



Nature Inspired Scalable Design Trends for Flapping Wing Configurations

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ABSTRACT

Flapping wing Micro Air Vehicles (MAVs) take precedence over their fixed wing counterparts by virtue of their ability to hover, take off from a constraint surface, maneuver in tight spaces and their perch and stare capability. This paper aims at establishing a scalable relationship between flapping frequency and the geometric parameters that include wingspan (b) and body mass (m) using real data of birds and apply that to determine the Strouhal No.'s relationship with efficiency for flapping wing MAVs. Surrogate models in the form of power laws are developed for systems to handle growing amount of data and generate governing laws for complex phenomena. Geometric data of 140 birds of various species is utilized to form scalable equations. Subsequently, regression analysis is applied to develop the power law model of flapping wing frequency from wingspan and body mass respectively, and subsequently, Strouhal No. is calculated. The derived equations are used to graphically represent the relationship between frequency and geometric parameters for birds and MAVs data and Strouhal No. versus propulsive efficiency for MAVs data. The results indicate a strong correlation of wingspan with flapping frequency. The findings of this study are compared to existing models suggested by previous researchers and futuristic design trends are assessed.

Keywords: Micro Aerial Vehicles, Flapping Frequency, Strouhal Number, Surrogate Models, Propulsive Efficiency

INTRODUCTION

According to DARPA Micro Air Vehicle (MAV) is an autonomous flying craft that takes advantage of increasingly miniaturized electromechanical technology. MAVs have numerous promising civil and military applications. In accordance with the mission, size and type of equipment installed, the category of MAVs generally differs. They can be employed for rigorous inspection of buildings, silent and inconspicuous surveillance and can be equipped with various micro-sensors, multiple micro-phones, cameras and gas detectors [1]. Two approaches may be adopted when designing MAVs; top down or bottom up. This research focuses on a segment of the top down approach since it is concentrated upon developing surrogate models to predict certain performance characteristics. The advantage of the top down approach is that it gives insight into pressing problems of lacking scientific knowledge [2]. MAVs are a relatively new territory and though research is vast in this domain; a few fixed / flapping designs have achieved operational status. This is owing to the challenges encountered with when developing on a miniature scale flying object and the problems associated with low Reynolds number aerodynamics and propulsion.

Bird flight has provided the inspiration for flapping wing MAV design. Research carried out in MAV realm reveals a magnitude of complexities present in designing flapping wing MAV as compared to fixed and rotary wing MAV [3]. According to Ryan and Su [4], to counteract the decreasing aerodynamic efficiency, high frequency flapping is required which demands an increase in the power to weight ratio. In addition, manufacturing and assembly techniques become challenging for small size. By discovering the trends in bird's flight, we can formulate empirical relationships using geometric parameters to assist in effective and efficient design.

Why do large birds flap their wings seldom and fly farther as compared to smaller birds; who, in order to remain airborne, to achieve the same speed, need to flap their wings raft? The answer lies in a dimensionless number called the Strouhal No. (St). The idea is that all MAVs have a Strouhal No. which predicts their propulsive efficiency. In order to determine the Strouhal No., one must know the frequency, amplitude and velocity. Frequency is the number of wing beats per second. One complete wingbeat is the downward and upward flapping stroke. The downward stroke

is for power and lift and the upward stroke is for recovery. Amplitude is the vertical distance travelled by the tip of wing during flapping stroke. The conditions assumed are symmetric flight, normalized angle of attack, and that the wing tip descends in smooth curvilinear waveform. Velocity is the forward speed of the bird at which it cruises. Some smaller birds use transitional flight patterns for alternate flapping with boundary and gliding motions.

