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BAT Synergetic Synchronization of Two Chua's Chaotic Systems

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ABSTRACT

In this paper we use optimized synergetic mode strategies for synchronization of different chaotic nonlinear systems. Stability is insured through Lyapunov synthesis. Two examples for non-identical chaotic systems are used in a simulation study to test the effectiveness of the proposed methodology which indicates good performance and satisfactory in synchronization of two different nonlinear chaotic systems.

Keywords: Chaotic nonlinear systems, synergetic control, BAT algorithm optimisation, synchronization, Lyapunov

INTRODUCTION

In recent years, research in nonlinear dynamics and chaotic systems has made remarkable progress. The dynamics of a chaotic system are highly time varying. Chaos is nonlinear phenomena which have been identified in physical, medical, biological, economical, electrical and mechanical systems [1]. Synchronization of chaotic systems has drawn much attention due to its wide application in various fields of physics and engineering [2]. In this context, control and synchronization of chaotic systems have attracted the attention of researchers [3-5]. Many approaches and methodologies have been proposed for the control and synchronization of chaotic systems such as, adaptive control [6], adaptive sliding control [7], fuzzy adaptive method [8], with coupling [9], feedback control [10-11], variable structure control [12-14] and some other methods based on sliding mode control have been successfully applied to chaotic systems [15]. On the other hand, sliding mode strategy has been widely used in the literature due to its robustness and its simplicity design. However, sliding mode approach suffers from several shortcomings. The first one is the presence of chattering while the second consists in the fact that system dynamics must be well-known and exactly modelled and the third the approaching phase [16-19]. The best well-known classes of the optimization algorithms are: particle swarm optimization (PSO), genetic algorithm (GA) and bat algorithm (BA) [20-21]. The Bat Algorithm, based on the echolocation behaviour of bats is a bio-inspired algorithm developed by Yang in 2010, was successfully used for many optimization problems [22-23]. Echolocation is an important feature of bat behaviour. It means that bats emit a very loud sound pulse and listen for the echo that bounces back from the surrounding objects whilst flying. Their pulses vary in properties and can be correlated with their hunting strategies, depending on the species [24].

In this paper, firstly, we propose a new approach to synchronize non-identical chaos systems. This approach based on synergetic control theory is presented without the previously mentioned drawbacks [25]. Secondly, Parameters of the synergetic control are optimized, we apply a BAT algorithm, according to an objective function to be defined and only easily measurable system variables used in this strategy of synchronization. The stability of the proposed approach will be guaranteed in the sense of second Lyapunov criteria. This paper introduces in the next section the problem formulation, followed by a brief recall of synergetic control technique. In next section synergetic approach design is presented and optimized by Bat algorithm and stability issue addressed followed by simulation and a presentation of results for different chaotic systems. The last section concludes the paper.

PROBLEM FORMULATION

In this paper, we study a class of chaotic n-dimensional systems having the system description in (1) and (2):

Master system

$$\begin{cases} \dot{x}_i = x_{i+1}, \quad 1 \le i \le n-1 \\ \dot{x}_n = g(x,t) \end{cases} \qquad x = [x_1, x_2, \ldots, x_n] \in \Re^n \qquad (1)$$

Slave system

$$\begin{cases} \dot{y}_i = y_{i+1}, & 1 \le i \le n-1 \\ \dot{y}_n = f(y,t) + d(t) + u \end{cases} \qquad y = [y_1, y_2, \ldots, y_n] \in \Re^n \tag{2}$$

where $f(y,t)$ and $g(x,t)$ are nonlinear functions, $d(t)$ is the disturbance of system and $u \in \Re$ is the control. In general, the uncertainty and disturbance are assumed to be bonded as (3):

$$\begin{cases} |g(x,t)| \le G \le \infty \\ |f(y,t)| \le F \le \infty \\ |d(t)| \le D \end{cases} \tag{3}$$

For all $x \in U_x \subset \Re^n$ and $y \in U_y \subset \Re^n$, where U_x and U_y are compact sets defined as $U_x = \{x \in \Re^n : \|x\| \le m_x < \infty\}$, $U_y = \{y \in \Re^n : \|y\| \le m_y < \infty\}$ and G, F, D are known constants [15]. where $\|.\|$ denotes the Euclidian norm.

It is also assumed that $f(y,t)$ and $g(x,t)$ satisfy all the necessary conditions, such as systems (1) and (2) having unique solution in the time interval $[t_0, +\infty]$, $t_0 > 0$, for any given initial condition $x_0 = x(t_0)$, $y_0 = y(t_0)$ and without control, system (1) displays chaotic motion [15].

