



## MATLAB Simulation of the Effect of Billet Height on Extrusion Load in Axis-Symmetric Backward Cold Cup Extrusion

ME Bashir<sup>1</sup> and JS Ajiboye<sup>2</sup>

<sup>1</sup>Department of Mechanical Engineering, University of Benin, Nigeria

<sup>2</sup>Department of Mechanical/Production Engineering, University of Lagos  
[esimadanmusa@yahoo.com](mailto:esimadanmusa@yahoo.com)

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### ABSTRACT

The magnitude of extrusion load and its response to varying lengths of aluminium (Al) billet material during a backward cold cup extrusion process was carried out using MATLAB simulation. The matrix laboratory (MATLAB) software as a robust mathematical software tool was effective in simulation of the extrusion process in conjunction with the upper bound element technique (UBET). Results obtained showed that as the billet height decreased during plastic deformation the extrusion load also decreased. The maximum extrusion load was observed at the highest billet height, while the minimum extrusion load was observed at billet height of 2mm beyond which the extrusion load tends to increase again possibly due to work hardening of the metal billet resulting in velocity discontinuity at which point the extrusion operation proceed to stop. The use of MATLAB simulation for the extrusion process is cost effective, time saving and useful in predicting extrusion variables when compared to carrying out the physical extrusion experiment. This study is significant in the selection of proper extrusion machine capacity as well as executing an efficient extrusion process.

**Keywords:** Backward extrusion, Extrusion load, Billet Height, Upper Bound, MATLAB

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### INTRODUCTION

The extrusion process is a plastic deformation process involving a block of metal (billet) being forced to flow by compression through a die of a smaller cross sectional area. Forces are developed by the reaction of the work piece (billet) with the container and die. The reaction of the extrusion billet with the container and die results in high compressive stresses which are effective in reducing cracking of materials during primary breakdown from the ingot. The extrusion process may be sub categorized as indirect or backward extrusion and direct or forward extrusion. In backward extrusion the die of the front end of the hollow stem moves relative to the container, but there is no relative displacement between the billet and the container while in forward extrusion, the direction of metal flow will be in the same direction as ram travel during this period, the billet slides relatively to the walls of the container [3]. Extrusion process is used to create objects of fixed cross-sectional profile and materials that can be extruded include metals, polymers, food, ceramics, concrete etc. The products of extrusion are generally called 'extrudates'. Studies from experiments and theories have been proposed to help engineers and industries predict or select process variables and materials for an optimal process to save on energy, cost and materials, however with the advent of some powerful software programs there has been ease of modelling and simulating this extrusion process variable without going through the time, rigors and cost of carrying out intensive experiments, though such results arising from these computer simulations are later verified by comparison with experiments for optimization [3]. Despite the increased interest in metal forming by researchers, the technology at the industrial level is still mostly based on experience and trial and error methodology [1]. A great variety of modelling techniques for simulating the extrusion process with respect to its variables are available in the present day to engineers in design and development. Among the various methods of solutions for extrusion process, the upper bound elemental technique and slip-line method have the advantages of being analytical but both approaches show many difficulties [1]. Another approach is the use of numerical methods where especially the use of Finite Element Method (FEM) offers the opportunity of getting important process information and simulation and provides a more accurate description of the deformation and stresses in the extrusion process more than do other methods, however it demands an expert's use of a lot of computer time [1].

Grizeli *et al* [4] presented a numerical simulation of combined forward and backward extrusion process where the paper highlighted the finite element simulation as a very useful technique in studying where there is a generally close correlation in the load or extrusion force results obtained with finite element method and those obtained experimentally. According to Ajiboye *et al* [1], the extrusion load or force term is essential in determining the capacity of the extrusion press and temperature rise is slightly higher for higher initial billet height. Hae *et al* [5] used the finite element software DEFORM to simulate the process design of a forward and backward axis symmetric extrusion of an extrude part. The effect of an increasing billet length is a corresponding increase in the extrusion pressure. Extrusion pressure also increases as extrusion ratio increase [1]. The force for extrusion depends on the flow stress of the billet material, extrusion ratio, friction condition at the billet- container interface, friction condition of the die material interface, initial billet temperature and the speed of extrusion. ( $F_g = P_T A_c$  is the extrusion force [6]. The force term is essential in determining the capacity of the extrusion press. The temperature rise is slightly higher for higher initial billet height. The net deformation load dissipated within the material of the sleeve is the sum of the various plastic, frictional and shear power dissipated at appropriate volumes and surface areas of the material during deformation [7].

