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New self similar solution behind exponential magneto radiative blast wave in a dusty gas

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Abstract

The self-similar solutions for point source magnetoradiative exponential shock wave moving into uniform atmosphere of dusty gas where total energy of wave depend on cube of shock radius has discussed. The effects of radiation on the discontinuities are also investigated. The ordinary differential equation obtained by similarity method are solved by software Matlab. The nature of flow variables are illustrated through graphs.

Keywords: blast waves, exponential shock, optically, self-similar.

AMS Subject Classification(2010): 76L05.

1 Introduction

In the earlier investigation, the self-similar solutions driven by a sudden point explosion of the core are investigated by Koch et.al [3]. Hellewell [2] and Ray [5] have discussed the propagation of plane shock waves in optically thick and thin limiting case of gas in detail. Gusev [1], Ranga Rao and Ramana [4] have studied the problem of unsteady self-similar motion of a gas displaced by a piston according to an exponential law. Verma, Singh [10] and Singh, Srivastava [6] have considered the problems of spherical shock waves in an exponentially increasing medium under the law of uniform pressure. Analytical solution in the three cases of plane, cylindrical, spherical symmetrically flows have been also discussed by Srivastava [7, 8, 9] of magneto radiative shock in conducting plasma.

In the present paper we discussed the strong exponential spherical, cylindrical and plane shock waves in a uniform atmosphere with magnetic radiative effects, the similarity solutions have been developed, when radiation heat is more important than the radiation pressure and radiation energy. It is assumed that the unsteady model of gas distributed around a nucleus with heavy mass and the gravitational effect of atmosphere can be neglected compared with the attraction of large mass of nucleus where gas is assumed to be opaque, transparent and also isothermal.

2 Equations of motion

The equations of flow behind a spherical, cylindrical and plane shock wave are

$$\frac{d\rho}{dt}+\rho\frac{\partial u}{\partial t}+J\frac{\rho u}{r}=0 \quad (1)$$

$$\frac{du}{dt}+\frac{1}{\rho}\frac{\partial \rho}{\partial r}+\frac{h}{r}\frac{\partial h}{\partial r}+\upsilon\frac{h^2}{\rho r}=0 \quad (2)$$

$$\frac{dh}{dt}+h\frac{\partial u}{\partial r}+\upsilon\frac{h}{r}=0 \quad (3)$$

$$\frac{dE}{dt}+P\frac{d}{dt}\left(\frac{1}{\rho}\right)+\frac{1}{\rho r^J}\frac{\partial}{\partial r}\left(Fr^J\right)=0 \quad (4)$$

$$E=\frac{P}{(\gamma-1)}\rho \quad (5)$$

$$P=\Gamma\rho T \quad (6)$$

where u, ρ, P, T, E, h and F are the velocity, density, pressure, temperature energy, magnetic field and radiant heat flux where J =0, 1,2 corresponds to plane, cylindrical and spherical shock and υ =0 for plane and υ =1 for cylindrical as well as spherical symmetry.

Assuming total thermodynamics equilibrium and taking Roseland's diffusion approximation.

$$F=-\frac{c\mu}{3}\frac{\partial}{\partial r}\left(\sigma T^4\right) \quad (7)$$

where σ is the Stefan Bolt Mann constant, c the velocity of light, u the mean flow path of radiation which is a function of density and temperature. Following Wang (1966), we take

$$\mu=\mu_0\rho^{\alpha}T^{\beta} \quad (8)$$

$\mu_{0,}\alpha,\beta$ are constant

The inner expanding surface moves with time according to exponential law.

$$\bar{r}=A\exp(mt) \quad (9)$$

Since we have assumed self-similarity, the shock waves are also move with time according to an exponential constant.

3 Boundary conditions

The disturbance is headed by an isothermal shock, therefore the boundary condition given by Singh [10] are

$$u_1 = \left(1 - \frac{1}{\gamma M^2}\right)v \tag{10}$$

$$\rho_1 = \gamma M^2 \rho_0 \tag{11}$$

$$P_1 = \rho_0 v^2 \tag{12}$$

$$F_1 = \frac{1}{2}\left(\frac{1}{\gamma^2 M^4} - 1\right)\rho_0 v^3 \tag{13}$$

$$h_1 = \gamma M^2 h_0 \tag{14}$$

where subscripts 0 and 1 denote the regions immediately ahead and behind the shock front respectively and v is the shock velocity where M denotes the Mach number.