The slave system is driven by the master system but the behavior of the master system is not affected by the slave system. The control objective is to design an appropriate control input u in slave system to synchronize both systems, under the constraint such that (4) is assured:

$$\lim_{t\to\infty} \|y(t) - x(t)\| \to 0 \tag{4}$$

Let us define state errors between the master and slave systems as

$$e_i = y_i - x_i, \quad i = 1,2,\cdots,n \tag{5}$$

Therefore, the error dynamics of the master system and the slave system can be obtained as

$$\begin{cases} \dot{e}_1 = e_2, \\ \dot{e}_2 = e_3, \\ \vdots \\ \dot{e}_{n-1} = e_n, \\ \dot{e}_n = f(y,t) - g(x,t) + d(t) + u \end{cases} \qquad e = [e_1, \dot{e}_1, \cdots, e_1^{(n-1)}] \tag{6}$$

The synchronized problem can be viewed as the problem of choosing an appropriate control law u such that the error states $e_i (i = 1,2,\cdots,n)$ in (6) converge to zero.

SYNERGETIC METHOD DESEIGN

Based on aggregated regulators analytical approach theory, synergetic control is a state space approach for the design of controls for complex highly connected nonlinear systems [25-27]. The synergetic control forces the system state variables to evolve on a designer chosen invariant manifold enabling for desired performance to be achieved despite uncertainties and disturbances without damaging chattering inherent to sliding mode technique [25].

Start by defining a macro-variable as a function of the state variables as in (7):

$$\Psi = \psi(x,t) \tag{7}$$

The control law will force the system to operate on the manifold $\Psi = 0$. The designer can select the characteristics of this macro-variable according to the control specifications. The macro-variable can be a simple linear combination of the state variables. The same process can be repeated, defining as many macro-variables as control channels. Introducing a typical constraint (8), the selected macro-variable is forced to evolve in a desired manner despite the uncertainties and/or disturbances.

$$T\dot{\Psi} + \Psi = 0, \quad T > 0 \tag{8}$$

T which is designer chosen determining the rate of convergence to the attractor and can be made arbitrary small considering only eventual control constraint.

By suitable selection of macro-variables, the designer can obtain many interesting characteristics for the best synchronization, stability, parameter insensitivity and noise suppression. It is interesting to note that the synergetic control law guarantees the global stability on the manifold meaning that once the manifold is reached, the system is not supposed to leave it even for large-signal variations.

SYNCHRONIZATION USING SYNERGETIC CONTROL

Consider the n-dimensional chaotic systems (1) and (2), introducing the following macro-variable represented in (9):

$$\Psi = e_n + \sum_{i=1}^{n-1} \lambda_i e_i \tag{9}$$

where the parameters vector $\lambda = [\lambda_1, \lambda_2, \cdots, \lambda_{n-1}, 1]$ are chosen such that form a Hurwitz polynomial.
Taking time derivative of the macro-variable leads to (10)

$$\dot{\Psi} = f(x,t) - g(x,t) + u + \sum_{i=1}^{n-1} \lambda_i \dot{e}_i \tag{10}$$

Using (10), and the constraint condition (8), we propose the following theorem:

Theorem 1. For the n-dimensional chaotic systems, if we choose the following control law:

$$u = -f(y,t) + g(x,t) - \sum_{i=1}^{n-1} \lambda_i \dot{e}_i - \frac{1}{T}\Psi \tag{11}$$

to synchronize between master and slave systems.

Proof
Choosing the Lyapunov function candidate to be

$$V = \frac{1}{2}\Psi^T \Psi \tag{12}$$

Therefore
$$\dot{V} = \Psi^T \dot{\Psi} \tag{13}$$

$$= \Psi\left(f(x,t) - g(x,t) + u + \sum_{i=1}^{n-1} \lambda_i \dot{e}_i \right) \tag{14}$$

$$= -\frac{1}{T}\Psi^2 \tag{15}$$

thus:
$$\dot{V} \le 0 \tag{16}$$

Based on Lyapunov theory, the control law (11) allows ensuring system stabilization, robustness and the error vector will asymptotically converge to zero that the synchronized of the chaotic systems is assured.

