_L, for\ g_L = \alpha_0(x)y_n + h\beta_j(x)f_{n+j}, j = 0(1)5.$

3 Analysis of the basic properties of the order six block integrator

3.1 Order of the new block integrator

Let the linear operator $L\{y(x);h\}$ associated with the block (8) be defined as:

$$L\{y(x);h\} = A^{(0)}Y_m - Ey_n - hdf(y_n) - hbF(Y_m) \tag{9}$$

Expanding (9) using Taylor series and comparing the coefficients of h gives:

$$L\{y(x);h\} = c_0y(x) + c_1hy'(x) + c_2h^2y''(x) + \cdots + c_ph^py^p(x) + c_{p+1}h^{p+1}y^{p+1}(x) + \cdots \tag{10}$$

Definition 3.1.1. *The linear operator L and the associated continuous linear multistep method (6) are said to be of order p if* $c_0 = c_1 = c_2 = \cdots = c_p = 0$ *and* $c_{p+1} \neq 0$. c_{p+1} *is called the error constant and the local truncation error is given by:*

$$t_{n+k} = c_{p+1}h^{(p+1)}y^{(p+1)}(x_n) + O(h^{p+2}) \tag{11}$$

For our integrator,

$$L\{y(x);h\} = \begin{bmatrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0\\ 0&0&0&1&0\\ 0&0&0&0&1 \end{bmatrix}\begin{bmatrix} y_{n+1}\\ y_{n+2}\\ y_{n+3}\\ y_{n+4}\\ y_{n+5} \end{bmatrix} - \begin{bmatrix} 1\\1\\1\\1\\1 \end{bmatrix}\left[y_n\right] - h\begin{bmatrix} \frac{95}{288} & \frac{1427}{1440} & \frac{-133}{240} & \frac{241}{720} & \frac{-173}{1440} & \frac{3}{160}\\ \frac{14}{45} & \frac{43}{30} & \frac{7}{45} & \frac{7}{45} & \frac{-1}{15} & \frac{1}{90}\\ \frac{51}{160} & \frac{219}{160} & \frac{57}{80} & \frac{57}{80} & \frac{-21}{160} & \frac{3}{160}\\ \frac{14}{45} & \frac{64}{45} & \frac{8}{15} & \frac{64}{45} & \frac{14}{45} & 0\\ \frac{95}{288} & \frac{125}{96} & \frac{125}{144} & \frac{125}{144} & \frac{125}{96} & \frac{95}{288} \end{bmatrix}\begin{bmatrix} f_n\\ f_{n+1}\\ f_{n+2}\\ f_{n+3}\\ f_{n+4}\\ f_{n+5} \end{bmatrix} = 0 \tag{12}$$

Expanding (12) in Taylor series we get,

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(h)^j}{j!}y_n^j - y_n - \frac{95h}{288}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{1427}{1440}(1)^j - \frac{133}{240}(2)^j + \frac{241}{720}(3)^j - \frac{173}{1440}(4)^j + \frac{3}{160}(5)^j\right\}\\ \sum_{j=0}^{\infty}\frac{(2h)^j}{j!}y_n^j - y_n - \frac{14h}{45}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{43}{30}(1)^j + \frac{7}{45}(2)^j + \frac{7}{45}(3)^j - \frac{1}{15}(4)^j + \frac{1}{90}(5)^j\right\}\\ \sum_{j=0}^{\infty}\frac{(3h)^j}{j!}y_n^j - y_n - \frac{51h}{160}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{219}{160}(1)^j + \frac{57}{80}(2)^j + \frac{57}{80}(3)^j - \frac{21}{160}(4)^j + \frac{3}{160}(5)^j\right\}\\ \sum_{j=0}^{\infty}\frac{(4h)^j}{j!}y_n^j - y_n - \frac{14h}{45}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{64}{45}(1)^j + \frac{8}{15}(2)^j + \frac{64}{45}(3)^j + \frac{14}{45}(4)^j\right\}\\ \sum_{j=0}^{\infty}\frac{(5h)^j}{j!}y_n^j - y_n - \frac{95h}{288}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{125}{96}(1)^j + \frac{125}{144}(2)^j + \frac{125}{144}(3)^j + \frac{125}{96}(4)^j + \frac{95}{288}(5)^j\right\} \end{bmatrix} = \begin{bmatrix} 0\\0\\0\\0\\0 \end{bmatrix} \tag{13}$$

Hence,

$$\bar{c}_0 = \bar{c}_1 = \bar{c}_2 = \bar{c}_3 = \bar{c}_4 = \bar{c}_5 = \bar{c}_6 = 0,$$

$\bar{c}_7 = [-1.43(-02), -9.79(-02), -1.29(-02), -8.47(-03), -2.27(-02)]^T$.

