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Nordhaus-Gaddum Type results in Majority Domination

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Abstract

In this paper we deal with Nordhaus-Gaddum type results in majority domination number of G and $\overline{G}$ and also majority domatic number of G and $\overline{G}$. We also establish bounds for $\gamma_M(G)+d_M(G)$ and $\gamma_M(G)d_M(G)$.

Keywords: Majority dominating set, majority domination number $\gamma_M(G)$, majority domatic number $d_M(G)$.
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1 Introduction and Definitions

Nordhaus and Gaddum[2] established the inequality for the chromatic numbers $\chi(G)$ and $\chi(\overline{G})$ as $2\sqrt{p} \leq \chi(G) + \chi(\overline{G}) \leq p + 1$ and $p \leq \chi(G)\chi(\overline{G}) \leq \frac{(p+1)^2}{4}$. In 1972, Jaeger and Payan[3] published the first Nordhaus-Gaddum type result involving domination. In this paper we find Nordhaus-Gaddum type results in majority domination.

By a graph we mean a finite undirected graph without loops or multiple edges. Let $G = (V, E)$ be a finite graph with p vertices and q edges and v be a vertex in V. The closed neighborhood of v is defined by $N[v] = N(v) \cup \{v\}$.

Definition 1.1. *[4] A subset $S \subseteq V(G)$ of vertices in a graph $G = (V, E)$ is called a majority dominating set if at least half of the vertices of $V(G)$ are either in S or adjacent to the elements of S. That is, $|N[S]| \geq \left\lceil \frac{|V(G)|}{2} \right\rceil$.*

Definition 1.2. *A majority dominating set S is minimal if no proper subset of S is a majority dominating set. The minimum cardinality of a minimal majority dominating set is called the majority domination number and is denoted by $\gamma_M(G)$. The maximum cardinality of a minimal majority dominating set is denoted by $\Gamma_M(G)$. Also, majority dominating sets have superhereditary property.*

Theorem 1.3. *[4] Let S be a majority dominating set of G. S is minimal if and only if for every $v \in S$ one of the following conditions holds:*
(i) $|N[S]| > \left\lceil \frac{p}{2} \right\rceil$ and $|Pn[v, S]| > |N[S]| - \left\lceil \frac{p}{2} \right\rceil$.
(ii) $|N[S]| = \left\lceil \frac{p}{2} \right\rceil$ and either v is an isolate of S or $Pn[v, S] \cap (V - S) \neq \phi$.

Definition 1.4. *[5] The majority domatic number $d_M(G)$ of a graph G is the maximum number of elements in a partition of $V(G)$ into majority dominating sets.*

2 Nordhaus-Gaddum type results in majority domination of G and $\overline{G}$

Theorem 2.1. *[6] Let G be a graph without isolates. Then $\gamma_M(G) \leq \lceil \frac{p}{4} \rceil$.*

Proposition 2.2. *[4] Let G be any graph with p vertices. Then $\gamma_M(G) = 1$ if and only if there exists a vertex of degree $\geq \lceil \frac{p}{2} \rceil - 1$.*

Theorem 2.3. *For any graph G, $\gamma_M(G) + \gamma_M(\overline{G}) \leq \lceil \frac{p}{2} \rceil + 1$ and equality holds if and only if $G = K_p$ or $\overline{K_p}$.*

