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On regular Pre Semi I closed sets in ideal topological spaces

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Abstract

In this paper, we introduce a new classes of sets namely $rpsI$- closed sets in ideal topological spaces and investigate their properties and relations.

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1 Introduction

The generalized closed sets in point set topology have been considerable interest among general topologists. Levine [12] introduced generalized closed (briefly g-closed) sets in topology. Researchers in topology studied several versions of generalized closed sets. The subject of ideals in topological spaces has been studied by Kuratowski [11] and Vaidyanathaswamy [16]. After that many topologists have contributed more on this topic. In 2010, T.Shyla Isac Mary and P.Thangavelu [15] introduced and investigated regular pre-semi-closed sets. An ideal on a set X is a non-empty collection of subsets of X with heredity property which is also closed under finite unions.

2 Preliminaries

In this section we summarize the definitions and results which are needed in sequel. By a space we always mean a topological space (X, τ) with no separation properties assumed. If $A \subseteq X$, $cl(A)$ and $int(A)$ denote the closure and interior of A in (X, τ) respectively. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\wp(X)$ is the set of all subsets of X, a set operator $(\cdot)^* : \wp(X) \rightarrow \wp(X)$, called a local function of A with respect to I and τ is defined as follows: for $A \subseteq X$, $A^*(\mathcal{I}, \tau) = \{x \in X/A \cap U \notin \mathcal{I}, \text{ for every } U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau/x \in U\}$ [11]. Note that $cl^*(A) = A \cup A^*$ defines a Kuratowski operator for a topology $\tau^*(\mathcal{I})$ (also denoted by τ^* if there is no ambiguity), finer than τ. A basis $\beta(\mathcal{I}, \tau)$ for $\tau^*(\mathcal{I})$ can be described as follows: $\beta(\mathcal{I}, \tau) = \{U \setminus I : U \in \tau \text{ and } I \in \mathcal{I}\}$. Note that β is not always a topology [8]. $cl^*(A)$ and $int^*(A)$ denote the closure and interior of A in (X, τ^*) respectively.

Definition 2.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called*

i. I-open [7] if $A \subseteq int(A^*)$

ii. regular I-open [9] if $A = int(cl^*(A))$

iii. pre I-open [2] if $A \subseteq int(cl^*(A))$

iv. semi I-open [6] if $A \subseteq cl^*(int(A))$

v. αI-open [6] if $A \subseteq int(cl^*(int(A)))$

vi. semi pre I-open [6] if $A \subseteq cl(int(cl^*(A)))$

The complement of the above mentioned generalized I-open sets are their respective I-closed sets.

The semipre I-closure (resp. semi I-closure, pre I-closure, αI-closure, I-closure, regular I-closure) of a subset A of $(X, \tau, \mathcal{I})$ is the intersection of all semi pre I-closed (resp. semi I-closed, pre I-closed, αI-closed, I-closed, regular I-closed) sets containing A and is denoted by $spIcl(A)$ (resp. $sIcl(A)$, $pIcl(A)$, $\alpha Icl(A)$, $Icl(A)$, $rIcl(A)$).

The following is useful in sequel.

Lemma 2.2. *[14] For any subset A of an ideal topological space $(X, \tau, \mathcal{I})$, the following results hold:*

$sIcl(A) = A \cup int(cl^*(A))$

$pIcl(A) = A \cup cl^*(int(A))$

$spIcl(A) = A \cup int(cl^*(int(A)))$

$cl^*(int(A \cup B)) = cl^*(int(A)) \cup cl^*(int(B))$

Lemma 2.3. *[13] Let $(X, \tau, \mathcal{I})$ be an ideal space and $A \subseteq X$. If $A \subseteq A^*$, then $A^* = cl(A) = cl^*(A)$.*

Remark 2.4. *A is open if and only if $int(A) = A$ and A is *-open if and only if $A = int^*(A)$.*

Definition 2.5. *A space X is called extremally disconnected [17] if the closure of each open subset of X is open.*

Definition 2.6. *A subset A of a space X is called generalized closed [12] (g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.*

3 $rpsI$-closed sets

Lemma 2.2 and Definitions 2.1 motivate us to introduce the concept of some generalized closed sets via ideals. In this section, we define some generalized closed sets and study their properties.