However, determining these three parameters is a cumbersome, time consuming and expensive task. In order to simplify this pursuit, there exists a need to have an empirical relationship which can predict these three parameters based on the geometric parameters of the MAV. There are certain design techniques which can help to circumvent the challenging design process and yield fair approximations [13]. One such approach is the use of surrogate models which can be used as a substitute for complex analytical models. Many researchers have worked on this problem and have come up with different formulas. A similar effort has been carried out in this paper as well. Strouhal No. is a dimensionless number that describes the oscillatory flow mechanism and represents the ratio of unsteady and steady motion. It describes tail and wing kinematics of swimming and flying animals and governs well defined series of vortex growth and shedding regimes for airfoils undergoing pitching and heaving motions [10]. Smaller Strouhal No. provides larger propulsive efficiency. For a lower Strouhal No. less flapping is required to achieve a certain velocity. Propulsive efficiency increases over a narrow range of Strouhal No. and peaks between 0.2-0.4 [5].

Power laws are used to determine an empirical relationship between frequency, velocity and the geometric parameters. Regression analysis can be applied to determine a best-fit line for quantities that are not linear. Pennycuick has conducted significant research in determining a scalable relationship between frequency and geometric parameters; first, in 1990 and then in 2001 through regression techniques and dimensional analysis. Graham Taylor has published significant work related to Strouhal No as well. Pennycuick (1990) derived the formula using multiple regression and dimensional analysis by first experimentally determining the frequency and then correlating it with geometric parameter [6]:

$$f = \frac{1.08}{b} \sqrt[3]{\frac{m}{\rho}} \sqrt{\frac{g}{\sqrt{S}}} \quad (1)$$

Then refined it as [7]:

$$f = m^{\frac{3}{8}} \frac{g^{\frac{1}{2}}}{q} b^{-\frac{23}{24}} S^{-\frac{1}{3}} \rho^{-\frac{3}{8}} \quad (2)$$

where m is the mass, b is the wing span, S is the surface area, ρ is the density at sea level taken as 1.23 kg/m^3 , g is the gravitational constant taken as 9.8 m/s^2 and q is the power fraction taken as 1 for steady level flight.

Taylor *et al* [8] observed frequency against predicted frequency for 60 bird species and the regression was highly significant. However, according to them, Pennycuick's equation fails to explain all the systematic variation in wing-beat frequency.

Bunget [8] has confirmed Pennycuick's work however he claims it gives a good fit when applied to small birds, bats and insects. Taylor [11] has measured the frequency, velocity, amplitude and tilt angle of 22 species of birds experimentally. As the mass of the bird increases, the angle decreases as is seen for the data of birds. Surrogate models are used because the outcome of interest cannot be easily measured, in this case frequency; instead a model of the outcome is used for prediction of results.

According to Pennycuick the forward velocity can be predicted by using the following formula:

$$U = 1.508m^{\frac{1}{6}} \quad (3)$$

Pennycuick's velocity formula does not provide a good estimate of the Strouhal No. because the range of Strouhal No. falls outside 0.2-0.4, which is the range prescribed for optimum propulsive efficiency.

Hassanalian [3] has given the following formula for velocity which predicts the velocity approximately four times more than as determined by Pennycuick:

$$U = 4.77m^{\frac{1}{6}} \quad (4)$$

Taylor (2004) determined the stroke angle to be [8]:

$$\Theta = 67b^{-0.24} \quad (5)$$

Greenwalt [3] has derived a power law relationship between frequency and wing span and so has Azima [3], however, they seemed to be flawed upon verification. Greenwalt has suggested the following formula:

$$f = 3.54b^{-1.15} \quad (6)$$

Azima [3] has given separate formulas for large and small birds:

$$\text{Large birds: } f = 116m^{-\frac{1}{6}} \quad (7)$$

$$\text{Small birds: } f = 287m^{-\frac{1}{3}} \quad (8)$$

According to Taylor [5]:

$$U = 3fA \quad (9)$$

which consequently makes the Strouhal No. 1/3 for every instance.