BAT OPTIMISATION METHOD

The BAT algorithm is new optimization method based metaheuristic approach proposed by Yang [20]. The algorithm exploits the so-called echolocation of the bats. The bats use sonar echoes to detect and avoid obstacles. It's generally known that sound pulses are transformed into a frequency which reflects from obstacles. The bats navigate by using the time delay from emission to reflection. After hitting and reflecting, the bats transform their own pulse into useful information to explore how far away the prey is. The pulse rate can be simply determined in the range from 0 to 1, where 0 means that there is no emission and 1 means that the bat's emitting is their maximum [28-29].

BAT Algorithm

In order to model this algorithm, Yang [21] has idealized some rules, as follows:

1) All bats use echolocation to sense distance, and they also "know" the difference between food/prey and background barriers in some magical way;
2) A bat fly randomly with velocity v_i at position x_i with a fixed frequency $f_{\min}$, varying wavelength τ and loudness A_0 to search for prey. They can automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission $r \in [0,1]$, depending on the proximity of their target;
3) Although the loudness can vary in many ways, Yang assume that the loudness varies from a large (positive) A_0 to a minimum constant value $A_{\min}$.

Firstly, the initial position x_i, velocity v_i and frequency f_i are initialized for each bat. For each time step t, being T the maximum number of iterations, the movement of the virtual bats is given by updating their velocity and position using Equations :

$$f_i = f_{\min} + (f_{\min} - f_{\max})\beta \tag{17}$$

$$v_i^j(t) = v_i^j(t-1) + \left[\hat{x}^j - x_i^j(t-1)\right] f_i \tag{18}$$

$$x_i^j(t) = x_i^j(t-1) + v_i^j(t) \tag{19}$$

where β denotes a randomly generated number within the interval [0, 1]. Recall that $x_i^j(t)$ denotes the value of decision variable j for bat i at time step t. The variable $\hat{x}^j$ represents the current global best solution for decision variable j, which is achieved comparing all the solutions provided by the all bats.

In order to improve the variability of the possible solutions, Yang [20-21] has proposed to employ random walks. Primarily, one solution is selected among the current best solutions, and then the random walk is applied in order to generate a new solution for each bat.

$$x_{new} = x_{old} + \varepsilon \overline{A}(t) \tag{20}$$

in which $\overline{A}(t)$ stands for the average loudness of all the bats at time t, and $\varepsilon \in [-1, 1]$ attempts to the direction and strength of the random walk. For each iteration of this algorithm, the loudness A_i and the emission pulse rate r_i are updated, as follows:

$$A_i(t+1) = \alpha A_i(t) \tag{21}$$

where α and γ are constants. Generally, in the initial step, the emission rate and the loudness are often randomly chosen.

We use in this study: $\alpha = \gamma = 0.9$, $A_0 = 1$, $A_{\min} = 0$, $f_{\min} = 0$, $f_{\max} = 50$.

Objective Function

To optimize the synergetic parameters (λ & t) an objective function is formulated, where in the synchronisation between master system and slave system is affected. As an objective function represented as (23):

$$J = \sum_{1}^{n} e_i^2 \tag{23}$$

Subjected to

$$\lambda_{\min} \le \lambda \le \lambda_{\max} \tag{24}$$

$$T_{\min} \le T \le T_{\max} \tag{25}$$

This paper focuses on optimal tuning of synergetic approach using BAT algorithm. The typical values of the optimized parameters are taken as $[0.5-1.5]$ for λ and $[0.01-0.3]$ for T. The aim of the optimisation is to minimize the objective function in order to improve the synchronisation of two different chaotic systems (Figure. 1).

SIMULATION

In this section, a synchronization of two non-identical chaotic systems is presented to evaluate the performances and the robustness of the optimized synergetic approach.

The mathematical model of two different Chua's chaotic circuits is given by:

The master system is represented in (26):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = \dfrac{14}{1805}x_1 - \dfrac{168}{9025}x_2 + \dfrac{1}{38}x_3 - \dfrac{2}{45}\left(\dfrac{28}{361}x_1 + \dfrac{7}{95}x_2 + x_3\right)^3 \end{cases} \tag{26}$$

The slave system is represented in (27):

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = \dfrac{140}{1805}y_1 - \dfrac{1680}{9025}y_2 + \dfrac{6}{38}y_3 - \dfrac{2}{45}\left(\dfrac{28}{361}y_1 + \dfrac{7}{95}y_2 + y_3\right)^3 \end{cases} \tag{27}$$

where: $x_0 = [0.2\ 0.5\ 0.3]^T$ and $y_0 = [0.1\ 0.1\ 0.1]^T$.