### Nomenclature

E	=	Total power of deformation	h	=	Current height of deforming Zone
w	=	Velocity (axial)	$r_1$	=	Internal radius of extruded component = 10.5
u	=	Velocity radial	t	=	incremental displacement of punch?
$\emptyset, y, r$	=	Cylindrical coordinates	L	=	length of material extruded
z	=	axial component	x	=	die land length = 2mm
m	=	friction factor	$\sigma$	=	mean basic yield stress for Aluminium
$V_o$	=	Steady velocity of punch	$V_e$	=	Velocity of extrusion upward = 2.40m/s
$r_2$	=	billet radius = 12.5	$V_p$	=	relative sliding velocity @ punch/material interface 20.40
$W_d$	=	Velocity discontinuity			

## MATERIALS AND METHODS

### Description of the Process

The experimental prototype set up of the backward extrusion tool press and billet is shown in Fig 1. The upper bound elemental analysis and finite difference analysis were used for the analysis of the deformation power and process analysis while the MATLAB program was used for the simulation of process variables and their inter-relationship.

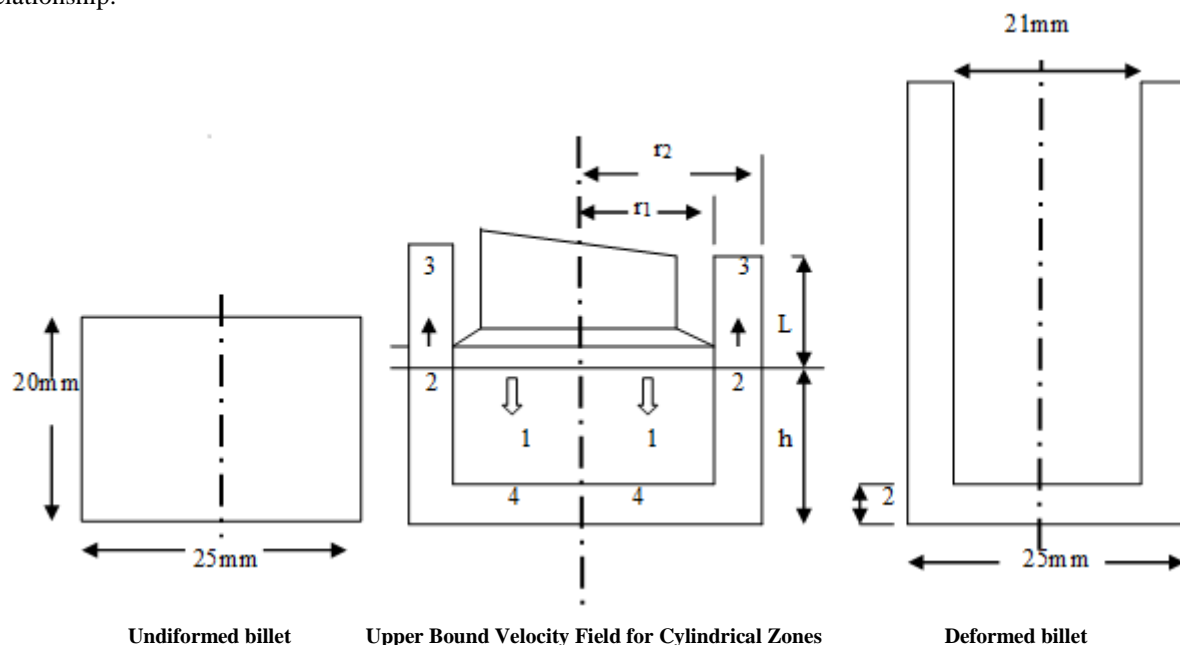


Fig. 1 Idealized backward extrusion of aluminium billet

### Total Power of Deformation or Extrusion Load

The net deformation power dissipated within the material of the sleeve is the sum of the various plastic, frictional and shear power dissipated at appropriate volumes and surface areas of the material of the sleeve during deformation.

