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A novel watershed image segmentation technique using graceful labeling

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Abstract

In this paper, we propose a novel watershed image segmentation technique based on absolute difference graph labeling. The main objective is to partition the image into distinct regions that are visually different, homogeneous and meaningful with respect to some characteristics or computed property, that is, such as grey level, texture or color to enable easy image analysis. A graph representing an image is derived from the similarity between the pixels and partitioned by a computationally efficient absolute difference graph method, which identifies representative vertices for each absolute difference and then expands them to obtain complete absolute difference graph labeling. As a result, the image is portioned into a set of disjoint labels (regions).

Keywords: graceful labeling, sum labeling.

AMS Subject Classification (2010): 05C78.

1 Introduction

Mathematical morphology [4, 10] is a well-known technique used in image processing and computer vision for manipulating the features present in the image based on shape analysis and description. Mathematical morphology is a collection of operators based on set theory and defined on an abstract structure known as an infinite lattice. The mathematical morphology is intended to analyze and recognize the geometrical properties and structure of the objects. Such analysis must be quantitative in order to provide a mathematical framework for describing the spatial organization.

Watershed transform [1] is an important morphological tool used for image segmentation which can be classified as a region-based segmentation approach. The watershed concept is one of the classic tools in the field of topography. It is the line that determines where a drop of water will fall into a particular region. In image processing [9], especially mathematical morphology, gray scale images are considered as topographic relieves. Initially the watersheds are computed in the field of topography where the topographic surfaces are handled by Digital Elevation Models (DEM's). The data structure of DEM’s is defined as a regular square grid where the elevations are equally spaced in two orthogonal directions which is similar to the representation of gray scale images in digital space. In the topographic representation of a given image *f*, the intensity value of each pixel stands for the elevation at this point [6].

As an interpretation in topography, the watershed can be imagined as the high mountain that separates two regions. Each region has its own minimum and, if a drop of water falls on one side of the watershed, it will reach the minimum of the regions. Points of contact between the propagation originating from different minima are defined as the boundaries of the regions and are used to create the final partition. A common difficulty in measuring the images occurs when features touch, and therefore cannot be separately identified, counted, or measured. This situation may arise where actual feature may overlap or when the particles resting on a surface tend to agglomerate and touch each other. One method of separating touching but most convex features in an image is called watershed segmentation. The regions that the watershed separates are called catchment basins. Lines separating catchment basins are watershed lines and are interpreted as boundaries between regions for the purposes of segmentation [5, 8]. This concept is shown in Figure 1.

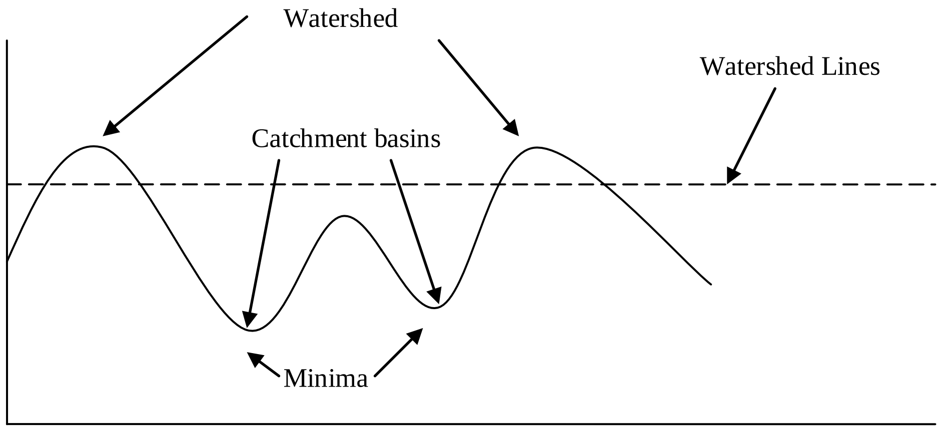


Figure 1: *Watershed Transform.*

An alternative approach is to imagine the landscape being immersed in a lake, with holes pierced in local minima basins (also called ‘catchment basins’) will fill up with water starting at these local minima and at points where water coming from different basins would meet, dams are built to avoid the regions to be merged. When the water level has reached the highest peak in the landscape, the process is stopped. As a result, the landscape is partitioned into regions or basins separated by dams, called watershed lines or simply watersheds. When simulating this process for image segmentation, two approaches may be used: either one first finds the basins, then watersheds by taking a set complement; or one computes a complete partition of the image into basins and subsequently finds the watersheds by boundary detection. Generally watershed transform denotes a labeling of the image, such that all the points of a given catchment basin will have the same unique label distinct from all the labels of other catchment basins. An example of watershed immersion for a simple image (a) with its watershed transform (b) and natural image (c) with its watershed transform (d) is given in Figure 2. Different basins are indicated by distinct grey values.