Simulation Cases Without Control

Objective function used in this work is presented in Fig.1. The simulation results of the master system without control are shown in Fig. 2 to Fig. 4, phase plane trajectory of the master system in Fig.5 and of slave system in Fig.6. The simulation results of the slave system without control are shown in Fig. 7 to Fig. 9.

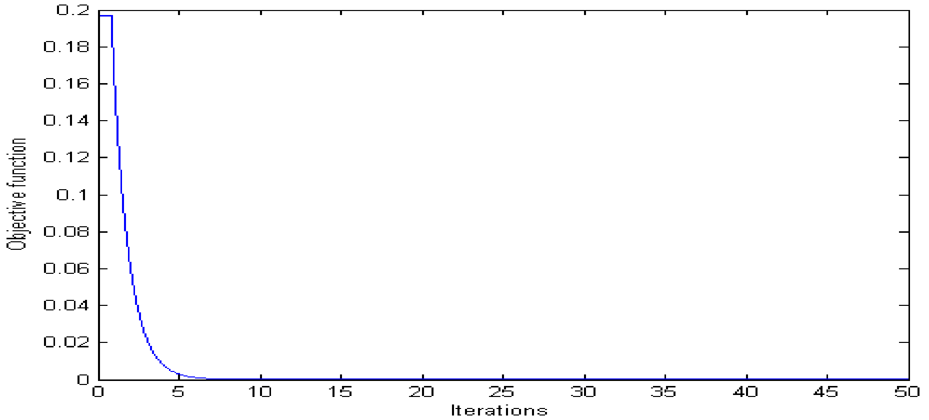

Fig. 1 Objective function.

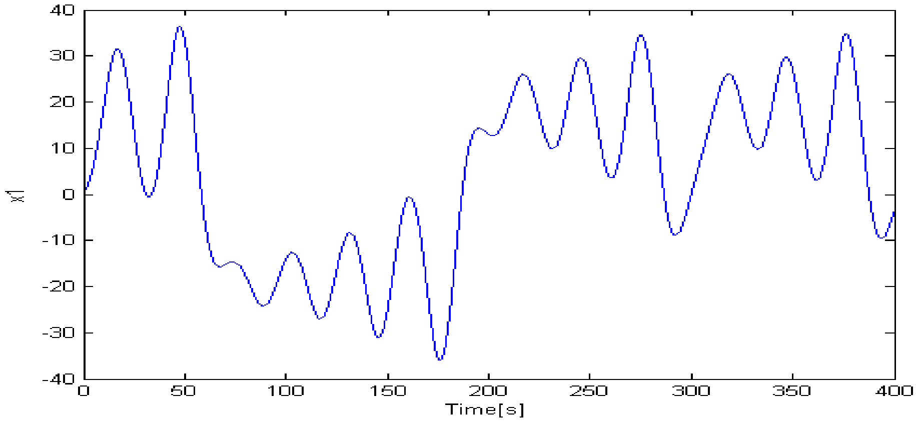

Fig. 2 Time response of state $x_1(t)$.

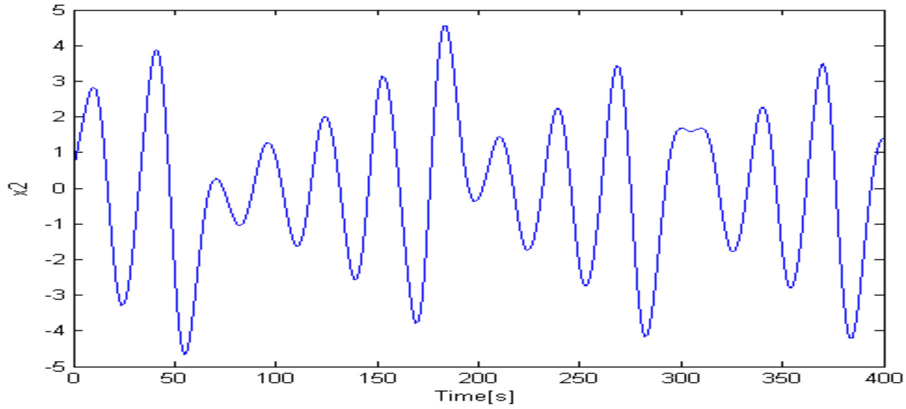

Fig. 3 Time response of state $x_2(t)$.