Proof. Case(i) : Let G or $\overline{G}$ has an isolate. If G has an isolate, then $\gamma_M(G) \leq \lceil \frac{p}{2} \rceil$ and $\gamma_M(\overline{G}) = 1$. A similar result holds when $\overline{G}$ has an isolate. Therefore, in both the cases $\gamma_M(G) + \gamma_M(\overline{G}) \leq \lceil \frac{p}{2} \rceil + 1$.
Case(ii) : Neither G nor $\overline{G}$ has isolates. By Theorem 2.1,
$\gamma_M(G) \leq \lceil \frac{p}{4} \rceil$ and $\gamma_M(\overline{G}) \leq \lceil \frac{p}{4} \rceil$. Hence $\gamma_M(G) + \gamma_M(\overline{G}) \leq \lceil \frac{p}{2} \rceil + 1$.
Let $\gamma_M(G) + \gamma_M(\overline{G}) = \lceil \frac{p}{2} \rceil + 1$.
Suppose G has an isolate, then $\gamma_M(\overline{G}) = 1$. Then $\gamma_M(G) = \lceil \frac{p}{2} \rceil \Rightarrow G = \overline{K_p}$.
Suppose $\overline{G}$ has an isolate, then $\gamma_M(G) = 1$. Then $\gamma_M(\overline{G}) = \lceil \frac{p}{2} \rceil \Rightarrow \overline{G} = \overline{K_p}$. Hence $G = K_p$.
Suppose G or $\overline{G}$ has no isolates. Then by Theorem, $\gamma_M(G) \leq \lceil \frac{p}{4} \rceil$ and $\gamma_M(\overline{G}) \leq \lceil \frac{p}{4} \rceil$. Then $\gamma_M(G) + \gamma_M(\overline{G}) \leq 2 \lceil \frac{p}{4} \rceil$.
Case(i) : When $p \equiv 0, 3 (mod\ 4)$.
Then, $2 \lceil \frac{p}{4} \rceil < \lceil \frac{p}{2} \rceil + 1$. Therefore, $\gamma_M(G) + \gamma_M(\overline{G}) < \lceil \frac{p}{2} \rceil + 1$, which is a contradiction.
Case(ii) : When $p \equiv 1, 2 (mod\ 4)$.
Suppose G or $\overline{G}$ is disconnected. Then $\gamma_M(G) + \gamma_M(\overline{G}) \leq \lceil \frac{p}{4} \rceil + 1 < \lceil \frac{p}{2} \rceil + 1$ $p \geq 3$, which is a contradiction. If $p = 2$ then $G = \overline{K_2}$. Therefore, G and $\overline{G}$ are connected graphs with out isolates. Then $\gamma_M(G) + \gamma_M(\overline{G}) \leq 2 \lceil \frac{p}{4} \rceil$.
By assumption, $\lceil \frac{p}{2} \rceil + 1 \leq 2 \lceil \frac{p}{4} \rceil$.
If $p \equiv 1, 2 (mod\ 4)$, then $\lceil \frac{p}{2} \rceil + 1 = 2 \lceil \frac{p}{4} \rceil$. This implies that $\gamma_M(G) \leq \lceil \frac{p}{4} \rceil$ and $\gamma_M(\overline{G}) \leq \lceil \frac{p}{4} \rceil$. Since G and $\overline{G}$ are connected graphs with $\gamma_M(G) \leq \lceil \frac{p}{4} \rceil$ and $\gamma_M(\overline{G}) \leq \lceil \frac{p}{4} \rceil$, we have $p = 3, 4, 7, 8$. But $p \equiv 1, 2 (mod\ 4)$, a contradiction. Therefore $G = K_p$ or $\overline{K_p}$.
The converse is obvious. ∎

Theorem 2.4. *For any graph G, $\gamma_M(G)\gamma_M(\overline{G}) \leq \lceil \frac{p}{2} \rceil$.*

Proof. For a set of vertices $X \subseteq V(G)$, define $D_e(X) = \{v \in V - X :$ v is adjacent to at least $\left\lceil \frac{|X|}{2} \right\rceil$ vertices in $X\}$ and $D_i(X) = \{u \in X : u$ is adjacent to at least $\left\lceil \frac{|X|}{2} \right\rceil$ vertices of X other than $u\}$.

Let $d_e(X) = |D_e(X)|$ and $d_i(X) = |D_i(X)|$. Let $S = \{x_1, x_2, x_3, \cdots, x_{\gamma_M(G)}\}$ be a γ_M-set of G.

Choose a set $W \subseteq V(G)$ such that $|W| = \lceil \frac{p}{2} \rceil$ and W contains S. Now partition W into subsets $B_1, B_2, ...B_{\gamma_M(G)}$ such that $x_j \in B_j$ and at least $\left\lceil \frac{|B_j|}{2} \right\rceil$ vertices of B_j are adjacent to x_j.

Choose a partition P for which $d_i(B_1)+d_i(B_2)+........+d_i(B_{\gamma_M(G)})$ is maximum. Suppose that $d_e(B_j)\geq 1$. Then there exists $x\in V-(B_j)$ such that x is adjacent to at least $\left\lceil\frac{|B_j|}{2}\right\rceil$ vertices of B_j. Then there exists an unique k such that $x\in B_k$. Suppose $x\in D_i(B_k)$. Then $(S-\{x_j,x_k\})\cup\{x\}$ is a majority dominating set, a contradiction to the assumption that S is a γ_M-set. Therefore, $x\notin D_i(B_k)$.