Therefore, the new block integrator is of order six.

3.2 Zero Stability

Definition 3.2.1. *The block integrator (8) is said to be zero-stable, if the roots $z_s, s = 1,2,\dots,k$ of the first characteristic polynomial $\rho(z)$ defined by $\rho(z) = det(z\boldsymbol{A}^{(0)} - \boldsymbol{E})$ satisfies $|z_s| \le 1$ and every root satisfying $|z_s| \le 1$ has multiplicity not exceeding the order of the differential equation. Moreover, as $h \longrightarrow 0$, $\rho(z) = z^{r-\mu}(z-1)^{\mu}$ where μ is the order of the differential equation, r is the order of the matrices $\boldsymbol{A}^{(0)}$ and $\boldsymbol{E}$* (refer [3]).

For the integrator,

$$\rho(z) = \left| z \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right| = 0 \tag{14}$$

$\rho(z) = z^4(z-1) = 0, \implies z_1 = z_2 = z_3 = z_4 = 0, z_5 = 1$. Hence, the new block integrator is zero-stable.

3.3 Consistency

The block integrator (8) is consistent since it has order $p = 6 \ge 1$.

3.4 Convergence

The new block integrator is convergent by the consequence of Dahlquist theorem given below.

Theorem 3.4.1. [6]

The necessary and sufficient conditions that a continuous LMM be convergent are that it be consistent and zero-stable.

3.5 Region of Absolute Stability

Definition 3.5.1. *The block integrator (8) is said to be absolutely stable if for a given h, all the roots z_s of the characteristic polynomial $\pi(z,\bar{h}) = \rho(z) + \bar{h}\sigma(z) = 0$ satisfies $z_s < 1$, s=1,2,..., n.where $\bar{h} = \lambda h$ and $\lambda = \frac{\partial f}{\partial y}$.*

We adopt the boundary locus method for the region of absolute stability of the block integrator. Substituting the test equation $y' = -\lambda y$ into the block formula gives,

$$\boldsymbol{A}^{(0)}\boldsymbol{Y}_m(r) = \boldsymbol{E}y_n(r) - h\lambda\boldsymbol{D}y_n(r) - h\lambda\boldsymbol{B}\boldsymbol{Y}_m(r) \tag{15}$$

Thus,

$$\bar{h}(r) = -\left(\frac{\boldsymbol{A}^{(0)}Y_m(r) - \boldsymbol{E}y_n(r)}{\boldsymbol{D}y_n(r) + \boldsymbol{B}Y_m(r)}\right) \tag{16}$$

We write (16) in trigonometric ratios,

$$\bar{h}(\theta) = -\left(\frac{\boldsymbol{A}^{(0)}Y_m(\theta) - \boldsymbol{E}y_n(\theta)}{\boldsymbol{D}y_n(\theta) + \boldsymbol{B}Y_m(\theta)}\right) \tag{17}$$

where $r = e^{i\theta}$. Equation (17) is our characteristic/stability polynomial. Applying (17) to our integrator gives,

$$\bar{h}(\theta) = \frac{(\cos 2\theta)(\cos 3\theta)(\cos 4\theta)(\cos\theta) - (\cos 2\theta)(\cos 3\theta)(\cos 4\theta)(\cos 5\theta)(\cos\theta)}{\frac{1}{6}(\cos 2\theta)(\cos 3\theta)(\cos 4\theta)(\cos\theta) - \frac{1}{6}(\cos 2\theta)(\cos 3\theta)(\cos 4\theta)(\cos 5\theta)(\cos\theta)} \tag{18}$$

which gives the stability region shown in Figure 1 below.

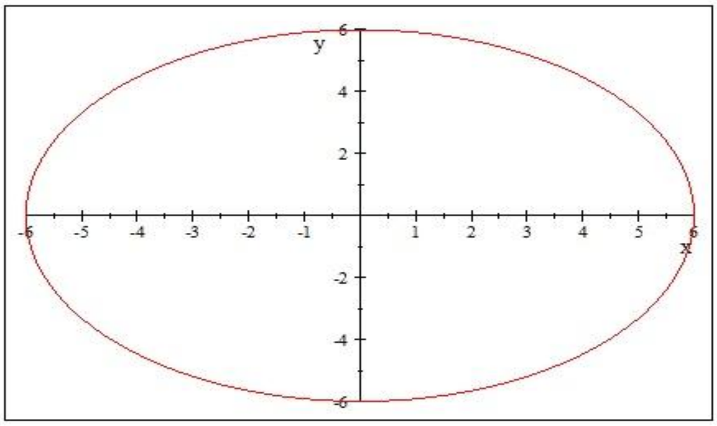

Figure 1: *Region of absolute stability of the order six block integrator.*

4 Numerical implementations

We use the following notation in tables 1 and 2.

