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Labeling of finite union of paths

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Abstract
In this paper we study the skolem difference mean labeling of finite union of paths.

Keywords: Skolem difference mean labeling, Skolem difference mean graphs, finite union of graphs.
AMS Subject Classification (2010): 05C78.

1 Introduction

Throughout this paper we consider only finite, undirected, simple graphs. Let *G* be a graph with *p* vertices and *q* edges. For all terminologies and notations we follow *[2]*. The following definitions are necessary for the present study. There are several types of labeling and a detailed survey can be found in *[3]*. The concept of skolem difference mean labeling was introduced in *[4]* and the results proved in *[1]* motivated the author to study the skolem difference mean labeling of finite union of paths.

Definition 1.1. A path is a walk if all the points and lines are distinct. A path on n vertices is denoted by P_n.

Definition 1.2. Let G_1 and G_2 be two graphs having point sets V_1 and V_2 and line sets X_1 and X_2 respectively. Then their union $G_1 U G_2$ has $V=V_1 U V_2$ and $X=X_1 U X_2$.

Definition 1.3. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions.

Definition 1.4. A graph *G(V,E)* with *p* vertices and *q* edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \epsilon V$ with distinct elements *f(x)* from *1,2,3…p+q* in such a way that the edge *e=uv* is labeled with $\frac{|f(u)-f(v)|}{2}$ if |*f(u)-f(v)*| is even and $\frac{|f(u)-f(v)|+1}{2}$ if |*f(u)-f(v)*| is odd and the resulting labels of the edges are distinct and from *1,2,3…q*. A graph that admits skolem difference mean labeling is called skolem difference mean graph.

The skolem difference mean labelings of the path P_6 is given in Figure 1.

.Figure 1: Skolem difference mean labeling of P_6 .

2 Main Results

Theorem 2.1. The graph mP_n is skolem difference mean for all m≥1 and n>1.

Proof. Let G be the graph mP_n

Let $V(G)=\{v_{ij}/1\le i\le m,\ 1\le j\le n\}$ and $E(G)=\{v_{ij}v_{ij+1}\ /1\le i\le m,\ 1\le j\le n-1\}$
Define $f:V(G)\rightarrow\{1,2\ldots 2mn-m\}$ as follows. Let $m\ge 1$ be odd or even.

Case (i). n is odd.

$v_{i2j+1}=2mn-m-n(i-1)-2j;\ 1\le i\le m,\ 0\le j\le \frac{n-1}{2}$ $\quad$ $v_{i2j}=m+(n-2)(i-1)+2(j-1);\ 1\le i\le m,\ 1\le j\le \frac{n-1}{2}$

Case (ii). n is even.

$v_{i2j+1}=2mn-m-(n-1)(i-1)-2j;\ 1\le i\le m,\ 0\le j< \frac{n}{2}$ $\quad$ $v_{i2j}=m+(n-1)(i-1)+2(j-1);\ 1\le i\le m,\ 1\le j\le \frac{n}{2}$

In both the cases the induced edge labels are distinct and from $1,2\ldots mn-m$. Hence the theorem. ■

The skolem difference mean labelings of the graphs $4P_5$ and $5P_4$ are given in Figure 2.

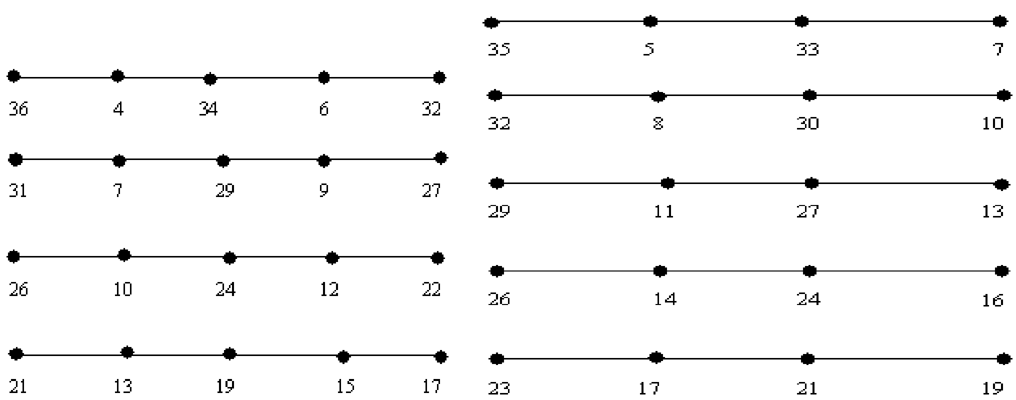

Figure 2: Skolem difference mean labelings of the graphs $4P_5$ and $5P_4$.

Theorem 2.2. The graph $mP_n\ U\ nP_m$ is skolem difference mean for all m,n>1.