Bruderer and Boldt [9] have experimentally measured the velocity of 140 birds of various species and additionally, they calculated the velocity of those species using Pennycuik's equation. Further calculation of frequency and ultimately Strouhal No. reveals Pennycuik's equation to give less accurate values as compared to determination by using actual velocity. Calculation of frequency and Strouhal No. using the velocity calculated through Pennycuik's equation for velocity gives less accurate value of the two parameters compared to when frequency and Strouhal No. are calculated using the actual velocity values as recorded by researchers [9].

For this research, the geometric parameters i.e. mass, wing span and the flight velocities is taken from Bruderer and Boldt [9] which has recorded the aforementioned parameters of 140 birds of various species. Secondary data analysis is done by using regression analysis technique to model scalability trends between frequency, Strouhal No. and geometric parameters. Scalability can be defined as a mathematical function, a relationship between independent and dependent variables. There are various methods by which scalability trends may be established and one such method is power law. Power law is a relationship between two quantities such that one is proportional to the fixed power of the other. Power laws can be used to scale any parameter with respect to another parameter it is dependent upon provided that the logarithm of the dependent variable is directly proportional to the logarithm of the independent variable.

Strouhal No. is a dimensionless number which can be used to describe oscillating flow mechanisms, and in this case the amount a MAV needs to flap and the velocity it needs to attain in order to remain airborne with respect to its mass and wing span. The importance of dimensionless numbers is their constancy can imply dynamic similarity between systems despite possible differences in medium and scale. From literature it deduced that low speed birds have high Strouhal No. and high-speed birds have low Strouhal. No. The range of Strouhal No. for optimum propulsive efficiency is 0.2-0.4 [8]. According to Hassanalian [3], 70% propulsive efficiency is achieved when Strouhal No. is between this range. Strouhal No. is determined by using the relationship between frequency, amplitude and velocity. Flapping actuates wings to generate lift and propulsive force. It converts input motion (rotational/translational) into beating motion at frequency ranging from several to hundreds of Hertz [9]. Lift is generated through leading edge vortices and wing rotational forces. For this paper nature is taken as a source of inspiration.

METHODOLOGY

Two-dimensional flapping motion consists of up and down motion of the wing (flapping) and the change in angle of incidence (feathering). Feathering phenomenon is taken to be constant for the purpose of this study and flapping motion is focused upon.

Bruderer and Boldt have recorded the mass, wing span, area and velocities of 140 birds of various species ranging from a mass of 0.001 kg to 10 kg and a wing span of 0.1 m to 3.5 m [9]. The data comprising of frequency, wing span and mass of 33 birds is acquired from a study carried out by Ashraf [12]. The data of 140 birds was used to derive the equations by using power law. The log of mass and wing span were plotted against the log of frequency respectively for the data of 140 birds. The coefficients 'a' and 'b' for the power law ($X=aY^b$) were found by the slope and the intercept of the equation of best fit separately. Using the power law relation, a formula for frequency with respect to wing span and mass, individually, was established. Frequency is our response variable; mass, wing span and area are the predictors. They are used to estimate the nature of the relationship. The candidate predictors are systematically removed with the highest p-value and the one left is the significant predictor. Similarly, mass, wing span are plotted against velocity. Empirical relationships between frequency, velocity and geometric parameters is determined. Subsequently, using the derived equations, for the data of 30 birds [12] mass and wing span, respectively, are plotted against frequency in order to depict graphically the relationship between the aforementioned parameters. Similarly, the equations are used to plot the relationship between mass and wing span, respectively, against frequency for the data of 17 MAVs taken from Hassanalian [3]. Furthermore, Strouhal No. for the MAVs is calculated which is further used to determine the propulsive efficiency and the relationship between the two is plotted.

RESULTS AND DISCUSSION

Surrogate model is formulated in order to determine significant predictors and their degree of relevance for frequency and Strouhal No. A confidence interval of 95% is used. Frequency is our response variable, while mass and wing span are the predictors. The independent variables are used to determine the dependent variable. Mass is not as significant a contributor in predicting frequency alone as is wing span. Hence, the power law for frequency can be based upon wing span alone, if wing span is the only parameter that is known.