Definition 3.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called*

i. *generalized I-closed (gI-closed) if $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is I-open.*

ii. *regular generalized I-closed (rgI-closed) if $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is regular I-open.*

iii. *pre generalized pre regular-I closed ($pgprI$-closed) if $pIcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular generalized I-open.*

iv. *regular pre semi I-closed ($rpsI$-closed) if $spIcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular generalized I-open.*

The complements of the above mentioned I-closed sets are their respective I-open sets.

Remark 3.2. *A subset of a $rpsI$-closed set need not be $rpsI$-closed set.*

Example 3.3. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space, the set $\{a, b, d\}$ is $rpsI$-closed but the subset $\{b\}$ is not $rpsI$-closed.*

Theorem 3.4. *Every semi pre I-closed set is $rpsI$-closed.*

Proof. Let A be a semi pre I-closed set in X. Let $A \subseteq U$ and U be rgI-open. Since A is semi pre I-closed we have $spIcl(A) = A \subseteq U$ and U is rgI-open. Therefore A is $rpsI$-closed. ■

The following example shows that the converse of the above theorem is not true.

Example 3.5. *In Example 3.3, $\{a, b, d\}$ is $rpsI$-closed but not semi pre I-closed.*

Theorem 3.6. *Every $pgprI$-closed set is $rpsI$-closed.*

Proof. Let A be a $pgprI$-closed in X. Let $A \subseteq U$ and U be rgI-open. Since A is $pgprI$-closed we have $pIcl(A) \subseteq U$. Also $spIcl(A) \subseteq pIcl(A) \subseteq U$. Therefore A is $rpsI$-closed. ■

The following example shows that the converse of the above theorem is not true.

Example 3.7. *In Example 3.3, $\{b, c\}$ is $rpsI$-closed but not $pgprI$-closed.*

Theorem 3.8. *Every pre I-closed set is $rpsI$-closed.*

Proof. Let A be a pre I-closed set in X. We know that pre I-closure of A is the smallest pre I-closed containing A. Therefore $pIcl(A) \subseteq A$. Suppose $A \subseteq U$ and U is rgI-open. Then $pIcl(A) \subseteq U$ and U is rgI-open. Therefore A is $pgprI$-closed. By Theorem 3.6, A is $rpsI$-closed. ■

The following example shows that the converse of the above theorem is not true.

Example 3.9. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the set $\{b, c\}$ is $rpsI$-closed but not pre I-closed.*

Theorem 3.10. *Every αI-closed set is $rpsI$-closed.*

Proof. Let A be a αI-closed set in X. We know that every αI-closed set is pre I-closed set. By Theorem 3.8, A is $rpsI$-closed. ■

The following example shows that the converse of the above theorem is not true.

Example 3.11. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the set $\{b, c\}$ is $rpsI$-closed but not αI-closed.*

Theorem 3.12. *Every rI-closed set is $rpsI$-closed.*

Proof. Let A be a rI-closed subset of X. Let $A \subseteq U$ and U be rgI-open. Since A is rI-closed we have $A = cl^*(int(A))$. Therefore $cl^*(int(A)) \subseteq U$ and U is rgI-open implies $int(cl^*(int(A))) \subseteq int(U) \subseteq U$ and U is rgI-open. $A \cup int(cl^*(int(A))) \subseteq A \cup U = U$ and U is rgI-open. By Lemma 2.2(iii), we have $spIcl(A) \subseteq U$ and U is rgI-open. Hence A is $rpsI$-closed. ■

The following example shows that the converse of the above theorem is not true.

Example 3.13. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the set $\{c\}$ is $rpsI$-closed but not rI-closed.*

Definition 3.14. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called SI set [6] if $cl^*(int(A)) = int(A)$.*

Theorem 3.15. *Every SI set is $rpsI$-closed.*

Proof. Let A be a SI subset of X. Let $A \subseteq U$ and U be rgI-open. Since A is SI set we have $cl^*(int(A)) = int(A)$.

Now, $A \subseteq U \Rightarrow int(A) \subseteq int(U) \subseteq U \Rightarrow cl^*(int(A)) \subseteq U \Rightarrow int(cl^*(int(A))) \subseteq int(U) \subseteq U \Rightarrow A \cup int(cl^*(int(A))) \subseteq A \cup U = U \Rightarrow spIcl(A) \subseteq U$. Hence A is $rpsI$-closed. ■

The following example shows that the converse of the above theorem is not true.