Proof. Let G be the graph mP_nUnP_m
Let $V(G)=\{v_{ij},\ u_{ij}\ /1\le i\le m,\ 1\le j\le n\}$ and
$E(G)=\{v_{ij}v_{ij+1}\ /1\le i\le m,\ 1\le j\le n-1\}\cup\{u_{ji}u_{ji+1}/1\le j\le n,\ 1\le i\le m-1\}$
Define $f:V(G))\rightarrow\{1,2\ldots 4mn-m-n\}$ as follows:

Case (i). Both m and n are odd.

$v_{i2j+1}=4mn-m-ni-2j;\ 1\le i\le m,\ 0\le j\le \frac{n-1}{2}$

$v_{i2j}=m+n-2+(n-2)(i-1)+2j;\ 1\le i\le m,\ 1\le j\le \frac{n-1}{2}$

$u_{i2j+1}=3mn-n-mi-2j;\ 1\le i\le n,0\le j\le \frac{m-1}{2}$

$u_{i2j}=m+n-2+m(n-2)+(m-2)(i-1)+2j;\ 1\le i\le n,\ 1\le j\le \frac{m-1}{2}$

Case (ii). n is odd and m is odd or even. Without loss of generality let us take m as even.

$v_{i2j+1}=4mn-m-ni-2j;\ 1\le i\le m,\ 0\le j\le \frac{n-1}{2}$

$v_{i2j}=m+n-2+(n-2)(i-1)+2j;\ 1\le i\le m,\ 1\le j\le \frac{n-1}{2}$

$u_{i2j+1}=3mn-n+1-(m-1)i-2j;\ 1\le i\le n,\ 1\le j\le \frac{m}{2}$

$u_{i2j} = m+ (n-2)(m-1)+2n-4+(m-1)(i-1)+2j;\ 1\le i\le,n,\ 1\le j\le \frac{m}{2}$

Case (iii). *m* and *n* are even.

$v_{i2j+1}=4mn-m-n-(n-1)(i-1)-2j;\ 1\le i\le m,\ 0\le j< \frac{n}{2}$

$v_{i2j}=m+n-2+(n-1)(i-1)+2j;\ 1\le i\le m,\ 1\le j\le \frac{n}{2}$

$u_{i2j+1}=3mn-n-(m-1)(i-1)-2j;\ 1\le i\le n,0\le j< \frac{m}{2}$

$u_{i2j}= mn+ m+(m-1)(i-1)+2j;1\le i\le n,0\le j< \frac{m}{2}$

In all the cases the edge labels are $1,2,3\ldots 2mn-m-n$. Hence the graph $mP_n \cup nP_m$ is skolem difference mean for all values of *m* and *n*. ■

The skolem difference mean labelings of the graphs $3P_5U5P_3$, $4P_5U5P_4$ and $4P_6U6P_4$ are given in Figures 3, 4 and 5.