Relationship between Frequency, Mass and Wing Span

In accordance with fig. 1. the relationship between frequency and mass is not significant because the data is scattered; the best fit line has an r^2 value of 50.3% which does not give a very good fit and the sum of square of errors is 106.5. Fig. 1. illustrates that frequency cannot be determined by mass alone since the data is very scattered. Following is the relationship determined between frequency and mass using power law:

$$f = 4.8m^{-0.228} \tag{10}$$

However, this relationship alone cannot be used to accurately determine the frequency.

The relationship of frequency with wing span is quite significant as is visible from fig. 2., having an r^2 value of 83.5% which gives a line of good fit. The sum of square of errors is 41.1. The data points lie in the vicinity of the best fit line. From the figure it can be observed that the data points vary moderately from the best fit line. Hence, frequency has a more direct relationship with wing span.

$$f = 5.12b^{-0.68} \tag{11}$$

The formulas given by Pennycuick and Greenwalt were tested against the formula suggested in Eq. (11) by calculating the frequency using each of the formulas and comparing it with the actual frequency as shown in Table 1. Frequency calculated by using Eq. (11) has the least cumulative standard deviation from the actual frequency as compared to the equations derived by Pennycuick and Greenwalt. The formula to estimate frequency as derived by this research proved to be superior than all other formulas suggested priorly since the frequency ratio (ratio of actual frequency and estimated frequency) was close to 1 for all MAVs when using Eq. (11). Hence, $f=5.12b^{-0.68}$ can be substituted for the complex method used to measure frequency, as it only requires the measurement of wing span of an MAV.

Relationship between Velocity and Mass

Power law was again applied to determine an empirical relation for velocity. However, the data was taken from Bruderer and Boldt [9] which has recorded the velocities of 140 birds; the wing span and mass of those birds were used as the predictors. The plot for frequency vs. wing span showed the data to be significantly scattered which resulted in the relationship between frequency and wing span as insignificant. Comparatively, the data was less scattered for frequency and mass. Hence, mass can be used to predict velocity using the following relationship:

$$U = 12.54m^{0.06678} \tag{12}$$

This formula does not give a good enough estimate of the velocities. However, it estimates velocity better than the formula suggested by previous researcher which is the reason it is used for further calculations.