ERR- |Exact Solution-Computed Result|

We consider a very important model in the refinery. This model examines the effect of additive on gasoline. The problem is stated thus;

Problem 4.1. (Mixture Model) *In an oil refinery, a storage tank contains 2000 gal of gasoline that initially has 100lb of an additive dissolved in it. In the preparation for winter weather, gasoline containing 2lb of additive per gallon is pumped into the tank at a rate of 40 gal/min. The well-mixed solution is pumped out at a rate of 45 gal/min. Using a numerical integrator, how much of the additive is in the tank 0.1 min, 0.5 min and 1min after the pumping process begins?*

Let y be the amount (in pounds) of additive in the tank at time$t$. We know that$y = 100$ $when\ t = 0$. Thus, the IVP modeling the mixture process is,

$$y' = 80 - \frac{45y}{(2000-5t)},\ y(0) = 100 \tag{19}$$

with the theoretical solution,

$$y(t) = 2(2000 - 5t) - \frac{3900}{(2000)^9}(2000 - 5t)^9 \tag{20}$$

Applying the block integrator (8) to Problem 4.1, we obtain the results in Table 1 at different values of time t.

Problem 4.2. (*SIR* Model) *The SIR model is an epidemiological model that computes the theoretical number of people infected with a contagious illness in a closed population over time. The name of this class of models derives from the fact that they involve coupled equations relating the number of susceptible people* $S(t)$, *number of people infected* $I(t)$ *and the number of people who have recovered* $R(t)$. *This is a good and simple model for many infectious diseases including measles, mumps and rubella [16]. It is given by the following three coupled equations,*

$$\frac{dS}{dt} = \mu(1 - S) - \beta IS \tag{21}$$

$$\frac{dI}{dt} = -\mu I - \gamma I + \beta IS \tag{22}$$

$$\frac{dR}{dt} = -\mu R + \gamma I \tag{23}$$

where $\mu, \gamma \ \ and\ \beta$ are positive parameters. Define y to be,

$$y = S + I + R \tag{24}$$

and adding equations (21), (22) and (23), we obtain the following evolution equation for y,

$$y' = \mu(1 - y) \tag{25}$$

Taking $\mu = 0.5$ and attaching an initial condition $y(0) = 0.5$ (for a particular closed population), we obtain,

$$y'(t) = 0.5(1 - y), y(0) = 0.5 \tag{26}$$

whose exact solution is,

$$y(t) = 1 - 0.5e^{-0.5t} \tag{27}$$

Applying the block integrator (8) to Problem 4.2, we obtain the results in Table 2 at different values of time t.

Table 1: *Performance of the Block Integrator (8) on Problem* 4.1.

t	**Exact solution**	**Computed solution**	**ERR**
0.1000	107.7662301168306800	107.7662301168309500	2.700062e-013
0.2000	115.5149409193027200	115.5149409193028400	1.278977e-013
0.3000	123.2461630508846600	123.2461630508845200	1.421085e-013
0.4000	130.9599271090915000	130.9599271090910700	4.263256e-013
0.5000	138.6562636455414600	138.6562636455413400	1.136868e-013
0.6000	146.3352031660151600	146.3352031660153300	1.705303e-013
0.7000	153.9967761305115300	153.9967761305114500	8.526513e-014
0.8000	161.6410129533037400	161.6410129533038300	8.526513e-014
0.9000	169.2679440029996800	169.2679440029996000	8.526513e-014
1.0000	176.8775996025960900	176.8775996025958600	2.273737e-013

Table 2: *Performance of the Block Integrator (8) on Problem* 4.2.

t	Exact solution	Computed solution	ERR
0.1000	0.5243852877496430	0.5243852877552174	5.574430e-012
0.2000	0.5475812909820202	0.5475812909859664	3.946177e-012
0.3000	0.5696460117874711	0.5696460117956543	8.183232e-012
0.4000	0.5906346234610092	0.5906346234953703	3.436118e-011
0.5000	0.6105996084642975	0.6105996086572718	1.929743e-010
0.6000	0.6295908896591411	0.6295908898470451	1.879040e-010
0.7000	0.6476559551406433	0.6476559553183269	1.776835e-010
0.8000	0.6648399769821803	0.6648399771546479	1.724676e-010
0.9000	0.6811859241891134	0.6811859243738679	1.847545e-010
1.0000	0.6967346701436833	0.6967346704442603	3.005770e-010

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