Example 3.16. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the set $\{b, c, d\}$ is $rpsI$-closed but not SI set.*

Definition 3.17. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be semi$^*I$-open [4] if $A \subseteq cl(int^*(A))$.*

Lemma 3.18. *[3] For an ideal topological space $(X, \tau, \mathcal{I})$ and a subset K of X, the following properties are equivalent:*

i. K is an rI-closed set.

ii. K is semi$^* - I$-open and closed.

Theorem 3.19. *For an ideal topological space $(X, \tau, \mathcal{I})$ and a subset K of X. If K is semi$^* - I$-open and closed then K is $rpsI$-closed.*

Proof. By Lemma 3.18 and Theorem 3.12, we have K is $rpsI$-closed. ∎

Lemma 3.20. *[3] For an ideal topological space $(X, \tau, \mathcal{I})$ and a subset K of X, the following properties are equivalent:*

i. K is an rI-closed set.

ii. there exists a *-open set L such that $K = cl(L)$.

Theorem 3.21. *For an ideal topological space $(X, \tau, \mathcal{I})$ and a subset K of X. Suppose there exists a *-open set L such that $K = cl(L)$ then K is $rpsI$-closed set.*

Proof. By Lemma 3.20 and Theorem 3.12, we have K is $rpsI$-closed. ∎

Definition 3.22. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be weakly semi I-open [5] (WSI-open) if $A \subseteq cl^*(int(cl(A)))$. The complement of weakly semi I-open set is weakly semi-I-closed.*

Lemma 3.23. *[5] If a subset A of a space $(X, \tau, \mathcal{I})$ is weakly semi I-closed then A is semi pre I-closed.*

Theorem 3.24. *Every weakly semi I-closed set is $rpsI$-closed.*

Proof. Let A be weakly semi I-closed. By Lemma 3.23, A is semi pre I-closed. By Theorem 3.4, we have A is $rpsI$-closed. ∎

The following example shows that the converse of the above theorem is not true.

Example 3.25. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the set $\{a, b, d\}$ is $rpsI$-closed but not weakly semi I-closed.*

Remark 3.26. *Consider the above example. In this ideal space,*

i. $\{a\}$ is semi I-closed but not closed.

ii. $\{c\}$ is semi I-closed but not rI-closed.

iii. *Every rI-closed set is αI-closed. But the converse need not be true, for example the set $\{c\}$ is αI closed but not rI-closed.*

iv. *$\{a\}$ is pre I-closed but not closed.*

v. *$\{a, c\}$ is weakly semi I-closed but not αI-closed.*

vi. *$\{b, c, d\}$ is weakly semi I-closed but not SI set.*

vii. *g-closed and SI sets are independent to each other. For example, the set $\{b, d\}$ is g-closed but not SI set and $\{a\}$ is SI set but not g-closed.*

viii. *g-closed and semi pre I-closed sets are independent to each other. For example, the set $\{a, b, d\}$ is g-closed but not semi pre I-closed and $\{a\}$ is semi pre I-closed but not g-closed.*

ix. *$\{b, c\}$ is semi pre I-closed but not pre I-closed.*

x. *Every closed set is $pgprI$-closed but the converse need not be true. The set $\{a\}$ is $pgprI$-closed but not closed.*

Remark 3.27. *The concepts of g-closed set and $rpsI$-closed set are independent. In Example 3.3, $\{b, d\}$ is g-closed but not $rpsI$-closed and $\{a\}$ is $rpsI$-closed but not g-closed. Similarly, The concepts of $pgprI$-closed set and semi I-closed set are independent. In Example 3.3, $\{a, b, d\}$ is $pgprI$-closed but not semi I-closed. $\{b, c\}$ is semi I-closed but not $pgprI$-closed.*

Theorem 3.28. *If A is $rpsI$-closed and $cl^*(int(A))$ is open. Then A is $pgprI$-closed.*

Proof. Let $A \subseteq U$ and U be rgI-open. Since A is $rpsI$-closed, $spIcl(A) \subseteq U$ whenever $A \subseteq U$ and U is rgI-open. By Lemma 2.2(iii), $A \cup int(cl^*(int(A))) \subseteq U$ which implies $A \cup (cl^*(int(A))) \subseteq U$ by Remark 2.4. Again by Lemma 2.2(ii), $pIcl(A) \subseteq U$ whenever $A \subseteq U$ and U is rgI-open. Therefore A is $pgprI$-closed. ∎

Remark 3.29. *The union of two $rpsI$-closed sets need not be a $rpsI$-closed set.*