Construct a new partition P' with $B'_r=B_r$ except $r=j,k$. $B'_j=B_j\cup\{x\}$ and $B'_k=B_k-\{x\}$, for which $d_i(B'_r)=d_i(B_r)$, $r\neq j,k$. $d_i(B'_j)=d_i(B_j)+1$ and $d_i(B'_k)\geq d_i(B_k)$. Then $\sum d_i(B'_l)>\sum d_i(B_j)$ which is a contradiction to the choice of P. Therefore, $d_e(B_j)=0$.

If $d_e(X)=0$ then X is a majority dominating set in $\overline{G}$. Hence B_j is a majority dominating set in $\overline{G}$ for all j. Therefore $\gamma_M(\overline{G})\leq|B_j|$, for all j. $\gamma_M(G)\gamma_M(\overline{G})\leq|B_1|+|B_2|+.........+\left|B_{\gamma_M(G)}\right|=\left\lceil\frac{p}{2}\right\rceil$. Hence, $\gamma_M(G)\gamma_M(\overline{G})\leq\left\lceil\frac{p}{2}\right\rceil$. ■

Corollary 2.5. *For any graph G, $d_M(\overline{G})\geq\gamma_M(G)$.*

Proof. The proof follows since $\{B_1,B_2,......,B_{\gamma_M(G)}\}$ is a partition of $V(\overline{G})$ into majority dominating sets of $\overline{G}$. ■

Corollary 2.6. *For any graph G, $d_M(G)\geq\gamma_M(\overline{G})$.*

Theorem 2.7. *Let G and $\overline{G}$ have no isolates. Then $\gamma_M(G)+\gamma_M(\overline{G})\leq\left\lceil\frac{p}{4}\right\rceil+1$. The bound is sharp.*

Remark 2.8. *If $\gamma_M(G)=\left\lceil\frac{p}{4}\right\rceil$ then $\gamma_M(\overline{G})$ need not be equal to 2. For example, let $G=C_4$. Then, $\gamma_M(G)=\left\lceil\frac{p}{4}\right\rceil$ where as $\gamma_M(\overline{G})=1$.*

3 Nordhaus-Gaddum type results in majority domatic number of G and $\overline{G}$

Proposition 3.1. *[5] Let G be any graph with p vertices. Then $1\leq d_M(G)\leq p$ and the bounds are sharp.*

Theorem 3.2. *[5] For any graph G, $d_M(G)=p$ if and only if $\delta(G)\geq\left\lceil\frac{p}{2}\right\rceil-1$.*

Theorem 3.3. *Let G be any graph with p vertices. Then*
(i) $d_M(G)+d_M(\overline{G})\leq 2p$.
(ii) $d_M(G)d_M(\overline{G})\leq p^2$ and equality holds if and only if all the vertices of G and $\overline{G}$ have degree $\left\lceil\frac{p}{2}\right\rceil-1$ or $\left\lfloor\frac{p}{2}\right\rfloor$.

Proof. The two inequalities can be obtained using Proposition 3.1.

Let $d_M(G)+d_M(\overline{G})=2p$. Then $d_M(G)=p=d_M(\overline{G})$. By Theorem 3.2, $\delta(G)\geq\left\lceil\frac{p}{2}\right\rceil-1$ and $\delta(\overline{G})\geq\left\lceil\frac{p}{2}\right\rceil-1$. It implies that $\left\lceil\frac{p}{2}\right\rceil-1\leq\delta(G)\leq\Delta(G)\leq p-1-\left\lceil\frac{p}{2}\right\rceil+1=\left\lfloor\frac{p}{2}\right\rfloor$. Hence all the vertices of G and $\overline{G}$ have degree $\left\lceil\frac{p}{2}\right\rceil-1$ or $\left\lfloor\frac{p}{2}\right\rfloor$. [When p is odd, every vertex of G as well as $\overline{G}$ has degree $(\frac{p-1}{2})$. When p is even, every vertex of G as well as $\overline{G}$ has degree $(\frac{p}{2})-1$ or $(\frac{p}{2})$]. Conversely, let all the vertices of G and $\overline{G}$ have degree $\left\lceil\frac{p}{2}\right\rceil-1$ or $\left\lfloor\frac{p}{2}\right\rfloor$. Then $\delta(G)\geq\left\lceil\frac{p}{2}\right\rceil-1$ and $\delta(\overline{G})\geq\left\lceil\frac{p}{2}\right\rceil-1$. Therefore $d_M(G)=p=d_M(\overline{G})$. Hence $d_M(G)+d_M(\overline{G})=2p$. ■