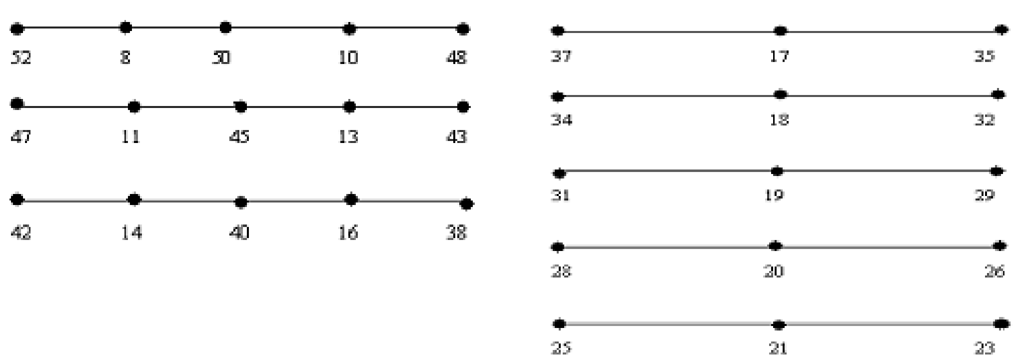

Figure 3: Skolem difference mean labelings of the graph $3P_5U5P_3$

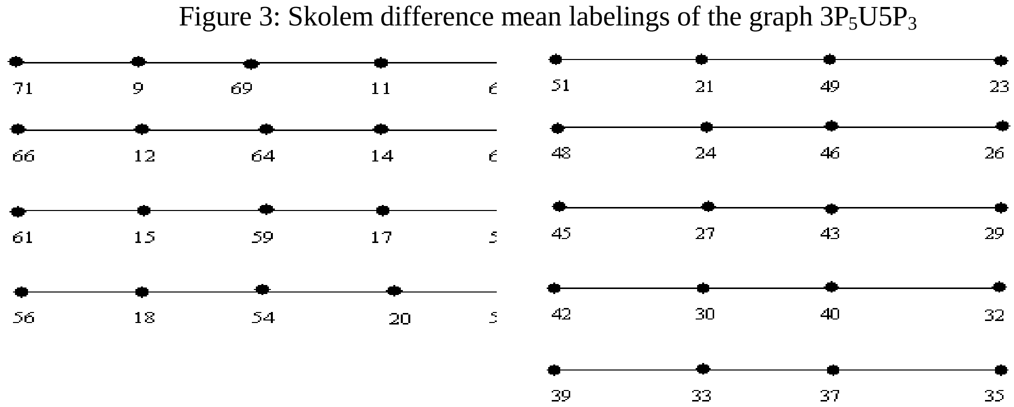

Figure 4: Skolem difference mean labelings of the graph $4P_5U5P_4$

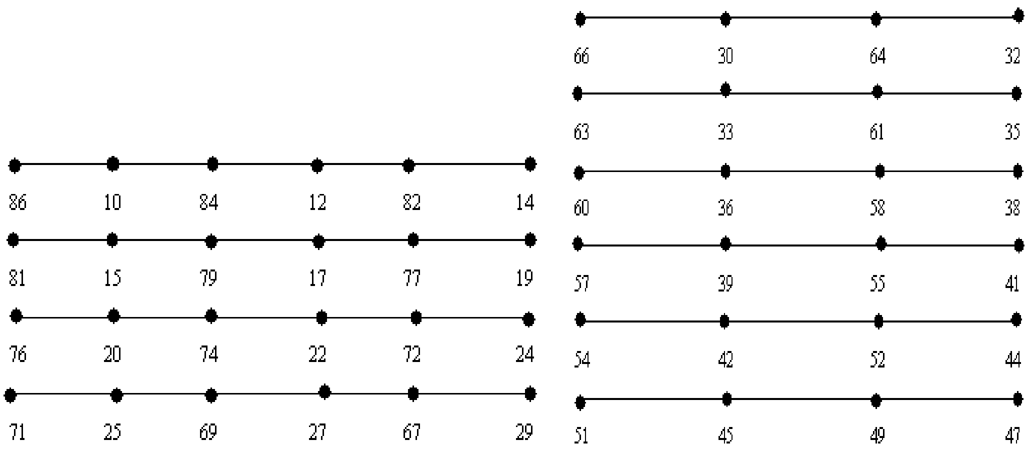

Figure 5: Skolem difference mean labelings of the graph $4P_6U6P_4$

Theorem 2.3. The graph $(m\text{-}1)P_n\, U\, (n\text{-}1)P_m$ is skolem difference mean for all $m,n>1$.

Proof. Let G be the graph $(m\text{-}1)P_n\, U\, (n\text{-}1)P_m$. Let $V(G)=\{v_{ij}, u_{kl} / 1\le i\le m\text{-}1, 1\le j\le n, 1\le k\le n\text{-}1, 1\le l\le m\}$ and $E(G)=\{v_{ij}v_{ij+1}, u_{kl}u_{kl+1} / 1\le i\le m\text{-}1, 1\le j\le n\text{-}1, 1\le k\le n\text{-}1, 1\le l\le m\text{-}1\}$

Define $f:V(G)\,)\rightarrow\{1,2\ldots 4mn\text{-}3m\text{-}3n\text{+}2\}$ as follows

Case (i). n and m are odd.

$v_{i2j+1}=4mn-3m-3n+2-n(i-1)-2j;\ 1\le i\le m-1,\ 0\le j\le \frac{n-1}{2}$

$v_{i2j}=m+n-4+(n-2)(i-1)+2j;\ 1\le i\le m-1,\ 1\le j\le \frac{n-1}{2}$

$u_{i2j+1}=3mn-3m-2n+2-m(i-1)-2j;\ 1\le i\le n-1, 0\le j\le \frac{m-1}{2}$

$u_{i2j}=2(n-m)+(m-2)n+(m-2)i+2j;\ 1\le i\le n-1, 1\le j\le \frac{m-1}{2}$

Case (ii). n is even and m is odd or even. Without loss of generality let us take m as odd.

$v_{i2j+1}=4mn-3m-3n+2-(n-1)(i-1)-2j;\ 1\le i\le m-1,\ 0\le j< \frac{n}{2}$

$v_{i2j}=m+n-4+(n-1)(i-1)+2j;\ 1\le i\le m-1,\ 1\le j\le \frac{n}{2}$

$u_{i2j+1}=3mn-2m-2n+1-m(i-1)-2j;\ 1\le i\le n-1, 0\le j\le \frac{m-1}{2}$

$u_{i2j}=m(n-1)-1+(m-2)i+2j;\ 1\le i\le n-1, 1\le j\le \frac{m-1}{2}$

Case (iii). n and m are even.