Example 3.30. *Consider the ideal topological space in Example 3.3. In this ideal topological space the sets $\{a\}$ and $\{b, c\}$ are $rpsI$-closed sets. But their union $\{a, b, c\}$ is not $rpsI$-closed set.*

Remark 3.31. *The intersection of two $rpsI$-closed sets need not be a $rpsI$-closed.*

Example 3.32. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the sets $\{b, c\}$ and $\{a, b, d\}$ are $rpsI$-closed sets, but their intersection $\{b\}$ is not $rpsI$-closed sets.*

Theorem 3.33. *Every closed set is $rpsI$-closed set.*

Proof. Let A be a closed set in X. Let $A \subseteq U$ and U be rgI-open. Since A is closed we have $A = cl(A)$, $cl(A) \subseteq U$. But $spIcl(A) \subseteq cl(A) \subseteq U$. Therefore A is $rpsI$-closed. ■

The following example shows that the converse of the above theorem is not true.

Example 3.34. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the set $\{a\}$ is $rpsI$-closed but not closed.*

Theorem 3.35. *Suppose A is rgI-open and A is $rpsI$-closed then A is semi pre I-closed.*

Proof. Since A is rgI-open and A is $rpsI$-closed and $A \subseteq A$, we have $spIcl(A) \subseteq A$. Therefore A is semi pre I-closed. ■

Theorem 3.36. *If A is $rpsI$-closed, then $spIcl(A) \setminus A$ does not contain a non empty rgI-closed set.*

Proof. Suppose A is $rpsI$-closed. Let F be a rgI-closed subset of $spIcl(A)\setminus A$. Then $F \subseteq spIcl(A)\cap (X \setminus A) \subseteq X \setminus A$ and $A \subseteq X \setminus F$. But A is $rpsI$-closed and since $X \setminus F$ is rgI-open, we have $spIcl(A) \subseteq X \setminus F$. Therefore $F \subseteq X \setminus spIcl(A)$. Since $F \subseteq spIcl(A)$, we have $F \subseteq (X \setminus spIcl(A)) \cap spIcl(A) = \phi$ implies $F = \phi$. Therefore $spIcl(A) \setminus A$ does not contain a non empty rgI-closed set. ■

Theorem 3.37. *If A is $rpsI$-closed and if $A \subseteq B \subseteq spIcl(A)$ then*

i. B is $rpsI$-closed.

ii. $spIcl(B) \setminus B$ contains no non-empty $rpsI$-closed sets.

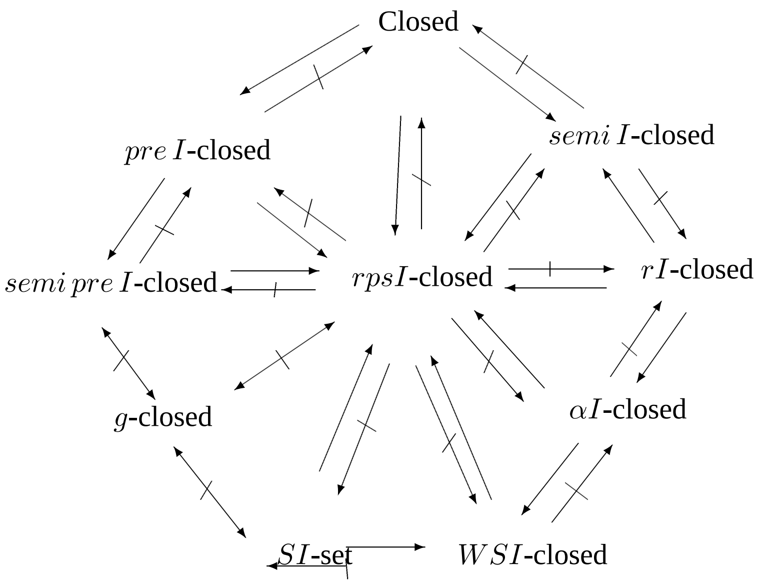
Proof. i. Given $A \subseteq B \subseteq spIcl(A)$. Then $spIcl(A) = spIcl(B)$. Suppose that $B \subseteq U$ and U is rgI-open. Since A is $rpsI$-closed and $A \subseteq B \subseteq U$, $spIcl(A) \subseteq U$ we have $spIcl(B) \subseteq U$. Therefore B is $rpsI$-closed.
ii. The proof follows from Theorem 3.34. ■