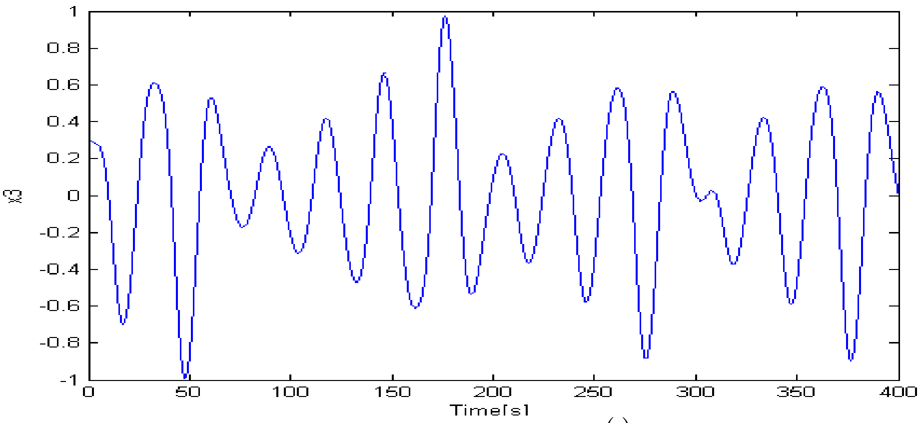

Fig. 4 Time response of state $x_3(t)$.

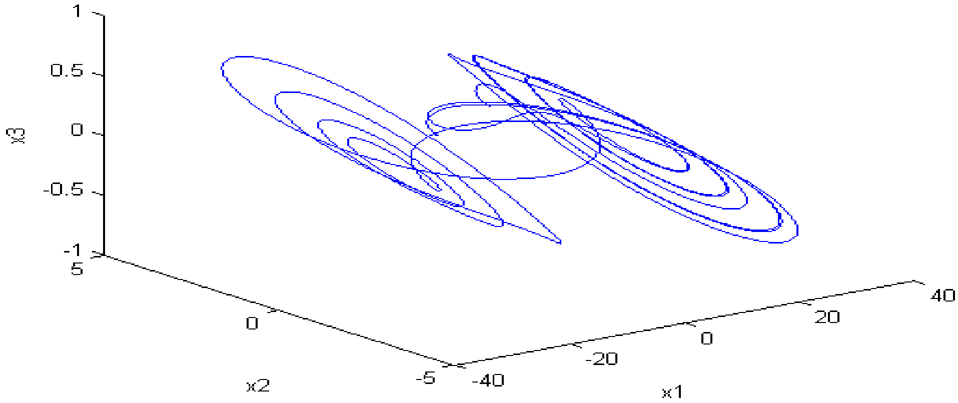

Fig. 5 Phase plane trajectory of the master system.

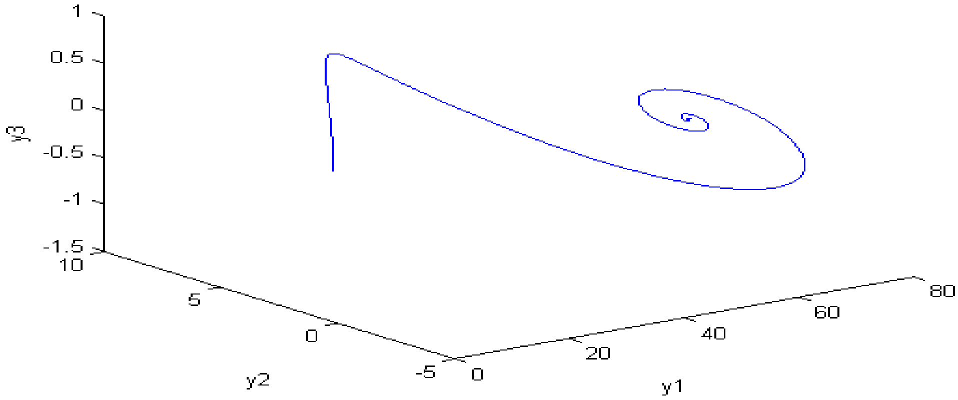

Fig. 6 Phase plane trajectory of the slave system.

