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# Connected point set domination of fuzzy graphs

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**Abstract**

A dominating set D of a fuzzy graph is said to be a point set dominating set if for every $S \subseteq V - D$ there exists a node $d \in D$ such that $< S \cup \{d\} >$ is a connected fuzzy graph. The minimum cardinality taken over all minimal point set dominating set is called the point set domination number of a fuzzy graph G and it is denoted by $\gamma_p(G)$. A point set dominating set D of any fuzzy graph G is a connected point set dominating set if the fuzzy subgraph$< D >$induced by D is connected. The minimum cardinality taken over all point set dominating set of a fuzzy graph G is said to be connected point set domination of a fuzzy graph G and is denoted by$\gamma_{cp}(G)$. In this paper we focus on connected point set domination number of fuzzy graph and obtain some results for new parameter of fuzzy graphs.

**Keywords:** Fuzzy graph, fuzzy star, point set dominating set of a fuzzy graph, connected point set dominating set of a fuzzy graph.

**AMS Subject Classification(2010):** 05C72.

## 1 Introduction

A Mathematical framework to describe the phenomena of uncertainty in real life situation is first suggested by L.A.Zadeh[10] in 1965. Rosenfeld[6] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. The study of dominating sets in graphs was begun by Orge and Berge. The domination number was introduced by Cockayne and Hedetniemi[1]. Sampathkumar and Pushpalatha[7] introduced the concept of point set domination in graphs. V.Swaminathan and R.Poovazhaki[9] introduced the concept of connected point set domination of graph. They also discussed point set domination with reference to degree in [5]. A.Somasundaram and S.Somasundaram[8] discussed domination in fuzzy graph using effective edges. Nagoorgani and Chandrasekeran[4] discussed domination in fuzzy graph using strong arcs. In this paper we introduce the concept of point set domination and connected point set domination in fuzzy graphs using effective edges. We concentrate on connected point set domination number of fuzzy graph and obtain some interesting results for new parameter of fuzzy graphs.

## 2 Preliminaries

In this section, the basic definitions and known results related to fuzzy graph are given.

**Definition 2.1.**[3] *Let E be the universal set. A fuzzy set A in E is represented by* $A = \{(x, \mu_A(x)) : \mu_A(x) > 0, x \in E\}$, *where the function* $\mu_A: E \to [0,1]$ *is the membership degree of x in the fuzzy set A.*

**Definition 2.2.**[3] *A fuzzy graph* $G(\sigma, \mu)$ *is a pair of function* $\sigma: V \to [0,1]$ *and* $\mu: V \times V \to [0,1]$, *where for all* $u, v \in V$, *we have* $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$.

**Definition 2.3.**[3] *The fuzzy graph* $H(\tau,\rho)$ *is called a fuzzy subgraph of* $G(\sigma,\mu)$ *if* $\tau(u) \le \sigma(u)$ *for all* $u \in V$ *and* $\rho(u,v) \le \mu(u,v)$ *for all* $u, v \in V$.

**Definition 2.4.**[3] *The fuzzy subgraph* $H(\tau, \rho)$ *is said to be a spanning fuzzy subgraph of* $G(\sigma, \mu)$ *if* $\tau(u) = \sigma(u)$ *for all* $u \in V$ *and* $\rho(u,v) \le \mu(u,v)$ *for all* $u, v \in V$.

**Definition 2.5.**[3] *The fuzzy subgraph of* $G(\sigma, \mu)$ *induced by* $\tau$ *is the maximal fuzzy subgraph of* $G(\sigma, \mu)$ *that has fuzzy node set* $\tau$. *Evidently, this is just the fuzzy graph* $(\tau,\rho)$ *where* $\rho(u,v) = \tau(u) \wedge \tau(v) \wedge \mu(u,v)$ *for all* $u, v \in V$.

**Definition 2.6.**[3] *Two nodes that are joined by a path are said to be connected. The relation connected is reflexive, symmetric and transitive. The equivalence classes of nodes under this relation are the connected components of the given fuzzy graph.*

**Definition 2.7.**[3] *Let* $G(V, \sigma, \mu)$ *be a fuzzy graph. Define the degree of a vertex v to be* $d(v) = \sum_{u \ne v} \mu(u,v)$. *The minimum degree of G is* $\delta(G) = \wedge \{d(v)/v \in V\}$ *and maximum degree of G is* $\Delta(G) = \vee \{ d(v)/v \in V\}$.

**Definition 2.8.**[8] *Let* $G(\sigma, \mu)$ *be a fuzzy graph. A subset D of V is said to be a dominating set of G if for every* $v \in V - D$ *there exists an element* $u \in D$ *such that* $\mu(u,v) = \sigma(u) \wedge \sigma(v)$. *A dominating set D of G is called the minimal dominating set of G if every node* $v \in D, D - \{v\}$ *is not a dominating set. The minimum scalar cardinality of D is called the domination number and is denoted by* $\gamma(G)$.

*Note that scalar cardinality of a fuzzy subset D of V is* $|D|_f = \sum_{v \in D} \sigma(v)$.

**Example 2.9.** *For the fuzzy graph G given in Figure 1, D*={*s,q*}is a *minimal dominating set and the domination number* $\gamma(G)=1.1$.