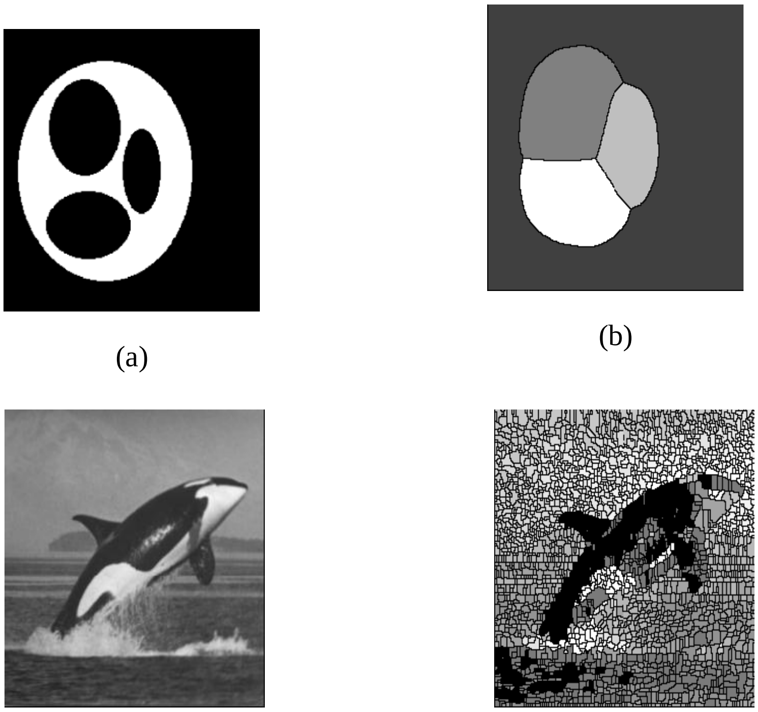


Figure 2: *Watershed Segmentation by Immersion.*

The purpose of this proposed algorithm is to overcome the difficulties encountered in the existing algorithms, where in the sequence case, the order of the pixels to be visited is not clearly specified. There are a number of issues concerning the watershed transform such as accuracy of watershed lines and dedicated hardware architectures for fast computation of watershed transforms and related operations, which are not discussed explicitly. Our interest here is in medium level image processing on general purpose architectures using graph labeling.

2 Preliminaries

In this section, we give some known definitions on graphs and digital images.

A graph $G=(V,E)$ consists of a set V of vertices and a set E of lines (edges). The set of vertices which are neighbours to a set of vertices W is denoted by $N_G(W)$. A path is called simple, if all its vertices are distinct. In an undirected graph $G=(V,E)$, a path $(v_0, v_1, ..., v_{n-1}, v_n)$ forms a cycle if $v_0 = v_n$ and $v_1, v_2, ..., v_{n-1}, v_n$ are distinct. A graph with no cycles is acyclic. A weighted graph is a triple $G=(V,E,W)$ where $W: E \rightarrow R$ is a weight function defined on the edges. A valued graph is a triple $G=(V,E,f)$ where $f: V \rightarrow R$ is a weight function defined on the vertices. A level component at level h of a valued graph is a connected component of the set of vertices V with the same value $f(V)=h$. A regional minimum at level h is a level component of P at which no point has neighbours with value lower than h. A valued graph is called lower complete when each vertex which is not in a minimum has a neighbouring vertices of lower value.

2.2. Digital grids

A digital grid is a special kind of graph. Usually one works with the square grid $D \subseteq Z^2$, where the vertices are called pixels. When D is finite, the size of D is the number of points in D. The set of pixels D can be endowed with a graph structure $G = (V, E)$ by taking for V the domain D and for E for certain subset of $Z^2 \times Z^2$ defining the connectivity. Usual choices are 4-connectivity, that is., each point has edges to its horizontal and vertical neighbors, or 8-connectivity where a point is connected to its horizontal, vertical and diagonal neighbours.