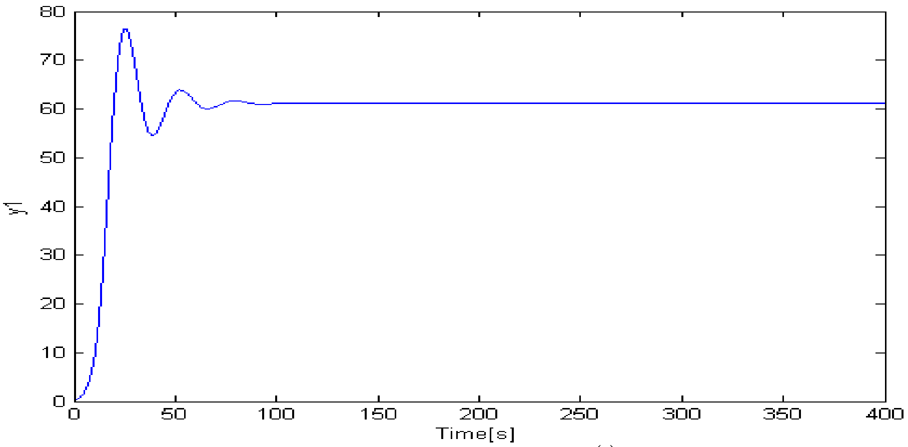

Fig. 7 Time response of state $y_1(t)$.

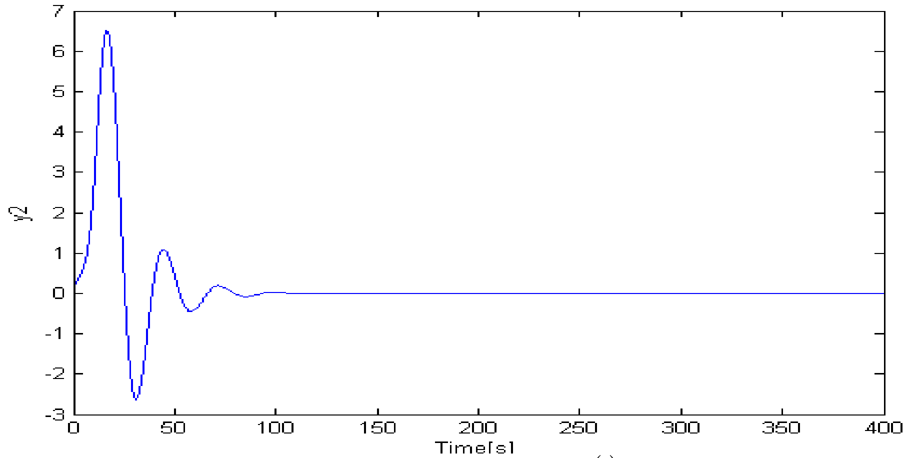

Fig. 8 Time response of state $y_2(t)$.

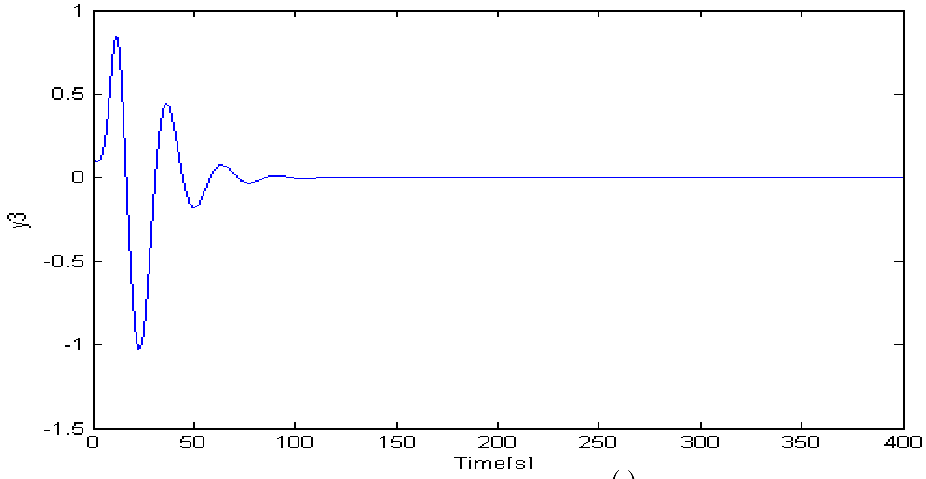

Fig. 9 Time response of state $y_3(t)$.