4 Similarity solution

The similarity transformations for the problem under consideration are

$$\eta = \frac{r}{B\exp(mt)} \tag{15}$$

$$u = vV(\eta) \tag{16}$$

$$\rho = \rho_0 G(\eta) \tag{17}$$

$$p = \rho_0 v^2 P(\eta) \tag{18}$$

$$F = \rho_0 v^3 Q(\eta) \tag{19}$$

$$h = \sqrt{\rho_0}\, vH(\eta) \tag{20}$$

the variable η assumes the value 1 at the shock and $\bar{\eta}$ on inner expanding surface. This enables us to express the radius of the inner expanding surface

$$\bar{r} = \bar{\eta} R.$$

Now using the equation (6), (8) and (15), (19) into equation (7)

$$Q = -G^{\alpha-\beta-4} P^{\beta+4}\left(\frac{P'}{P} - \frac{G'}{G}\right) \tag{21}$$

with $\beta = -2, \alpha$ remaining arbitrary $(0 \le \alpha \le 2)$ and

$$N = \frac{4mc\mu_0 \sigma \rho_0^{\alpha-1}}{3T^{\beta-4}} \quad \text{is a dimensionless parameter.} \tag{22}$$

5 Solution of equations of motion

Equations (1) – (5) are transformed by using equations (15) – (20).

$$G' = \frac{G(V + Jv)}{(\eta - v)} \tag{23}$$

$$P' = \left[HH'\eta - \sigma H^2\right] - G\left[V'(\eta - V) - V\right] \tag{24}$$

$$H' = (1 - V)\left[HV' + \frac{JVH}{\eta}\right] \tag{25}$$

$$Q' = \frac{P}{(\gamma - 1)} + \frac{PG'(2 + V - \gamma)}{G\eta(\gamma - 1)} - \frac{QJ}{\eta} - \frac{VP}{\eta(\gamma - 1)} \tag{26}$$

$$V' = \frac{QG^{1-\alpha}(\eta - P) - JNPV + (\eta - V)NP\left[HH'^{\eta} - VH^2 - G\right]}{NPG} \tag{27}$$

where primes denotes differentiation with respect to η. The appropriate transformed shock conditions are

$$V = \left(1 - \frac{1}{\gamma M^2}\right) \tag{28}$$

$$G = \gamma M^2 \tag{29}$$

$$P = 1 \tag{30}$$

$$Q = \frac{1}{2}\left(\frac{1}{\gamma M^4} - 1\right) \tag{31}$$

$$H = \gamma M \tag{32}$$

6 Result and Discussion

For exhibiting the numerical solutions it is convenient to write the flow variables in the non-dimensional forms as

$$\frac{u}{u_1} = \frac{V}{V_1}$$

$$\frac{\rho}{\rho_1} = \frac{G}{G_1}$$

$$\frac{p}{p_1} = \frac{P}{P_1}$$

$$\frac{h}{h_1} = \frac{H}{H_1}$$

The numerical integration of ordinary differential equations (23) – (27) are carried out until the kinematics condition is satisfied and it has been performed by using the software Matlab for $\gamma = 1.4, m^2 = 20$, N = 10 ,100 and $\alpha = 1.$. The nature of the flow variables is illustrated through graphs; it is clear from graphical analysis that profile of velocity and density are minimum at shock front but increases exponentially towards the center of explosion. We also observe that the increase in spherical symmetry is more comparing to plane and cylindrical symmetry. Where as in the case of pressure distribution profile, there is a slight change. But magnetic field distribution is maximum at shock front and decreases rapidly towards center of explosion. This fall in magnetic field is more in spherical case compared to cylindrical case.