Theorem 3.4. *Let G be any graph with p vertices. Then $d_M(G) + d_M(\overline{G}) \geq p+1$ and equality holds if and only if $G = K_p$ or $\overline{K_p}$, (p odd).*

Proof. Suppose there exists t-vertices with degree $\geq \lceil \frac{p}{2} \rceil - 1$. When $t = 0$ or $t = p$ then $d_M(G) \geq 1$ and $d_M(\overline{G}) = p$ or $d_M(G) = p$ and $d_M(\overline{G}) \geq 1$ respectively. In either case, $d_M(G) + d_M(\overline{G}) \geq p+1$. Let $0 < t < p$. Then $d_M(G) \geq t+1$. In $\overline{G}$, $p-t$ vertices have degree greater than $\lfloor \frac{p}{2} \rfloor$. That implies $d_M(\overline{G}) \geq (p-t)+1$. Hence, $d_M(G) + d_M(\overline{G}) \geq p+2 > p+1$. Let $G = K_p$ or ($\overline{K_p}$, p is odd). Then $d_M(G) + d_M(\overline{G}) = p+1$.
Conversely, suppose that $d_M(G) + d_M(\overline{G}) = p+1$ and there exist t-vertices with degree $\geq \lceil \frac{p}{2} \rceil - 1$. Clearly $t = 0$ or $t = p$, which is a contradiction.
Suppose $t = 0$, then $d_M(\overline{G}) = p$ and $d_M(G) = 1$. It implies that $G = \overline{K_p}$, p odd. Suppose $t = p$, then $d_M(G) = p$ and $d_M(\overline{G}) = 1$ which implies that $\overline{G} = \overline{K_p}$, p is odd. Hence, $G = K_p$, p is odd. ■

4 Bounds for $\gamma_M(G) + d_M(G)$ and $\gamma_M(G) d_M(G)$

Cockayne and Hedetniemi [1] established a bound on the sum and product of the domination number and the domatic number of a graph G. In this section, we establish a bound for the sum (product) of majority domination number and majority domatic number of a graph G.

Theorem 4.1. *[5] $d_M(G) = 1$ if and only if $G = \overline{K}_p$, p odd.*

Theorem 4.2. *[5] For any graph G, $d_M(G) \leq \left\lfloor \frac{p}{\gamma_M(G)} \right\rfloor$ and $\gamma_M(G) \leq \left\lfloor \frac{p}{d_M(G)} \right\rfloor$.*

Theorem 4.3. *[4] For any graph G, $\left\lceil \frac{p}{2(\Delta(G)+1)} \right\rceil \leq \gamma_M(G) \leq \lceil \frac{p}{2} \rceil - \Delta(G)$. The bounds are sharp.*

Theorem 4.4. *For any graph G,*
(i) $\gamma_M(G) + d_M(G) \leq \lceil \frac{p}{2} \rceil + \left\lfloor \frac{3\Delta(G)}{2} \right\rfloor + 2$ if $\Delta(G) \geq \lceil \frac{p}{2} \rceil - 1$.
(ii) $\gamma_M(G) + d_M(G) \leq \lceil \frac{p}{2} \rceil + \Delta(G) + 2$ if $\Delta(G) < \lceil \frac{p}{2} \rceil - 1$ and equality holds if and only if $G = \overline{K}_p$, p even.