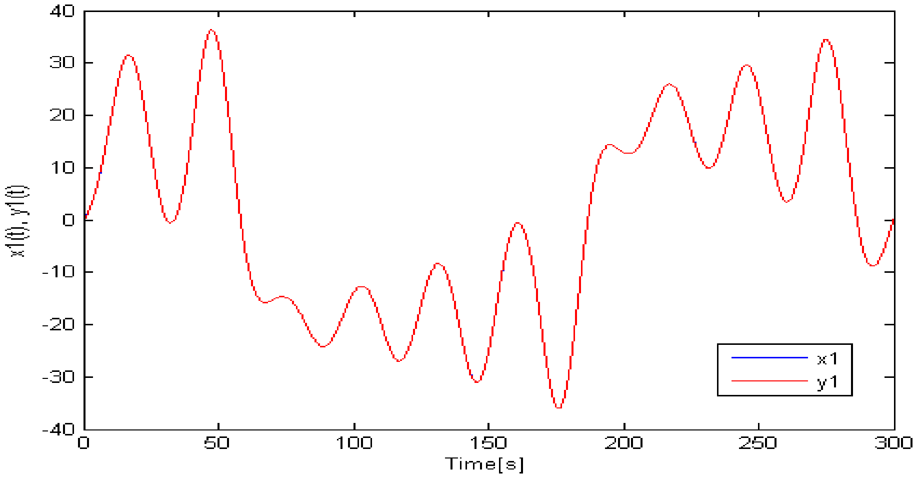

Fig. 10 Synchronization of output trajectories $x_1(t)$ **and** $y_1(t)$

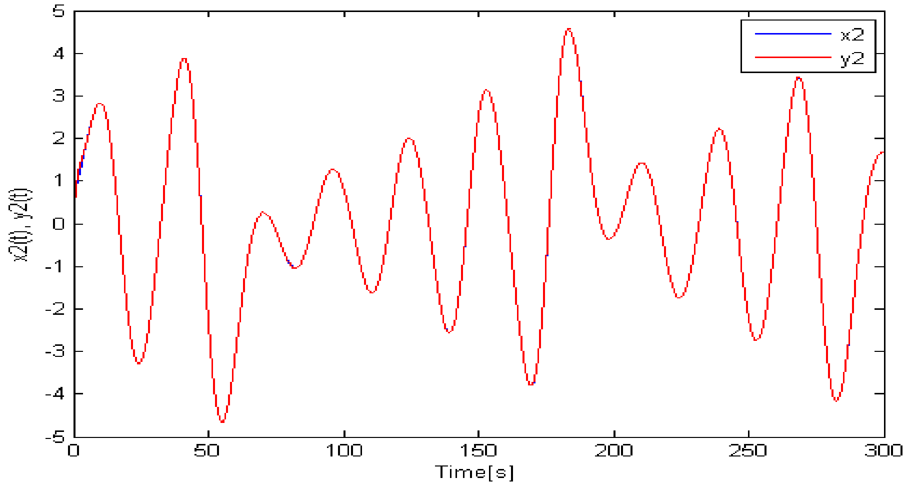

Fig. 11 Synchronization of output trajectories $x_2(t)$ **and** $y_2(t)$

Simulation Cases of Synchronization of Two Non-Identical Chaotic Systems

In order to force salve system to track master system, the optimized synergetic control by Bat algorithm is added to the slave system as follows:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_3 \\ \dot{y}_3 &= \frac{140}{1805}y_1 - \frac{1680}{9025}y_2 + \frac{6}{38}y_3 - \frac{2}{45}\left(\frac{28}{361}y_1 + \frac{7}{95}y_2 + y_3\right)^3 + d(t) + u \end{aligned} \tag{28}$$

where external disturbance $d(t)=0.2\sin(2t)$. The parameters of the synergetic approach after optimisation by Bat algorithm are : $\lambda=1$ and $T=0.02$.

Figures 10 to 12 indicate convergence of the states $x_1(t)$, $x_2(t)$ and $x_3(t)$ to the references $y_1(t)$, $y_2(t)$ and $y_3(t)$ respectively, in order to illustrate the accuracy of the synergetic methodology in synchronization of the chaotic systems (Fig. 13). In fig. 14 show the steady state error between the outputs and its references, it can be seen clearly that the error is converge asymptotically to zero.