$$E = E_{p1} + E_{p2} + E_{d1} + E_{d2} + E_{d3} + E_{f1} + E_{f21} + E_{f3} + E_{f32} + E_{f33} + E_{f4} \quad (1)$$

The relation in (1) is the deformation power and the various E subscripts were analyzed using Upper bound elemental technique (UBET) during a backward cold cup extrusion by Ajiboye *et al* [2]. The deformation power dissipated within the work piece is supplied by the external applied power of the forming machine.

### Upper Bound Element Analysis (UBET) of the Extrusion Load during a Backward Cold Cup Extrusion

In other to conveniently analyse the backward extrusion process using (UBE) the following assumptions are made.

- the longitudinal velocity  $V_y$  is uniform at each cross section of the material in the die and is equal to inlet velocity denoted by  $V_0$  at the entry
- the von mises yield criterion is assumed to be applicable

During the operation of forming the punch is assumed to be approaching the work piece in the stationary bottom die with a steady velocity of unity. After contact with the work piece, material will be displaced downwards for eventual extrusion upwards at the clearance between the punch cap and the outer die surface. This is the only option of flow for the deforming material since the tooling arrangement does not permit any forward extrusion of the material. Therefore, the material in the annular region 4 remains un-deformed i.e. rigid throughout the deformation process. However, its annular interface with region 1 and 2 is a surface of velocity discontinuity where deformation takes place throughout the operation of forming. Therefore, at an instant during the operation of forming the product, its deforming shape is as exemplified in Fig 1. With the dimensions and various zones of extrusion clearly indicated for the backward extrusion process of aluminium billet. The material displaced by the punch cap will have to leave the region through the surface of velocity discontinuity into region 2 since the other surface of velocity discontinuities is an interface with a region of static material [1]. Therefore, with a downward steady punch velocity of unity the axial velocity distribution is given as;

$$W = -z/h \quad (2)$$

From the equation 1 - E = the net deformation,

$E_{Pi}$  = Plastic Power at the regions, Where i = regions 1, 2 and  $E_{p1}$  = Plastic Power at region 1

For the region 1 indicated in figure 1, the circumferential velocity is zero, using the axial and radial velocity distributions. The strain rate field for region 1 is also considered. The material is assumed to obey the Von-Mises yield criterion and that it is rigid plastic with a flow stress of  $\sigma_0$ .

$E_{f1}$  = Frictional Power dissipated at region 1 at the punch/material interface

### Analysis of Region 2

Region 2 is the annular deformation region. Material flows into the region from region 1 through the surface of velocity discontinuity, and flows out into region 3 through the interface of velocity discontinuity. By incompressibility condition, all the material displaced by the Punch must leave region 2 for region 3. The  $r_1$  which is the radial velocity distribution in region 2 gives the same value as that at  $r_1$  in region 1. This thus satisfies the condition at the surface of velocity discontinuity, that the velocity of approach to it and exit from it must be the same in order to avoid material accumulating at the surface.

$E_{p2}$  = Plastic Power of deformation at region 2

$E_{f21}$  = The frictional power dissipated at the outer die/material interface due to relative axial sliding velocity at  $r_2$  between the material in the region and the surface of the outer die.

$E_{di}$  = Power dissipated at Discontinuity surface for the regions, Where i = regions 1, 2, 3

The velocity discontinuity at interface between region 1 and 2 is due to the axial velocity of material at this surface in region 1 relative to that of material at the surface in region 2.