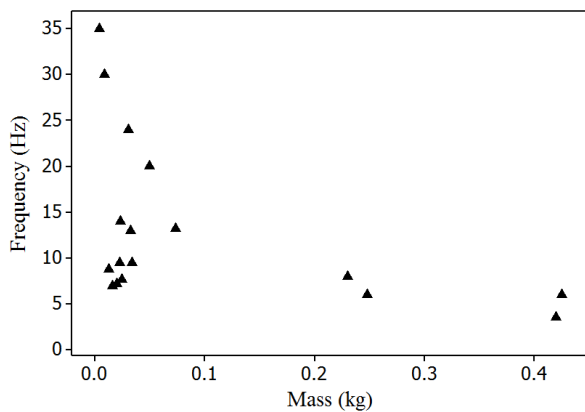


Fig.1. (a) Frequency vs mass for MAVs

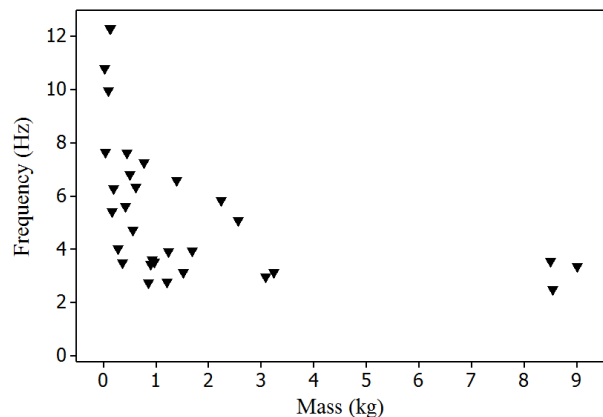


Fig. 1. (b) Frequency vs. mass for birds

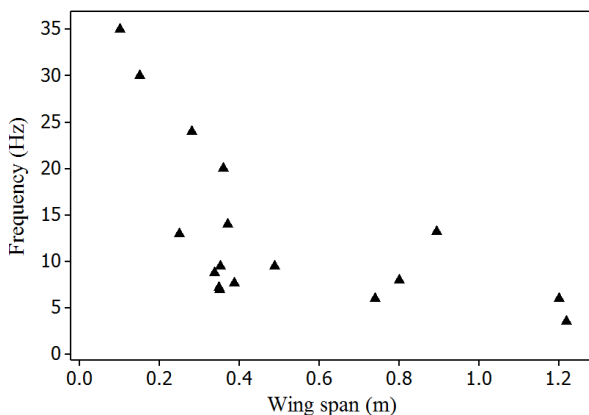


Fig. 2. (a) Frequency vs. wing span for MAVs

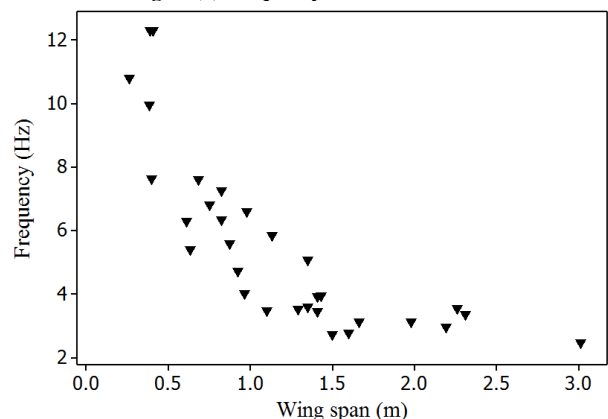


Fig. 2. (b) Frequency vs. wing span for birds (right)

Determination of Strouhal No.

In order to determine the Strouhal No. Eq. (11), Eq (12) are used respectively for frequency and velocity. Amplitude is calculated using Eq. (15), where the value of θ is determined using Eq. (5).

$$St = \frac{\text{Flapping velocity}}{\text{Forward velocity}} \tag{13}$$

$$St = \frac{fA}{U} \tag{14}$$

$$A = b \sin\left(\frac{\theta}{2}\right) \tag{15}$$

Strouhal No. is plotted against wing span in fig. 4. For MAVs Strouhal No. lies between 0.17 - 0.24, which falls below the suggested optimum value. The reason can lie in the equations used for calculating Strouhal No. However, for birds' data, the Strouhal No. lies between 0.2 - 0.25 which negates this premise. Fig. 4. (a) depicts a straightforward trend- as the wing span increases so does the Strouhal No. until 1.2 m after which any further increase in wing span does not bring about a commensurate increase in Strouhal No. In fig. 4. (b) there is no general trend visible since the data is scattered, hence, no outward relationship between Strouhal No. and wing span of birds is apparent.

Determination of Propulsive Efficiency

In order to determine propulsive efficiency, Paranjape [11] has provided a relationship between Strouhal No., amplitude and propulsive efficiency using the formula:

$$\varepsilon = \frac{1}{\pi} \left[\frac{2A + Sr_o - \frac{4ASr_o}{9}}{\frac{Sr_o A}{4} + \frac{Sr_o^2}{6} + \frac{3}{32} Sr_o^3 A + 2Sr_o A_o + \frac{2Sr_o^3 A_o}{3}} \right] \tag{16}$$