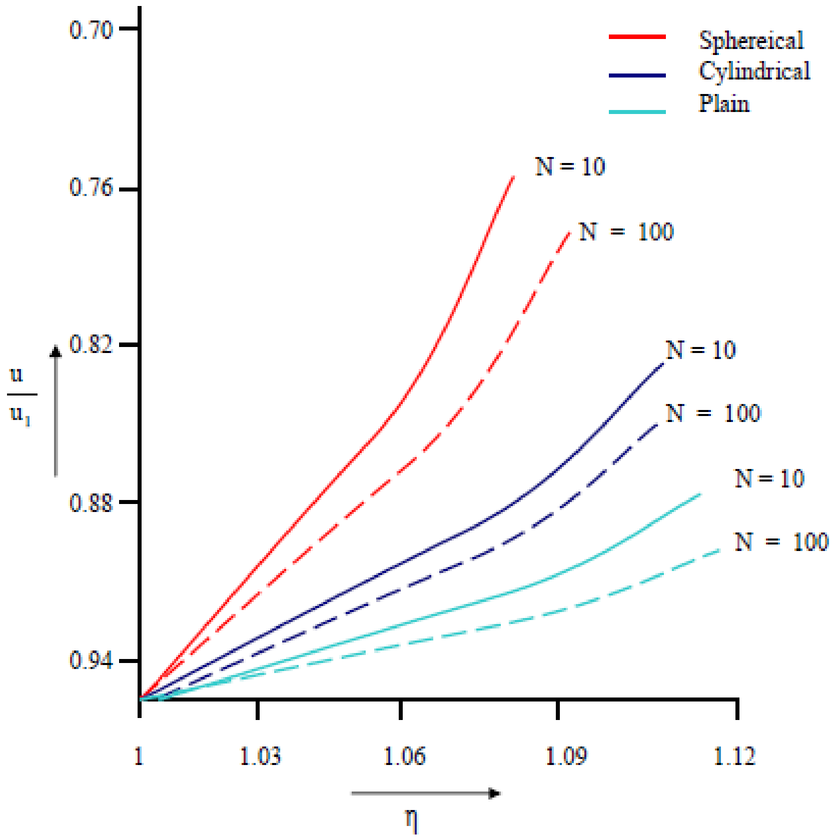

Figure 1: *Velocity distribution.*

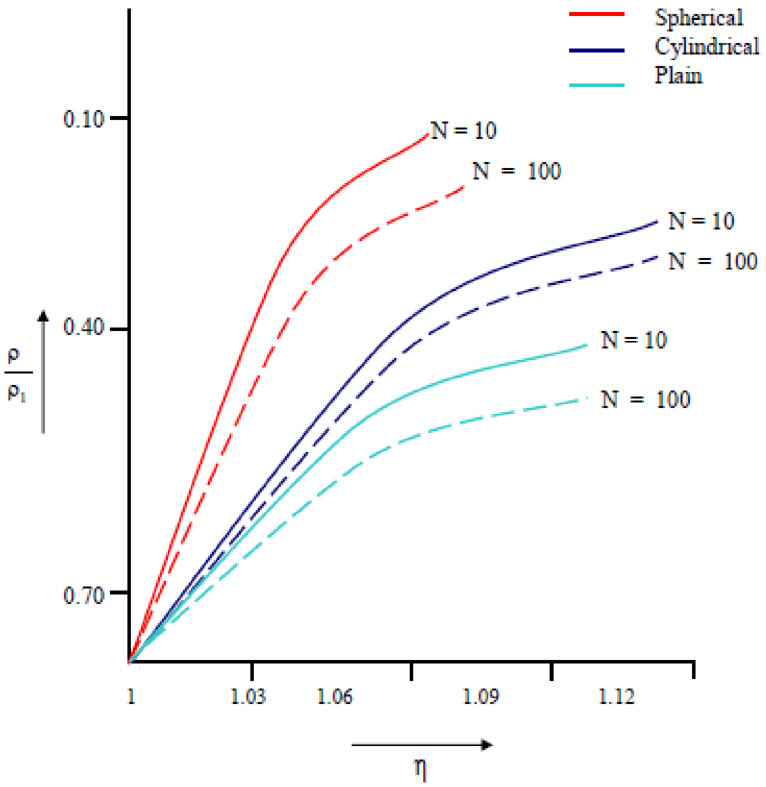

Figure 2: *Density distribution.*

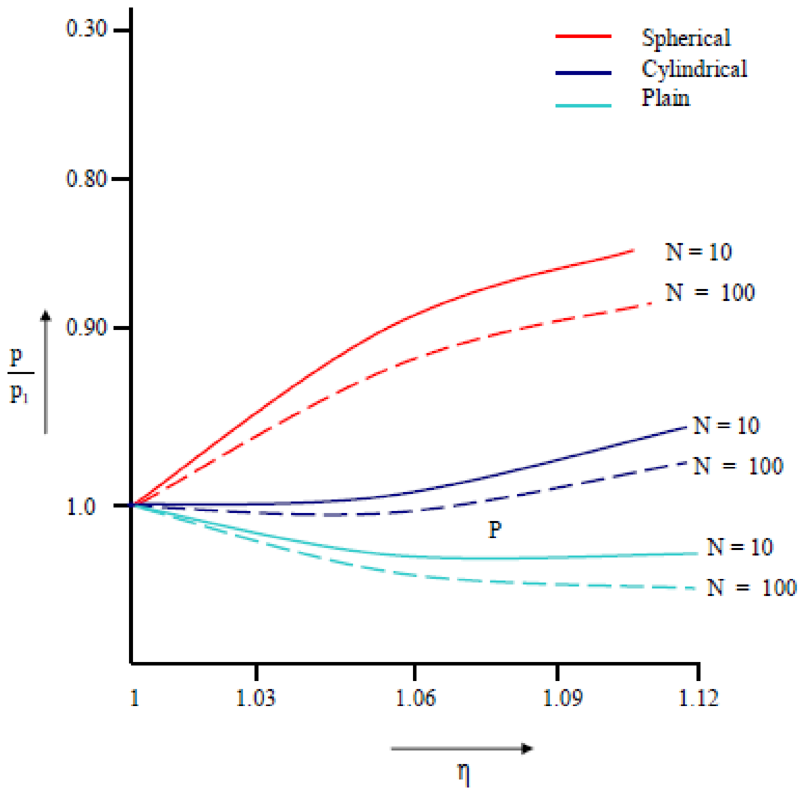