The material in region 2 crosses the surface of velocity discontinuity to region 3 where no deformation takes place and the material is constrained to move axially upwards as a rigid body. Therefore, the velocity discontinuity at the surface is the same as the radial velocity of material in region 2  $z = h$ ,

The velocity discontinuity at the interface between region 2 and region 4 (dead metal zone) is due to the radial velocity at  $z = 0$  in region 2 since the material at the surface in region 4 is static, being dead material. But the radial velocity distribution in region 2 is not a function of  $z$  therefore the shear power dissipated at the surface of velocity discontinuity is the same as that dissipated at the interface between region 2 and region 3, hence:

$$Ed_3 = Ed_2 \quad (3)$$

### Analysis of Region 3

This region is the extruded material. Material flows into the region from region 2 to be part of a rigid body of extruded material constrained to slide uniformly upwards in contact with the stationary outer die surface and also in contact with the punch cap body moving downwards with unit velocity.

$E_{f31}$  = the frictional power dissipated per incremental displacement,  $t$ , of the punch at the outer die/material interface

$E_{f32}$  = frictional power dissipated at the punch/material interface

$E_{f33}$  = the additional frictional power dissipated at the die land length (x) and the extruded

**Analysis of Region 4**

Region 4 is a dead metal zone. The material at the die insert/material interface is stationary with the die insert hence there is no frictional power dissipated at the interface. However, while the material at punch/material interface is stationary the punch moves with a steady unit velocity.

$E_{f4}$  = the frictional power dissipated at the interface punch /materials interface

A formulation of values using the UBET mathematical relations of the equation 1 can be done.

Let the velocity with which the material extrudes = $V_e$

$$\text{and } V_e = \frac{r^2}{(r_2^2 - r_1^2)} \tag{4}$$

$$\text{That is } V_e \frac{10.5^2}{(12.5^2 - 10.5^2)} = 2.40$$

Where  $r_2$ = radius of die container and  $r_1$ = radius of billet

$$\text{Also the length of material extruded is given by the expression; } L = V_e t \tag{5}$$

Where t = punch displacement

Hence for various punch displacement t, from 0 to 18mm, the corresponding height of the un-deformed billet is such that h is = 18mm at maximum height and tends to 0 during vertical deformation or compression; it then follows that using the expressions from equations 4, and 5

$$\text{i.e. } v_e = \frac{r_1^2}{r_2^2 - r_1^2} \text{ and } l = v_e t$$

Computation of corresponding values of punch displacement t(mm), extrusion length l(mm), and billet height h(mm) can be developed as shown in Table -1. Consequently, the varying extrusion load or deformation powers (E) at instants of varying billet height (h) and extrusion length (l) as shown in table 2 can be computed from equation (1) by inputting all corresponding values of the sub-functional equations

**RESULTS AND DISCUSSION**

**Effect of Billet Height on Extrusion Load**

From the graph in Fig. 2, billet height against extrusion load is plotted from the values in Table 2. It is observed that there is a decrease in the extrusion load as the billet height decreases during the extrusion process. The reason is that at the billets maximum or initial height, there is a rapid rise in the extrusion load due to compression. However, as the billet extrudes, this extrusion load required to maintain flow progressively decreases with decreasing length of the billet. This is supported by Domphol [6] who asserted that at initial ram displacement, there is a rapid rise in the extrusion load as a result of the compression of the billet during extrusion. This compression will have excited the internal molecular structure of the billet, resulting in deformation and surface to surface sliding which results to frictional forces. All these contribute to an increase in the temperature and internal energy of the material. The relationship in the graph also corroborates with [1], where they found both experimentally and theoretically that for a given percentage reduction in area, the temperature rise is higher for higher billet height during extrusion, and as this percentage reduction in area increases further, the temperature increased. They further assert that there was an increased energy required to deform the material as well as increased frictional work.