Definition 2.1. *A vertex labeling of a graph $G = (V, E)$ is an assignment f of labels to the vertices of $V(G)$ that induces for each edge uv in $E(G)$, depending on the vertex values $f(u)$ and $f(v)$. For a digital image $f(u)$ and $f(v)$ will represent the intensity values at u and v .*

Definition 2.2.[2] *A graph $G = (V, E)$ is numbered if each vertex v is assigned a non – negative integer $f(v)$ and each edge uv is assigned the absolute value of the difference of the numbers at its end points, that is $|f(u) - f(v)|$.*

Definition 2.3.[2] *Let $G = (V, E)$ be a graph. Then G is said to be graceful if there exists an injection f from V into $\{0,1,2,......,q\}$ such that when each edge uv is labeled with $|f(u) - f(v)|$, the resulting edge labels are distinct.*

Definition 2.4.[2] *A graph G is called a sum graph if there exists a vertex labeling function $f : V(G) \to Z$, where Z is a set of distinct positive integers so that $e = uv$ is an edge of G if and only if $f(u)$ and $f(v) = f(w)$, for some w in $V(G)$.*

The above concepts motivate us to define the following:

Let $G = (V, E)$ be a graph. G is said to be an absolute difference graph if there exists an injection from V into Z such that $|f(u) - f(v)|$ is distinct for all edges uv in E.

Let I be a two-dimensional gray scale image of size $m \times n$ and D be the set of intensity values of size $m \times n$, a difference vertex (pixel) labeling D is the range of (0, 255).

1. Let u and v be the neighbourhood pixels. u and v have the same label value if there is a move from u to v and
2. If there is no move from u to v, leave the pixel v.

Definition 2.5. *Let $f \in C(D)$ have minima $\{f(u_i)\}_{i \in I}$, for some index set I. The catchment basin $CB(f(u_i))$ of a minimum u_i is defined as the set of points $u_i \in D$ which are absolute difference closer to u_i than to any other regional minimum v_j :*

$$CB(u_i) = \begin{cases} \text{move to the neighbourhood pixel}, & \text{if } |f(u_i) - f(v_j)| \leq \max_{f(u_i) \prec f(v_j)} \{f(u_i), f(v_j)\} \\ \text{leave the pixel}, & otherwise \end{cases}$$

The watershed of f is the set of points which do not belong to any catchment basin:

$$wshed(f)=\left(\bigcup_{i\in I} CB(u_i)\setminus D\right)$$

Let w be some label and does not belong to I . The absolute difference graph of f is a mapping $g: D \to I \cup \{w\}$, such that $g(v)=i$ if $v \in CB(u_i)$ and $g(v)=w$ if $v \notin CB(u_i)$.

So the absolute difference of f assigns labels to the points of D , such that

i) Different catchment basins are uniquely labeled.

ii) A special label w is assigned to all the points of the watershed of f .

Definition 2.6. *Let f be a grey value image with $f^*=f_{LC}$ the lower completion of f. Let $\{m_i\}_{i\in I}$ be the collection of minima of f. The basin CB(m_i) of f corresponding to a minimum m_i is defined as the basin of the lower completion of f :*

$CB(m_i)=\{p \in D \,/\, f^*(m_i)+Tf^*(p,m_i) < f^*(m_j)+Tf^*(p,m_j) \text{ for all } j \in I-\{i\}\}$ *and the watershed of f is defined as in Definition 2.5.*

Definition 2.7. *For any image without plateaus, a function L assigning a label to each pixel is called a watershed segmentation if*

1. $L(m_i) \neq L(m_j)$ for all $i \neq j$, with $\{m_k\}_{k\in I}$ the set of minima of f ;

2. for each pixel p with $\Gamma(p) \neq \Phi$, there exists $p' \in \Gamma(p)$ with $L(p) = L(p')$.

Here the condition $\Gamma(p) \neq \Phi$ means that p has at least one lower neighbour. The new element is that for a given input image, many labeling exist which qualify as a watershed segmentation. Pixels which would have been labeled as watershed points according to Definition 2.6 are now merged by random choice with a basin belonging to some minimum m_k.

3 Watershed image segmentation technique using Graceful labeling

A novel watershed image segmentation algorithm using absolute difference graph labeling to partition an image is proposed in this section. The schematic diagram of this algorithm is given below.