$v_{i2j+1}=4mn-3m-3n+2-(n-1)(i-1)-2j;\ 1\le i\le m-1,\ 0\le j< \frac{n}{2}$

$v_{i2j}=m+n-4+(n-1)(i-1)+2j;\ 1\le i\le m-1,\ 1\le j< \frac{n}{2}$

$u_{i2j+1}=3mn-2m-2n+1-(m-1)(i-1)-2j;\ 1\leq i\leq n-1, 0\leq j<\frac{m}{2}$

$u_{i2j}=m(n-1)-2+(m-1)i+2j;\ 1\leq i\leq n-1, 1\leq j\leq\frac{m}{2}$

In all the cases the edge labels are *1,2,3…2mn-2m-2n+2.*Hence the graph $(m-1)P_n\ U\ (n-1)P_m$ is skolem difference mean for all the values of *m* and *n*. ■

The skolem difference mean labelings of the graphs $2P_5U4P_3$, $4P_4U3P_5$ and $3P_8U7P_4$ are given in Figures 6, 7 and 8.

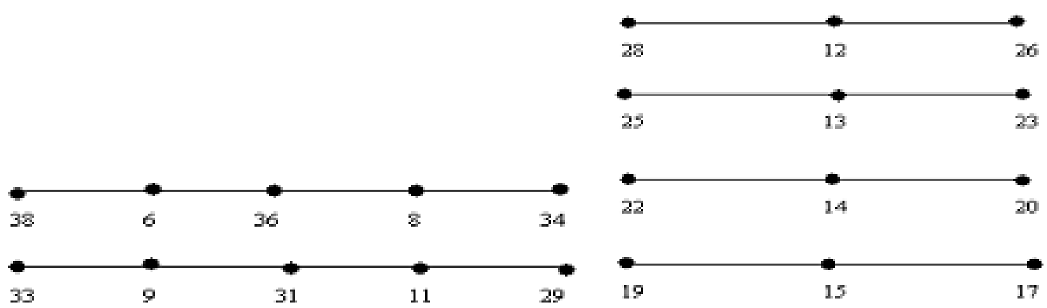

Figure 6: Skolem difference mean labelings of the graph $2P_5U4P_3$.

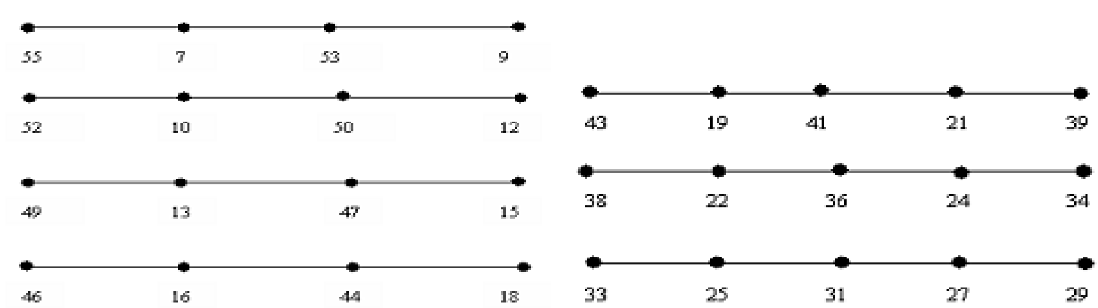

Figure 7: Skolem difference mean labelings of the graph $4P_4U3P_5$

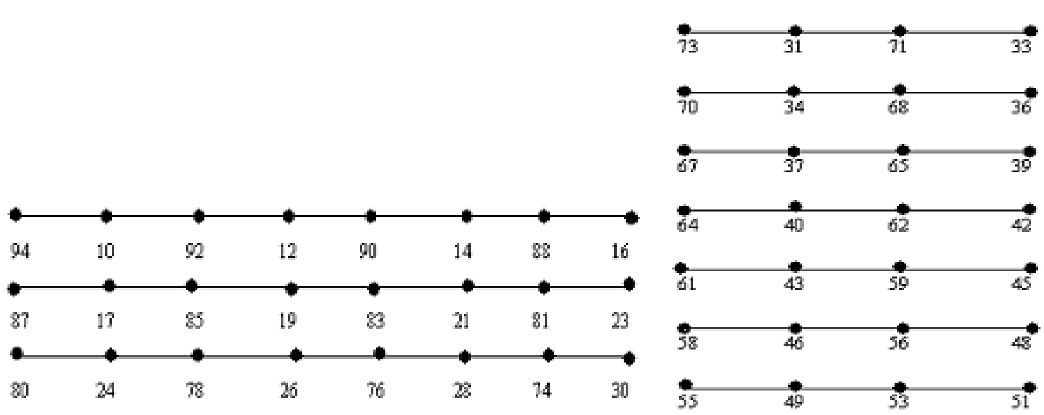

Figure 8: Skolem difference mean labelings of the graph $3P_8U7P_4$

Theorem 2.4. The graph $nP_n \cup mP_m$ is skolem difference mean for all $m,n>1$.