Figure 3: *Pressure distribution.*

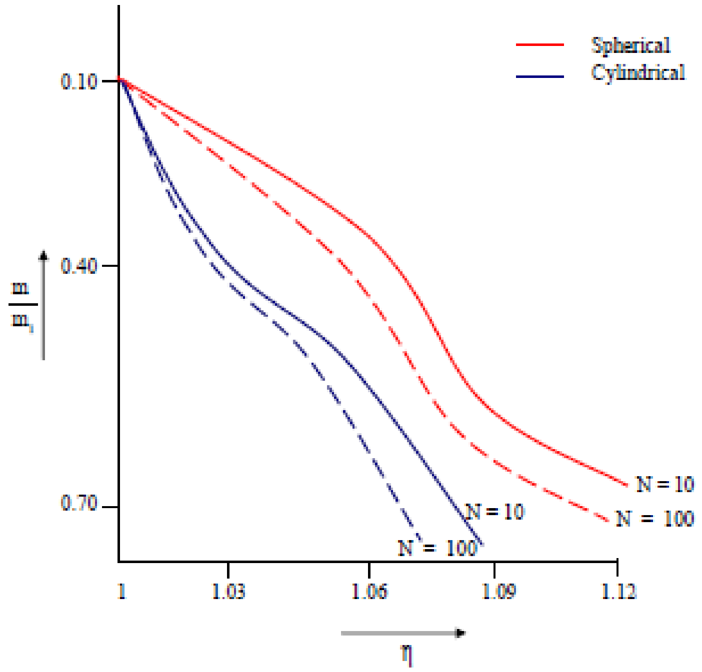

Figure 4: *Magnetic Field distribution.*

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