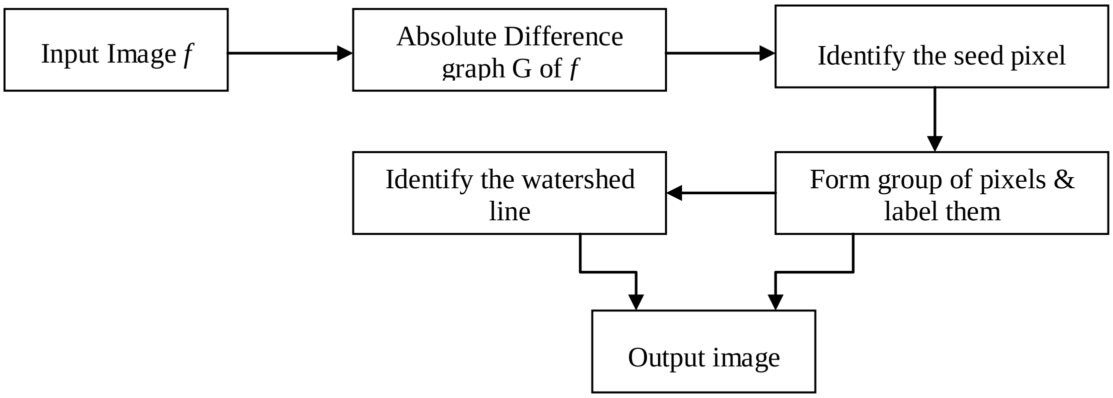


Figure 3: *Absolute difference graph labeling algorithm.*

Procedure:

The complete procedure for the proposed technique is given below.

Step 1. Let f is the input gray scale image where $f(x,y)$ will be the intensity at (x,y) and initialize the labeling variable as $i=0$

Step 2. Let u and v be the two adjacent pixels of f. The weight of the edge uv is the absolute difference between $f(u)$ and $f(v)$, that is $|f(u) - f(v)|$.

Step 3. Since watershed algorithm is a region growing method, we have to select the starting pixel that is seed point.

Step 4. Sort the intensity values of $f(x, y)$ and select the first minimum intensity as a local minimum.

Step 5. Let $f(u)$ be the local minimum so as to start from the pixel u.

Step 6. Increment $i = i + 1$ and label the pixel u as $'i'$.

Step 7. From u move to next adjacent pixel $'v'$ (either 4-connected or 8-connected) if $|f(u) - f(v)| = \max_{f(u) \le f(v)} (f(u), f(v))$ and assign label (v) = label (u). Otherwise select the next adjacent pixel.

Step 8. Repeat steps 6 and 7 until no more move is possible.

Step 9. Identity the pixels with no label value and assign them as watershed pixels and label them as $'w'$.

Step 10. The pixels with label $'w'$ will be the edges of the regions, that is, watershed lines.

Remark 3.1. *Any pixel in the upstream of a watershed pixel itself is a watershed pixel.*

4 Experimental analysis and results

The proposed method is applied on various Cervical Cytology images of both ‘BLOB’ and ‘MOSAIC’ types. Some of the examples of cytology images are shown below.

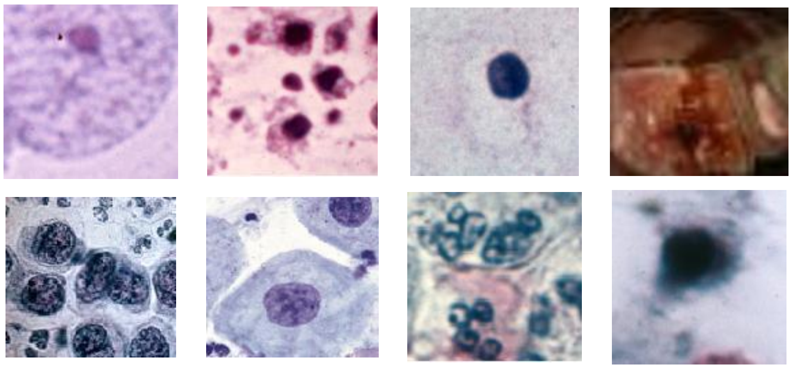


Figure 4: *Cervical Cytology images.*

The step by step analysis of the proposed method applied on cervical cytology image of both ‘Mosaic’ and ‘blob’ type images are shown below.

Step 1: Read the Input Cytology Image (Mosaic)

Figure 5: *Input cytology image.*

Since it is very difficult to take the pixel values of the entire image, the image is cropped to the size of 5×8.