Proof. Let G be the graph $nP_n \cup mP_m$

Let $V(G)=\{v_{ij}, u_{kl} / 1\le i,j\le n, 1\le k,l\le m\}$and
$E(G)= \{v_{ij}v_{ij+1} / 1\le i\le n, 1\le j\le n-1\}\cup\{u_{kl}u_{kl+1}/1\le j\le m, 1\le i\le m-1\}$.
Define $f:V(G)) \to \{1,2,3\ldots 2n^2+2m^2-n-m\}$as follows.

Case (i). n and m are odd.

$v_{i2j+1}=2n^2+2m^2-n-m-n(i-1)-2j;\ 1\le i\le n,\ 0\le j\le \frac{n-1}{2}$

$v_{i2j}=m+n-2+(n-2)(i-1)+2j;\ 1\le i\le n,\ 1\le j\le \frac{n-1}{2}$

$u_{i2j+1}=n^2+2m^2-n-m-m(i-1)-2j;\ 1\le i\le m, 0\le j\le \frac{m-1}{2}$

$u_{i2j} = m+2n-4+(n-2)(n-1)+(m-2)(i-1)+2j;\ 1\le i\le m, 1\le j\le \frac{m-1}{2}$

Case (ii). n is odd and m is odd or even.

Without loss of generality let us take m as even.

$v_{i2j+1}=2n^2+2m^2-n-m-n(i-1)-2j;\ 1\le i\le n,\ 0\le j\le \frac{n-1}{2}$

$v_{i2j}=m+n-2+(n-2)(i-1)+2j;\ 1\le i\le n,\ 1\le j\le \frac{n-1}{2}$

$u_{i2j+1}=n^2+2m^2-n-m-(m-1)(i-1)-2j;\ 1\le i\le m, 0\le j\le \frac{m}{2}$

$u_{i2j} = m+2n-4+(n-2)(n-1)+(m-1)(i-1)+2j;\ 1\le i\le m, 1\le j\le \frac{m}{2}$

Case (iii). n and m are even.

$v_{i2j+1}=2n^2+2m^2-n-m-(n-1)(i-1)-2j;\ 1\le i\le n,\ 0\le j< \frac{n}{2}$

$v_{i2j}=m+n-2+(n-1)(i-1)+2j;\ 1\le i\le n,\ 1\le j\le \frac{n}{2}$

$u_{i2j+1}=m+3n-2+(n-2)(n-1)+(m-1)(i-1)+2j;\ 1\le i\le m, 0\le j< \frac{m}{2}$

$u_{i2j} = n^2+2m^2-m+2-(m-1)(i-1)-2j;\ 1\le i\le m, 1\le j\le \frac{m}{2}$

Hence the graph $nP_n \cup mP_m$ is skolem difference mean for all the values of m and n. ■

The skolem difference mean labelings of the graphs $3P_3 \cup 7P_7$, $7P_7 \cup 4P_4$ and $4P_4 \cup 6P_6$ are given in Figures 9, 10 and 11.