Theorem 3.38. *Let A be $rpsI$-closed. Then A is semi pre I-closed iff $spIcl(A) \setminus A$ is rgI-closed.*

Proof. If A is semi pre I-closed, then $spIcl(A) = A$. Therefore $spIcl(A)\setminus A = \phi$ which is rgI-closed. Conversely, suppose that $spIcl(A) \setminus A$ is rgI-closed. Since A is $rps - I$ closed, by Theorem 3.34 we have $spIcl(A) \setminus A = \phi$. Thus $spIcl(A) = A$. Hence A is semi pre I-closed. ■

Remark 3.39. *Every semi I-closed set is $rpsI$-closed. But the converse is not true.*

In Example 3.3 the set $\{a, b, d\}$ is $rpsI$-closed but not semi I-closed.



Theorem 3.40. *In an extremally disconnected space X, every $rpsI$-closed set is $pgprI$-closed.*

Proof. In an extremally disconnected space X, $cl^*(int(A))$ is open for every subset A of X. Then the proof follows from Theorem 3.28. ∎

Theorem 3.41. *For every point x of a space X, $X \setminus \{x\}$ is $rpsI$-closed or rgI-open.*

Proof. Suppose $X \setminus \{x\}$ is not rgI-open. Then X is the only rgI-open set containing $X \setminus \{x\}$. This implies that $spIcl(X \setminus \{x\}) \subseteq X$. Hence $X \setminus \{x\}$ is $rpsI$-closed set in X. ∎

4 $rpsI$-open sets

Definition 4.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called $rpsI$-open if its complement is $rpsI$-closed.*

Remark 4.2. *Every I-open set is $rpsI$-open. The following example shows that the converse is not true.*

Example 4.3. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the set $\{a, b\}$ is $rpsI$-open but not I-open.*

Remark 4.4. *The union of two $rpsI$-open sets need not be $rpsI$-open. In Example 4.3, $\{a\}$ and $\{c\}$ are $rpsI$-open sets but their union $\{a, c\}$ is not $rpsI$-open set.*

Remark 4.5. *The intersection of two $rpsI$-open sets need not be $rpsI$-open. For example consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the sets $\{a, d\}$ and $\{b, d\}$ are $rpsI$-open sets but their intersection $\{d\}$ is not $rpsI$-open set.*

Theorem 4.6. *Let $(X, \tau, \mathcal{I})$ be an ideal space and $A \subseteq X$. If $A \subseteq A^*$ and A^* is $rpsI$-closed. Then $X \setminus cl^*(A)$ is $rpsI$-open.*

Proof. Given $A \subseteq A^*$ then by Lemma 2.3, $A^* = cl(A) = cl^*(A)$. Also A^* is $rpsI$-closed, $X \setminus A^*$ is $rpsI$-open. Therefore $X \setminus cl^*(A)$ is $rpsI$-open. ■

Definition 4.7. *[10] Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. Then $A^*(I, \tau) = \{x \in X / A \cap U \notin I$ for every $U \in SO(X, x)\}$ is called the semi local function of A with respect to I and τ, where $SO(X, x) = \{U \in SO(X) / x \in U\}$.*

Theorem 4.8. *Let $(X, \tau, \mathcal{I})$ be an ideal space. Then $A \cup (X - A^*)$ is $rpsI$-closed iff $A^* - A$ is $rpsI$-open.*

Proof. Suppose $A \cup (X - A^*)$ is rpsI closed. Since $X - (A^* - A) = A \cup (X - A^*)$, we have $A^* - A$ is rpsI open. Converse part is obviously true. ■

Definition 4.9. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is quasi I-open [1] if $A \subseteq cl(int(A^*))$.*

Remark 4.10. *Every I-open set is quasi I-open [13] and every quasi I-open set is $rpsI$-open.*

$$I\text{-open} \longrightarrow \text{quasi } I\text{-open} \longrightarrow rpsI\text{-open}$$

The following example shows that the reverse implications need not be true.

Example 4.11. *Consider the ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. In this ideal space the set $\{a, b\}$ is $rpsI$-open but not quasi I-open. Also $\{b, d\}$ is quasi I-open but not I-open and $\{c\}$ is $rpsI$-open but not I-open.*

Remark 4.12. *A subset of a $rpsI$-open set need not be $rpsI$-open. In the ideal topological space in Example 3.3, $\{b, c, d\}$ is $rpsI$-open but the subset $\{d\}$ is not $rpsI$-open set.*

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