Step 2: Display the intensity values of the Input Image. Table 1 shows the intensity values of the input image of size 5×8.

Table 1: *Intensity value.*

93	95	96	100	108	112	113	109
94	96	103	111	120	126	126	121
99	103	113	120	129	13	134	127
106	110	118	123	130	134	134	128
109	114	124	128	133	133	129	126

Step 3: Sort the intensity values to select the seed points. Table 2 shows the sorted intensity values.

Table 2: *Sorting the intensity value.*

X	*Y*	Intensity *f*(*x*,*y*)
3	6	13
1	1	93
2	1	94
1	2	95
1	3	96
2	2	96
3	1	96
1	4	10
2	3	103
3	2	103
4	1	106
1	5	108
1	8	109
5	1	109
4	2	110
2	4	111
2	6	112
2	7	113
3	3	113
5	2	114

X	*Y*	Intensity *f*(*x*,*y*)
4	3	118
2	5	120
3	4	120
2	8	121
4	4	123
5	3	124
2	6	126
2	7	126
5	8	126
3	8	127
4	8	128
5	4	128
3	5	129
5	7	129
4	5	130
5	5	133
5	6	133
3	7	134
4	6	134
4	7	134

Step 4: Take the minimum intensity value as a seed point for the first region. Apply the absolute difference graph to label the regions and find the watershed line pixels as shown in Table 3. This process is repeated until all the pixels in the image are visited. The result of the proposed method is compared with other watershed segmentation methods which are defined in the definitions 2.6 and 2.7.

93	95	96	100	108	112	113	109
94	96	103	111	120	126	126	121
99	103	113	120	129	13	134	127
106	110	118	123	130	134	134	128
109	114	124	128	133	133	129	126

Table 3: *Proposed method*

1	1	1	1	1	1	W	W
1	1	1	1	1	1	W	W
1	1	1	1	1	1	1	W
1	1	1	1	1	1	W	2
1	1	1	1	1	W	2	2

Watershed labeling consistent with the local condition [Definition 2.7]

2	2	2	2	2	2	2	3
2	2	2	2	2	1	1	3
2	2	2	2	1	1	1	3
2	2	2	2	1	1	1	3
2	2	2	2	1	1	4	4

Topographical distance [Definition 2.6].

B	B	B	B	B	W	W	W
B	B	B	B	W	A	A	W
B	B	B	W	A	A	A	W
B	B	B	W	A	A	A	W
B	B	B	W	A	A	W	W

Input Image of BLOB type

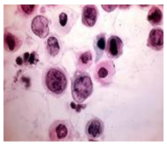


Figure 6: *Original image value.*

20	182	11	227	236	13	85	155	146
7	22	21	2	21	13	11	16	17
20	22	1	20	10	22	8	22	18

Absolute difference labeling

3	3	W	4	W	2	2	W	5
W	1	1	W	W	2	2	2	2
W	1	1	A	W	2	2	2	2

Topographical Distance [Definition 2.6].

C	C	W	D	D	W	B	W	E
C	W	A	W	W	B	B	B	W
W	A	A	A	W	B	B	B	W

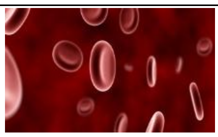
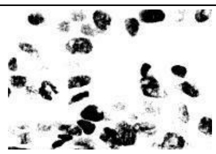
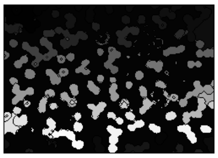
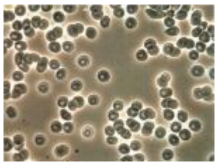
Result of Definition 2.7.

3	3	3	4	4	2	2	5	5
1	1	1	1	4	2	2	5	5
1	1	1	1	4	2	2	2	2

4 Comparative analysis

The experimental results shows that the proposed method well segments the original image into individual regions without any overlapping when compared to other two methods in terms of both number of regions and time. And at the same time the images are segmented without any over or under segmentation.

Table 4: *Comparative analysis*.

			Proposed method		**Topographical distance**	
S.No	**Image**	**Size**	**No. of regions**	**Time Taken (in seconds)**	**No. of regions**	**Time taken (in seconds)**
1		640×256	108	0.59	115	0.8
2		190×131	497	0.61	525	0.72
3		204×199	124	0.68	125	0.67
4		150×111	596	0.62	605	0.75

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