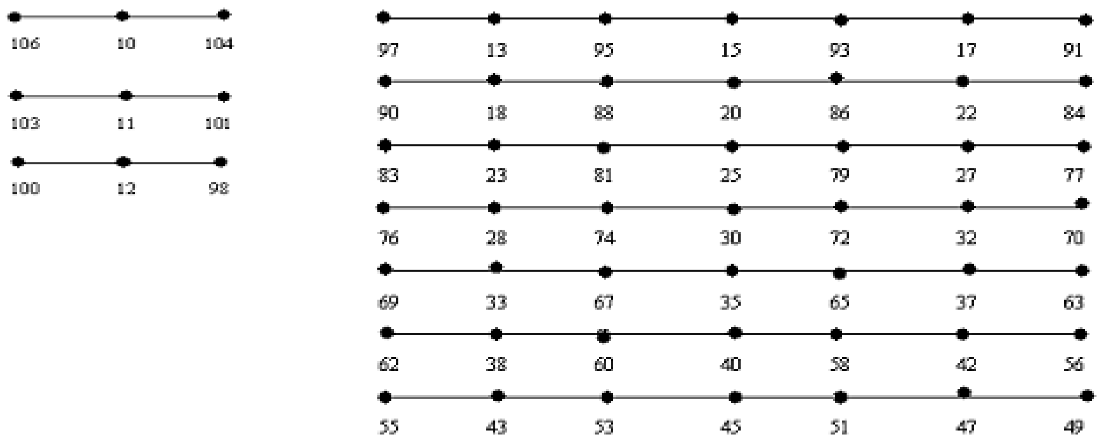

Figure 9: Skolem difference mean labelings of the graph $3P_3$U $7P_7$

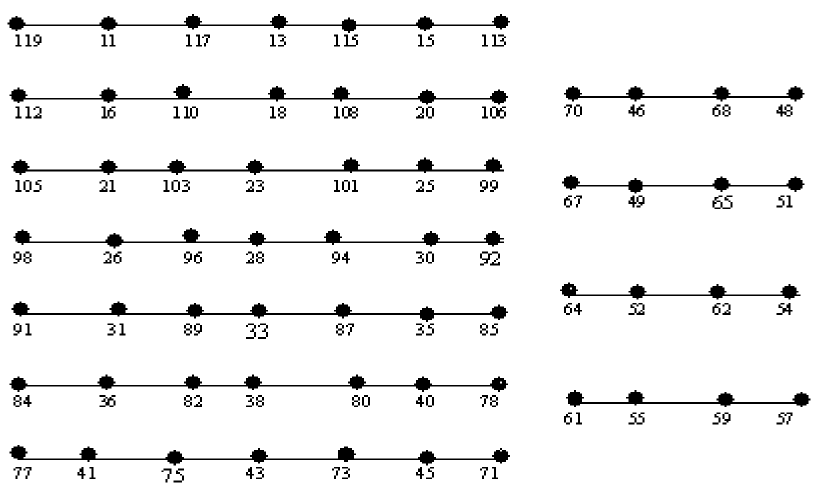

Figure 10: Skolem difference mean labelings of the graph $7P_7$U $4P_4$

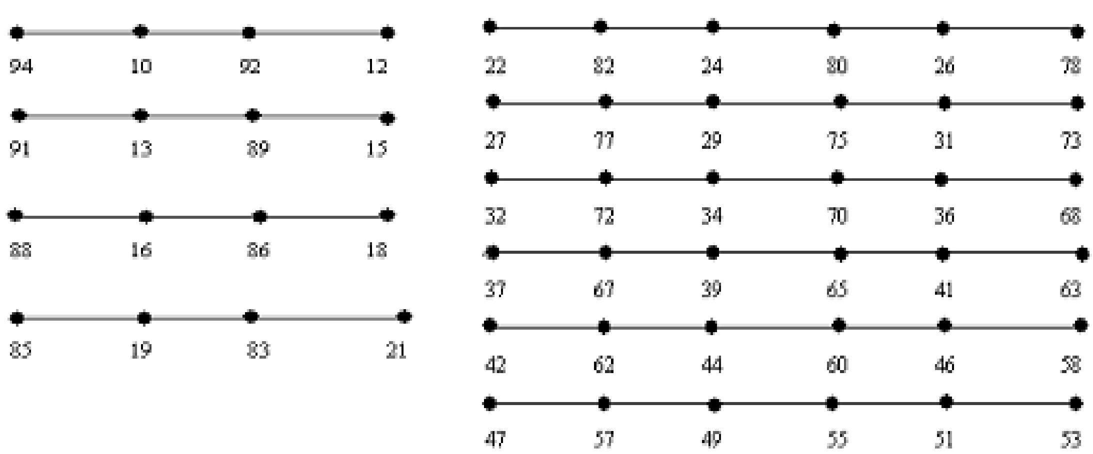

Figure 11: Skolem difference mean labelings of the graph $4P_4$U $6P_6$.

Theorem 2.5. The graph $(n\text{-}1)P_n U (m\text{-}1)P_m$ is skolem difference mean for all the values of *m* and *n*.

Proof. Let *G* be the graph $(n\text{-}1)P_n U (m\text{-}1)P_m$

Let $V(G)=\{v_{ii}, u_{kl} / 1\leq i\leq n\text{-}1, 1\leq j\leq n, 1\leq k\leq m\text{-}1, 1\leq l\leq m\}$ and

$E(G)= \{v_{ij}v_{ij+1} / 1\leq i\leq n\text{-}1, 1\leq j\leq n\text{-}1\} U \{u_{kl}u_{kl+1} / 1\leq k\leq m\text{-}1, 1\leq l\leq m\text{-}1\}$.

Define $f: V(G)) \rightarrow \{1,2\ldots 2n^2+2m^2\text{-}3n\text{-}3m+2\}$ as follows:

Case (i). *n* and *m* are odd.

$v_{i2j+1}=2n^2+2m^2-3n-3m+2-n(i-1) -2j;\ 1\leq i\leq n-1,\ 0\leq j\leq \frac{n-1}{2}$

$v_{i2j}=n+m-4+(n-2)(i-1)+2j;\ 1\leq i\leq n-1,\ 1\leq j\leq \frac{n-1}{2}$

$u_{i2j+1}=n^2+2m^2-2n-3m+2-m(i-1)-2j;\ 1\leq i\leq m-1, 0\leq j\leq \frac{m-1}{2}$

$u_{i2j} = n^2+m-2n-2+(m-2)(i-1)+2j;\ 1\leq i\leq m-1, 1\leq j\leq \frac{m-1}{2}$

Case (ii). *n* is odd and *m* is odd or even.

Without loss of generality let us take *m* as even.