Proof. By Theorem 4.2, $d_M(G) \leq \left\lfloor \frac{p}{\gamma_M(G)} \right\rfloor$.
Since $\gamma_M(G) \geq \left\lceil \frac{p}{2(\Delta(G)+1)} \right\rceil$, we have $d_M(G) \leq \left\lfloor p \, \frac{2(\Delta(G)+1)}{p} \right\rfloor$.
Therefore, $d_M(G) \leq 2\Delta(G) + 2$.
If $\Delta(G) \geq \lceil \frac{p}{2} \rceil - 1$ then $\gamma_M(G) = 1 \leq \left\lceil \frac{p-\Delta(G)}{2} \right\rceil$.
Adding we get, $\gamma_M(G) + d_M(G) \leq \left\lceil \frac{p-\Delta(G)}{2} \right\rceil + 2\Delta(G) + 2$.
Hence, $\gamma_M(G) + d_M(G) \leq \lceil \frac{p}{2} \rceil + \left\lfloor \frac{3\Delta(G)}{2} \right\rfloor + 2$.
If $\Delta(G) < \lceil \frac{p}{2} \rceil - 1$, then $\gamma_M(G) \leq \lceil \frac{p}{2} \rceil - \Delta(G)$.
Therefore, $\gamma_M(G) + d_M(G) \leq \lceil \frac{p}{2} \rceil - \Delta(G) + 2\Delta(G) + 2$, which gives $\gamma_M(G) + d_M(G) \leq \lceil \frac{p}{2} \rceil + \Delta(G) + 2$. Hence (i) and (ii) are proved.
Next we prove equality holds if and only if $G = \overline{K}_p$, p even.
Let $\gamma_M(G) + d_M(G) = \lceil \frac{p}{2} \rceil + \Delta(G) + 2$. Then $\gamma_M(G) = \lceil \frac{p}{2} \rceil$ and $d_M(G) = \Delta(G) +$

2, since $\gamma_M(G) \leq \lceil \frac{p}{2} \rceil - \Delta(G)$.

Suppose that $G \neq \overline{K_p}$, p even. Then $G = \overline{K_p}$, p odd and $\Delta(G) = 0$. By Theorem 4.1, $G = \overline{K}_p$, p odd if and only if $d_M(G) = 1$. Hence, $\gamma_M(G) + d_M(G) < \lceil \frac{p}{2} \rceil + 2$, which is a contradiction.

Suppose $G \neq \overline{K_p}$, then $\Delta(G) \geq 1$. If G has no isolates, then $\gamma_M(G) \leq \lceil \frac{p}{4} \rceil < \lceil \frac{p}{2} \rceil$ and if $\gamma_M(G) < \lceil \frac{p}{2} \rceil$ then $d_M(G) \geq 3$, since $d_M(G) \leq \left\lfloor \frac{p}{\gamma_M(G)} \right\rfloor$. Hence $\gamma_M(G) + d_M(G) \leq \lceil \frac{p}{4} \rceil + \Delta(G) + 2 < \lceil \frac{p}{2} \rceil + \Delta(G) + 2$, a contradiction.
Suppose G has isolates and $\Delta(G) \geq 1$. Then $\gamma_M(G) < \lceil \frac{p}{2} \rceil$ and $d_M(G) \geq 2$. Hence, $\gamma_M(G) + d_M(G) < \lceil \frac{p}{2} \rceil + \Delta(G) + 2$, which is a contradiction.

Therefore, $G = \overline{K}_p$, p even, is the required graph to satisfy $\gamma_M(G) = \lceil \frac{p}{2} \rceil$ and $d_M(G) = 2$. Thus $G = \overline{K}_p$, p even.
The converse is obvious. ∎

Theorem 4.5. *For a graph G, $\gamma_M(G) d_M(G) \geq 3$ if and only if $G \notin \{K_1, K_2, \overline{K}_2, \overline{K}_3\}$.*

Proof. If $\gamma_M(G) \geq 3$ or $d_M(G) \geq 3$, then $\gamma_M(G) d_M(G) \geq 3$.
Suppose $\gamma_M(G) < 3$. (i. e.), $\gamma_M(G) \leq 2$.
Case (i): $\gamma_M(G) = 1$. Let $\{u\}$ be a majority dominating set. Then
$|N(u)| \geq \lceil \frac{p}{2} \rceil - 1 \geq \lfloor \frac{p}{2} \rfloor$.
Subcase (i): Let $|N(u)| > \lfloor \frac{p}{2} \rfloor$.
Suppose that $|N(u)| = \lfloor \frac{p}{2} \rfloor$ + t ($t \geq 1$). Let $A = \{u\}$, B be a subset of $N(u)$ containing exactly $\lfloor \frac{p}{2} \rfloor$ vertices. Let C = $V(G) - A - B$. Then $|C| = \lfloor \frac{p}{2} \rfloor$. Since u belongs to $N(B)$ as well as $N(C), A, B, C$ are majority dominating sets. Therefore $d_M(G) \geq 3$.
Subcase (ii): Let $|N(u)| = \lfloor \frac{p}{2} \rfloor$ and $V(G) - \{u\}$ is totally disconnected.
Let $A = \{u\}$, B be subset of $N(u)$ containing exactly $\lceil \frac{p}{4} \rceil$ vertices and $\lceil \frac{p}{4} \rceil$ K_1 vertices. Since u belongs to $N(B)$ and $\{N(C), A, B, C\}$ are all majority dominating sets, $d_M(G) \geq 3$.
Here the following three cases arise.
(i) If $B \neq \phi$, $C = \phi$ then $G = K_2$.
(ii) If $B = \phi$, $C = \phi$ then $G = K_1$.
(iii) If $B = \phi$, $C = \phi$ then $G = \overline{K}_2$.