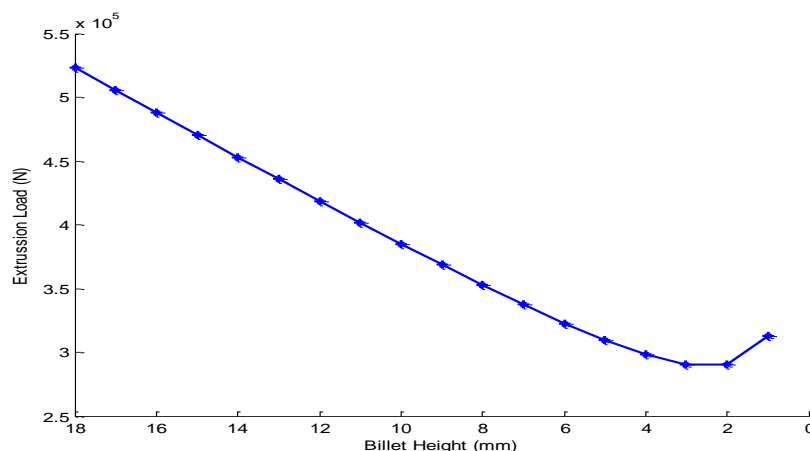


Fig. 2 Graph of billet height against Extrusion load

Table -1 Values for Punch Displacement t, Billet Height h and Extrusion Length L

t(mm)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
h(mm)	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
L(mm)	0	2.4	4.6	7.2	9.6	12	14.4	16.8	19.2	21.6	24	26.4	28.8	31.2	33.6	36	38.4	40.8	43.2

Table -2 Values of Extrusion Load at Varying Billet Height

h	E_p1	E_p2	E_d1	E_d2	E_d3	E_f1	E_f2	E_f31	E_f32	E_f33	E_f4	E_total
0	16339.49	42756.41	401728.2	16732.47	16732.47	42.8222	25767.04	0	0	3484.987	0	0.9
1			379410	17217.57	17217.57	44.06368	24335.54	585.7121	4181.984		204.9992	505778.3
2			357091.7	17747.46	17747.46	45.4198	22904.04	1171.424	8363.969		409.9985	488062.4
3			334773.5	18329.5	18329.5	46.90937	21472.53	1757.136	12545.95		614.9977	470450.9
4			312455.3	18972.83	18972.83	48.55581	20041.03	2342.848	16727.94		819.9969	452962.2
5			290137	19689.04	19689.04	50.38875	18609.53	2928.56	20909.92		1024.996	435619.4
6			267818.8	20493	20493	52.44627	17178.03	3514.273	25091.91		1229.995	418452.3
7			245500.6	21404.24	21404.24	54.77834	15746.52	4099.985	29273.89		1434.995	401500.1
8			223182.3	22448.96	22448.96	57.45201	14315.02	4685.697	33455.87		1639.994	384815.2
9			200864.1	23663.28	23663.28	60.55974	12883.52	5271.409	37637.86		1844.993	368469.9
10			178545.9	25098.7	25098.7	64.2333	11452.02	5857.121	41819.84		2049.992	352567.4
11			156227.6	26831.64	26831.64	68.66829	10020.52	6442.833	46001.83		2254.992	337260.6
12			133909.4	28981.48	28981.48	74.17023	8589.013	7028.545	50183.81		2459.991	322788.8
13			111591.2	31747.62	31747.62	81.24941	7157.511	7614.257	54365.8		2664.99	309551.1
14			89272.94	35494.92	35494.92	90.83961	5726.009	8199.969	58547.78		2869.989	298278.3
15			66954.7	40986	40986	104.8925	4294.507	8785.681	62729.76		3074.988	290497.4
16			44636.47	50197.4	50197.4	128.4666	2863.004	9371.393	66911.75		3279.988	290166.7
17			22318.23	70989.84	70989.84	181.6792	1431.502	9957.106	71093.73		3484.987	313027.8
18			0	0	0	0	0	10542.82	75275.72		3689.986	152089.4

## CONCLUSION

The result obtained from the research work is in agreement with literature reviews and experimental works cited. The use of MATLAB for the simulation of the extrusion variables was successful indicative of the robustness and viability of the software tool in analysing mathematical functions. The coding of the extrusion functions carried out using MATLAB 2007. The use of mathematical software tools in process simulations saves time and cost and is convenient and flexible for use in iterating and predicting process variables.

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