$v_{i2j+1}=2n^2+2m^2-3n-3m+2-n(i-1) -2j;\ 1\leq i\leq n-1,\ 0\leq j\leq \frac{n-1}{2}$

$v_{i2j}=n+m-4+(n-2)(i-1)+2j;\ 1\leq i\leq n-1,\ 1\leq j\leq \frac{n-1}{2}$

$u_{i2j+1}=n^2+2m^2-2n-3m+2-(m-1)(i-1)-2j;\ 1\leq i\leq m-1, 0\leq j< \frac{m}{2}$

$u_{i2j} = n^2-2n+m-2+(m-1)(i-1)+2j;\ 1\leq i\leq m-1, 1\leq j\leq \frac{m}{2}$

Case (iii). *n* and *m* are even.

$v_{i2j+1}=2n^2+2m^2-3n-3m+2-(n-1)(i-1) -2j;\ 1\leq i\leq n-1,\ 0\leq j< \frac{n}{2}$

$v_{i2j}=n+m-4+(n-1)(i-1)+2j;\ 1\leq i\leq n-1,\ 1\leq j\leq \frac{n}{2}$

$u_{i2j+1}=n^2+2m^2-n-3m+1-(m-1)(i-1)-2j;\ 1\leq i\leq m-1, 0\leq j< \frac{m}{2}$

$u_{i2j} = n^2-n+m-3+(m-1)(i-1)+2j;\ 1\leq i\leq m-1, 1\leq j\leq \frac{m}{2}$

In all the cases the edge labels are $1,2,3\ldots n^2+m^2\text{-}2n\text{-}2m+2$.

Hence, the graph $(n\text{-}1)P_n U (m\text{-}1)P_m$ is skolem difference mean for all the values of *m* and *n*. ■

The skolem difference mean labelings of the graphs $2P_3U4P_5$, $2P_3U5P_6$ and $3P_4U7P_8$ are given in Figures 12, 13 and 14.