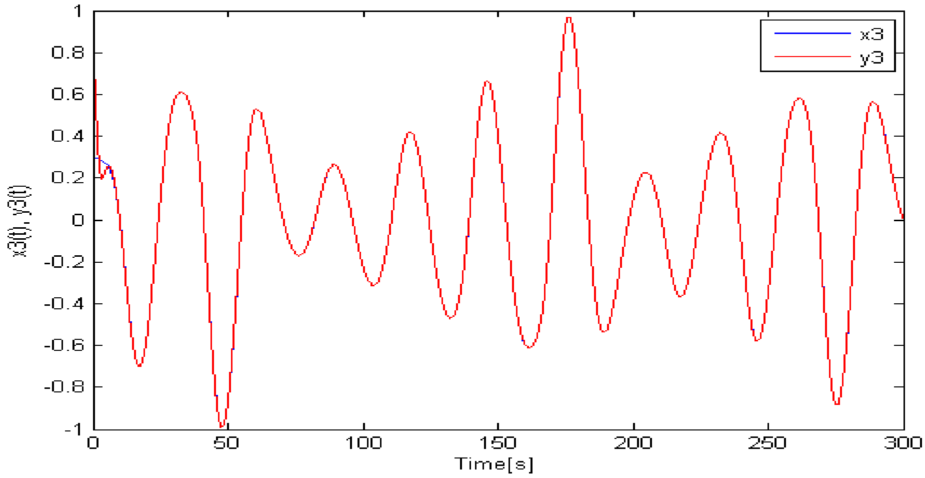

Fig. 12 Synchronization of output trajectories $x_3(t)$ and $y_3(t)$.

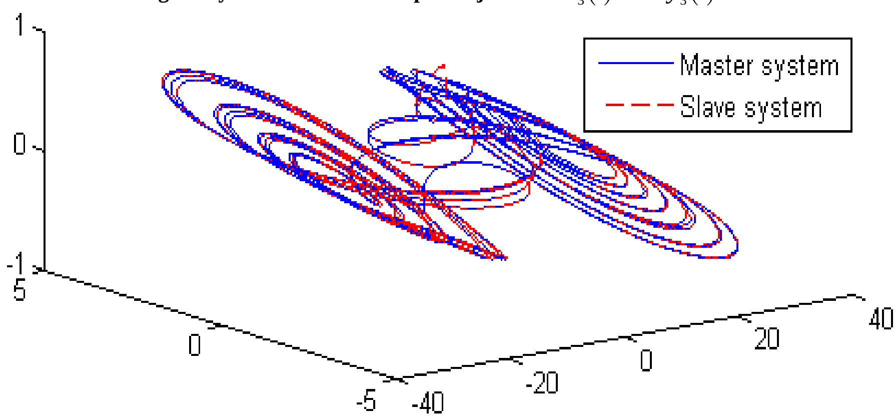

Fig. 13 Phase plane trajectory of the master-slave systems.

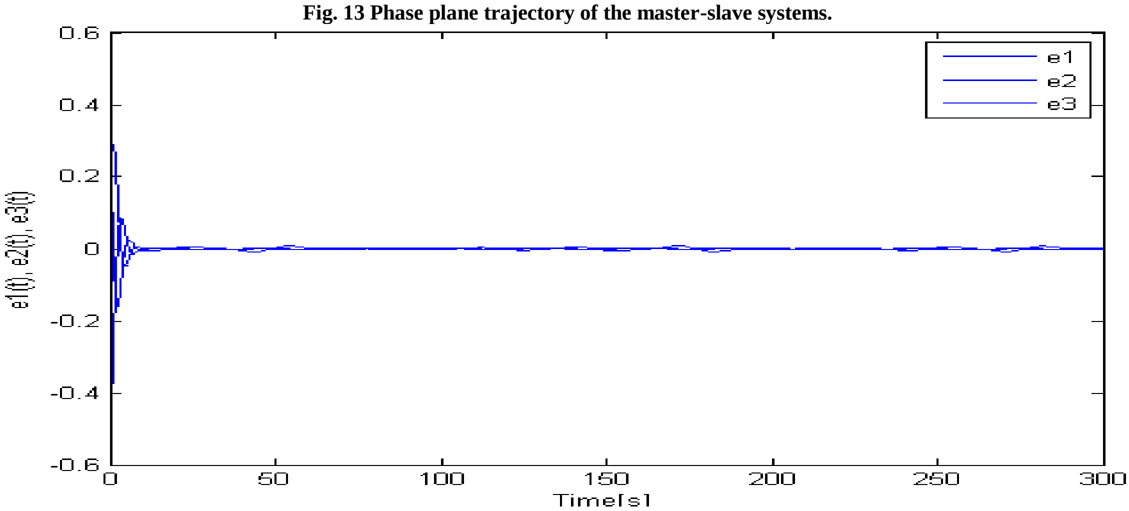

Fig. 14 Errors between trajectories of the master-slave systems

CONCLUSION

This paper presents an optimized approach by BAT algorithm based on synergetic strategy for the synchronization of non-identical chaotic systems. Simulation results without and under control reveal that the performance of a synergetic control in synchronization and stabilization of two nonlinear systems uncertainties. Lyapunov theorem is using for demonstrated the effectiveness of the proposed approach to synchronize two Chua's chaotic circuits.

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