$$Sr_o = 2\pi Sr \tag{17}$$

A_o is ignored because the feathering phenomenon is not taken into account; therefore, amplitude is considered to be constant. The Strouhal No. is calculated for the MAV data given in Table 1. and is plotted against efficiency which is calculated using Eq. (16). As Strouhal No. increases propulsive efficiency decreases. This is owing to the fact that birds which require a higher flapping velocity in order to remain airborne use more power and hence, their flight is less efficient. However, there is an apparent discrepancy in the results. According to literature propulsive efficiency is optimum between a Strouhal No. of 0.2 – 0.4, but this study provides results contrary to the popular assumption, since the most optimum efficiency falls below Strouhal No. 0.2 and is in fact between 0.17 and 0.18.

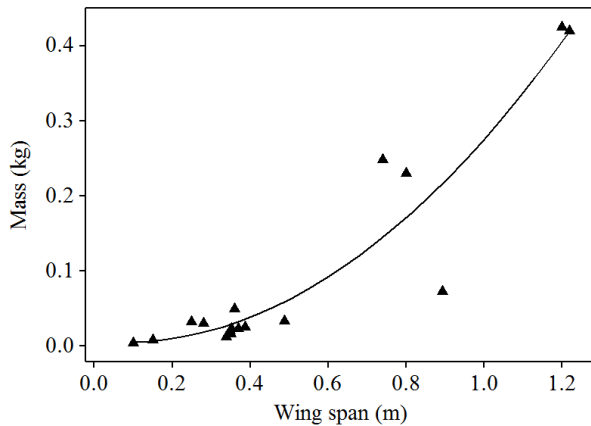


Fig. 3. (a) Mass vs. wing span for MAVs

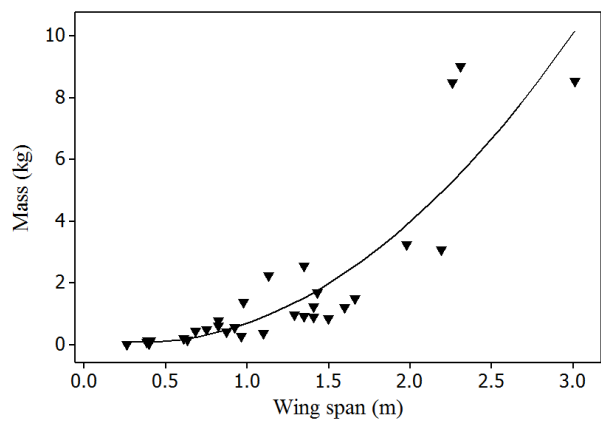


Fig. 3. (b) Mass vs. wing span for birds

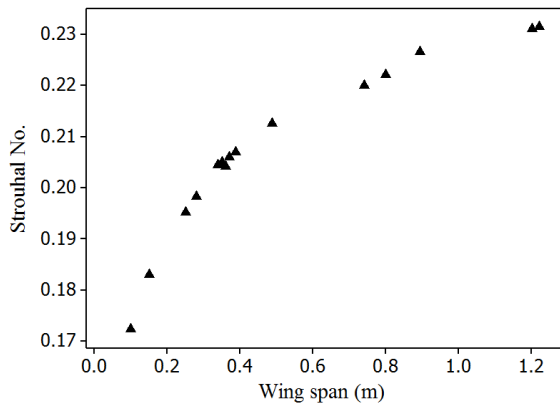


Fig. 4. (a) Strouhal No. vs wing span for MAVs

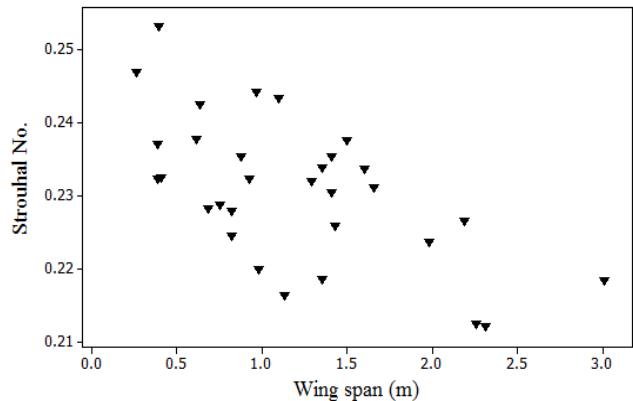


Fig. 4. (b) Strouhal No. vs wing span for birds

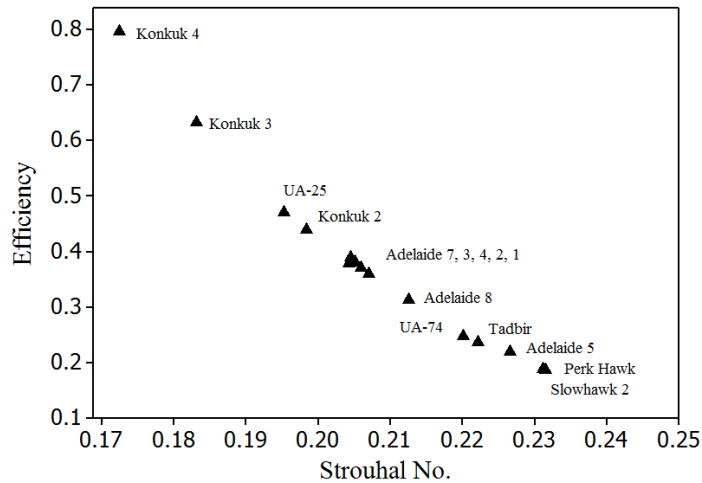


Fig. 5. Efficiency vs Strouhal No. for MAVs

Table -1 Comparison of Predicted Frequency using Estimated Equation, Pennycuick's Equation and Greenwalt's Equation

MAV	Mass (g)	Wing span (m)	Area (m ²)	Frequency (Hz)			
				Actual	Estimated Eq (11)	Pennycuick Eq (2)	Greenwalt Eq (6)