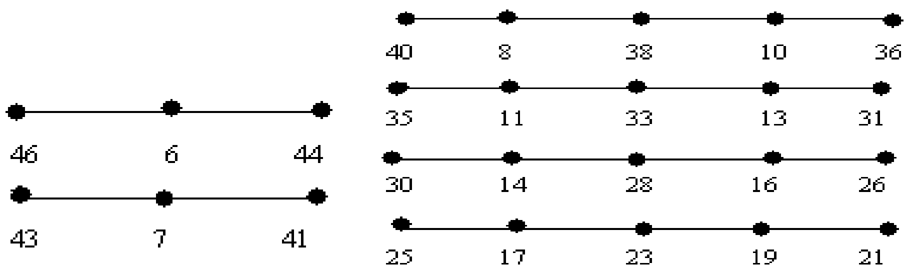

Figure 12**:** Skolem difference mean labelings of the graph $2P_3U4P_5$

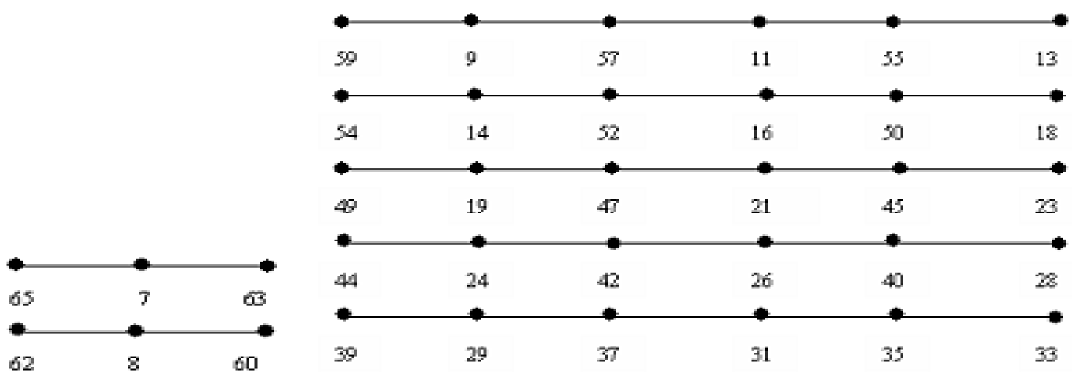

Figure 13**:** Skolem difference mean labelings of the graph $2P_3U5P_6$

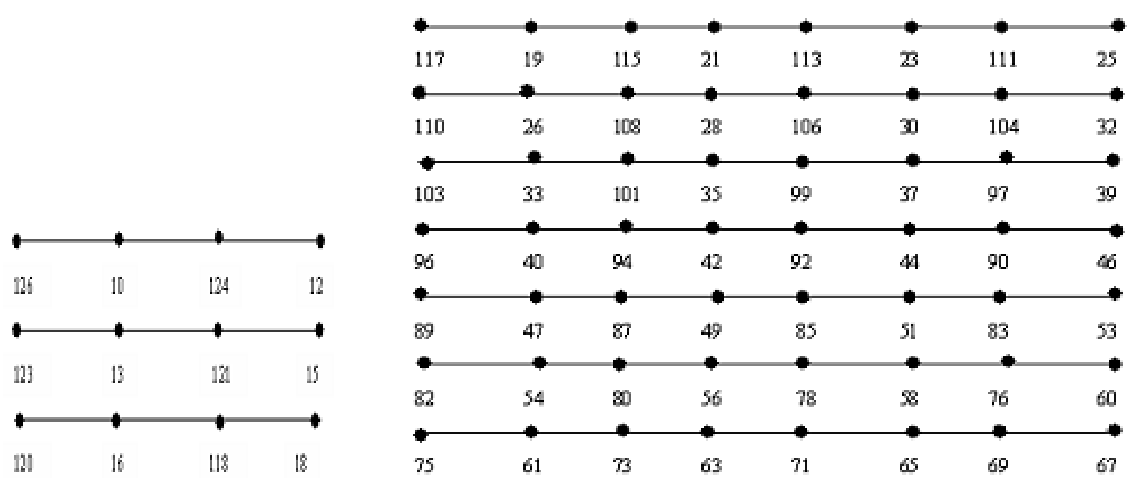

Figure 14: Skolem difference mean labelings of the graph $3P_4U7P_8$

Theorem 2.6. $\bigcup_{i=2}^{n} Pi$ is skolem difference mean for all the values of *n*.

Proof. Let $V(\bigcup_{i=2}^{n} Pi)=\{u_{ij} / 2\le i\le n, 1\le j\le i\}$ and $E(\bigcup_{i=2}^{n} Pi)=\{u_{ij}u_{ij+1} / 2\le i\le n, 1\le j\le i-1\}$

Define $f: V(\bigcup_{i=2}^{n} Pi) \rightarrow \{1,2,3\ldots n^2-1\}$ as follows.

$f(u_{i2j+1}) = n^2-2-(i+1)(\frac{i-1}{2}-1)-2j$; when i is odd; $0\le j\le \frac{i-1}{2}$

$= n^2-1-i\,(\frac{i}{2}-1)-2j$; when i is even; $0 \le j < \frac{i}{2}$

$f(u_{i2j}) = n+(i-1)(\frac{i-1}{2}-1)+2j$; when i is odd; $0 \le j < \frac{i-1}{2}$

$= n-1+(i-2)(\frac{i-2}{2})+2j$ when i is even; $0 \le j < \frac{i}{2}$

Then the induced edge labels are $1,2,3 \ldots \frac{n(n-1)}{2}$

Hence, the graph $\bigcup_{i=2}^{n} Pi$ is skolem difference mean for all the values of *n*. ■

The skolem difference mean labelings of $\bigcup_{i=2}^{8} Pi$ is given in Fgure 15.

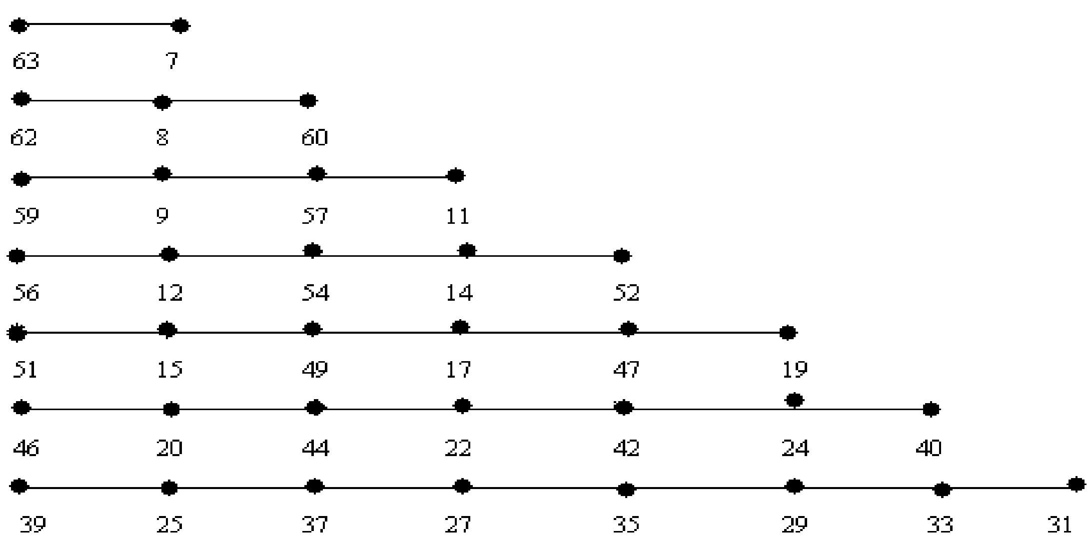

Figure 15**:** Skolem difference mean labelings of $\bigcup_{i=2}^{8} Pi$

Acknowledgement

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