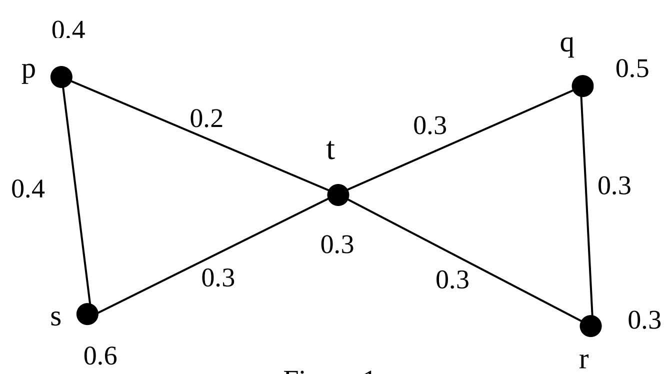


Figure 1

**Definition 2.10.**[9] *A dominating set D is a connected dominating set of a fuzzy graph G if the fuzzy subgraph <D> induced by D is connected. The minimum cardinality taken over all minimal connected dominating set is called the connected domination number of G and is denoted by* $\gamma_c(G)$.

## 3 Main Results

**Definition 3.1.** *A dominating set $D \subseteq V$ of a fuzzy graph G is said to be a point set dominating set of G if for every $S \subseteq V - D$ there exists a node $d \in D$ such that $< S \cup \{d\} >$is a connected fuzzy graph. The minimum cardinality taken over all minimal point set dominating set is called the point set domination number of the fuzzy graph G and it is denoted by $\gamma_p(G)$.*

**Definition 3.2.** *A point set dominating set $D \subseteq V(G)$ of any fuzzy graph G is a connected point set dominating set of G if the subgraph $< D >$induced by D is a connected fuzzy graph. The minimum cardinality taken over all minimal connected point set dominating set is called the connected point set domination number $\gamma_{cp}(G)$.*

**Definition 3.3.** *A fuzzy graph $G(\sigma, \mu)$ is said to be a fuzzy star if every vertex of G has exactly one strong neighbour u (say) in $V(G)$.*

**Proposition 3.4.** *A fuzzy graph G has a connected point set dominating set if and only if G is a connected fuzzy graph.*

**Proof.** Let G be a fuzzy graph with connected point set dominating set D. Since $D$ is a fuzzy dominating set and $< D >$ is connected for every $v \in V - D$ there exists some $u$ in $D$ such that $u$ dominates $v$. That is, $G$ is a connected fuzzy graph.
Conversely, let G be a connected fuzzy graph. If G is a block, then $D = V(G) - \{u\}$ is a connected fuzzy point set dominating set for any $u \in V(G)$. If G is a separable graph then $D = V(G) - \{u\}$ is a connected point set dominating set for any non fuzzy cut node $u \in V(G)$. Hence every connected fuzzy graph has a connected fuzzy point set dominating set. ■

**Theorem 3.5.** *If T is a fuzzy tree, $u \in V(T)$ is a support and if W denotes the set of all pendant vertices at u, then $D = V(T) - W$ is a connected point set dominating set of the fuzzy tree T.*

**Proof.** Let T be a fuzzy tree, and $u \in V(T)$ be a support. W denotes the set of all end nodes at $u$. Suppose $D = V(T) - W$ is not a connected point set dominating set of any fuzzy tree. Then we have the following two cases.

**Case (i):** $< D >$ contains an isolated vertex.

Let $u \in V(< D >) = V(T) - W$ be an isolated vertex. Let $v \neq u$ be any node in the fuzzy tree *T*. Then there is no path between u and v. If G is connected, it remains connected when any end node is removed from it. If G is not connected then G is not a fuzzy tree. This implies T is not a fuzzy tree, which is a contradiction. Therefore, D is a connected point set dominating set of the fuzzy tree *T*.

**Case (ii):** $< D >$contains more than two components.

Let $G_1$, $G_2$ be two components in $< D >$. Let $u \in G_1$ and $v \in G_2$ be any two vertices in $< D >= V(T) - W$. There is no path between the vertices $u$ and $v$. This implies that T is not a fuzzy tree,which is a contradiction. Therefore, D is a connected point set dominating set of the fuzzy tree *T*. ■

**Theorem 3.6.** *A subset D of V(G) is a connected point set dominating set of a fuzzy graph G if and only if G has a spanning fuzzy tree TG satisfying the following two conditions.*

- *i.* *Each $v \in V - D$ is a pendant vertex in TG.*
- *ii.* *For every subset $S \subseteq V - D$ with $< S >$ independent in G, there exists a non pendant vertex u in TG such that $S \subseteq N(u)$.*

**Proof.** Let $G$ be a fuzzy graph and $D \subseteq V(G)$ be a connected point set dominating set of $G$. Then $< D >$ is connected. Let $T < D >= (\tau, \rho)$ be a fuzzy tree of $< D >$. D is a point set dominating set implies that for every independent set $S \subseteq V - D$, there exists $u \in$D such that $< S \cup \{u\} >$ is connected, Therefore,$S \subseteq N(u)$. Hence $D = V(T < D >)$and for every independent set $S \subseteq V - D$ there exists a vertex $u \in V(T < D >)$ such that $S \subseteq N(u)$.

We obtain a fuzzy spanning tree TG of G as follows:

i. If there exists a vertex $v \in V - D$ which has more than one strong neighbour in $V(TD)$, then delete all such edges except one.
ii. Delete all edges in $< V - D >$ (or) reduce the arc weights of the edges (u,v) such that no pair $(u,v)$ has strong neighbour for $u, v \in < V - D >$.

Then TG is a spanning fuzzy tree of fuzzy graph G with V-D as the set of pendant vertices such that for every set $S \subseteq V - D$ with <S> independent in G, there exists a non pedant vertex $u$ in TG such that $S \subseteq N(u)$.

Conversely, if $G$ has a spanning fuzzy tree TG satisfying conditions (i) and (ii), then $< D >$ is connected (by condition (i)) and D is a point set dominating set of a fuzzy graph G (by condition (ii) ) That is, $D$ is a connected point set dominating set of the fuzzy graph $G$.

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