UA-74	248	74	0.0991	6	6.28	4.95	5
Slowhawk	420	122	0.3590	3.60	4.47	2.43	2.82
Adelaide3	20.09	34.768	0.0700	7.20	10.49	4.47	11.92
Adelaide2	16.09	35	0.0700	6.93	10.45	4	11.84
Adelaide1	25	38.66	0.0800	7.65	9.77	4.22	10.56
Tadir	230	80	0.1070	8	5.95	4.36	4.58
Adelaide4	23	35.30	0.0720	9.48	10.39	4.60	11.72
Park Hawk	425	120	0.0700	6	4.52	2.70	2.87
UA-25	32.40	25	0.0137	13	13.14	5.80	17.43
Adelaide7	12.75	33.84	0.0700	8.80	10.69	3.86	12.31
Adelaide8	34.06	48.83	0.0700	9.50	8.33	3.90	8.07
Konkuk2	30.60	28	0.0280	24	12.16	8.85	15.30
Konkuk1	50	36	0.0432	20	10.25	7.14	11.46
Adelaide6	23.44	37	0.0700	14.03	10.07	4.46	11.10
Konkuk4	4.32	10	0.0600	35	24.51	8.74	50
Konkuk3	8.70	15	0.0850	30	18.60	6.85	31.37
Adelaide5	73.34	89.40	0.0800	13.25	5.25	2.80	4.03

CONCLUSION

This study presents an improved empirical relationship between frequency and wing span to its predecessors. Through this relationship Strouhal No. has been predicted for a set of birds and MAVs; subsequently the propulsive efficiency has been determined for a set of MAVs. However, this study was limited to flight in cruise condition and ignored other factors such as feathering and spanning. A refinement of this study is essential, which considers feathering phenomenon as well. A fine tuning of the relationship between velocity and mass is required. Likewise, the formula needs to be adjusted so that the propulsive efficiency falls within the range as suggested by preceding researchers.

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