In all these cases, $\gamma_M(G) = 1$ and $d_M(G) = 1$ or 2. $\gamma_M(G) d_M(G) = 1$ or 2.
Hence $G \notin \{K_1, K_2, \overline{K}_2\}$.
Subcase (iii): Let $|N(u)| = \lfloor \frac{p}{2} \rfloor$ and $V(G) - \{u\}$ be not totally disconnected.
Let $A = \{u\}$. Since $V(G) - \{u\}$ is not totally disconnected, there exists an edge $e = xy$, where $x, y \in V(G) - \{u\}$. Partition $V(G) - \{u, x, y\}$ into two disjoint subsets B, C such that $|B| = \left\lceil \frac{p-3}{2} \right\rceil$ and $|C| = \left\lfloor \frac{p-3}{2} \right\rfloor$. Then $A, B \cup \{x\}$ and $C \cup \{y\}$ form a majority domatic partitions of G. Therefore $d_M(G) \geq 3$.
From sub cases (i), (ii) and (iii), if $\gamma_M(G) = 1$ then $d_M(G) \geq 3$ if and only if $G \notin \{K_1, K_2, \overline{K_2}\}$.
Case (ii): Let $\gamma_M(G) = 2$. If $d_M(G) \geq 2$ then we are through. Suppose $d_M(G) = 1$. Then $G = \overline{K}_p$, where p is odd. Since $\gamma_M(G) = 2$ and $d_M(G) = 1$, then $G = \overline{K_3}$ and $\gamma_M(G) d_M(G) \leq 2$, a contradiction. It implies that, if $G \neq \overline{K}_3$ then $d_M(G) = 1$ implies $\gamma_M(G) \geq 3$. Therefore,

$\gamma_M(G)d_M(G) \geq 3$.
Hence, $\gamma_M(G)d_M(G) \geq 3$ if and only if $G \notin \{K_1, K_2, \overline{K_2}, \overline{K}_3\}$. ■

Proposition 4.6. *Let G be a graph of order p. Then $\gamma_M(G)d_M(G) = p$ if and only if $\delta(G) \geq \lceil \frac{p}{2} \rceil - 1$.*

Proof. By Proposition 2.2, $\gamma_M(G) = 1$ if and only if $\delta(G) \geq \lceil \frac{p}{2} \rceil - 1$ and by Theorem 3.2, $\gamma_M(G)d_M(G) = p$ if and only if $\delta(G) \geq \lceil \frac{p}{2} \rceil - 1$. ■

References

[1] E. J. Cockayne, S. J. Hedetniemi , *Towards a theory of domination in graphs,* Networks - 7(1977), 247 - 261.

[2] F. Harary, Teresa W. Haynes, *Nordhaus-Gaddum inequalities for domination in graphs,* Discrete Math. 155(1996), 99-105.

[3] T.W. Haynes, S.T. Hedetniemi, Peter J. Slater, *Fundamentals of domination in graphs*, 1998 by Marcel Dekker, Inc., New york.

[4] J. Joseline Manora, V. Swaminathan, *Majority dominating sets in graphs* , Jamal Academic Research Journal, Vol. 3, No. 2(2006), 75 - 82.

[5] J. Joseline Manora, V. Swaminathan, *Majority domatic number of a graph* , Global Journal of Pure and Applied Mathematics, Vol. 6, No. 3(2010), 275 - 283.

[6] J. Joseline Manora, V. Swaminathan, *Results on majority dominating set,* Science Magna, North-West University, X'tian, P.R.China, Vol. 7, No. 3(2011), 53 - 58.