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Heat and Mass Transfer Effects on Unsteady MHD Flow through an Accelerated Isothermal Vertical Plate Embedded in Porous Medium in the Presence of Heat Source and Chemical Reaction

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ABSTRACT

An attempt is made to study the effect of heat and mass transfer of an unsteady rotating flow of an incompressible and electrically conducting fluid past an uniformly accelerated infinite isothermal vertical plate embedded in a porous medium, in the presence of a uniform transverse magnetic field, heat sink and chemical reaction. The dimensionless governing equations are solved using Laplace transform method. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. The permeability of the porous medium accelerates the velocity whereas the heat absorbing coefficient decelerates the velocity of flow field at all points. Further, it is found that the chemical reaction parameter has a retarding effect on velocity and concentration profile.

Key words: Mass and heat transfer, porous medium, heat sink, chemical reaction, Laplace transformation

INTRODUCTION

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field. MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations are in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

MHD flow with heat and mass transfer has been a subject of interest of many researchers because of its varied application in science and technology. Such phenomenon is observed in buoyancy induced motions in the atmosphere, in water bodies, quasi -solid bodies such as earth etc. In many transport processes existing in nature and in industrial applications in which heat and mass transfer is a consequence of buoyancy effects caused by diffusing of heat and chemical species. An important class of two dimensional time dependent flow problem dealing with the response of boundary layer to external unsteady fluctuations of the free stream velocity about a mean value attracted the attention of many researchers.

In natural processes and industrial applications many transportation processes exist where transfer of heat and mass takes place simultaneously as a result of thermal diffusion and diffusion of chemical species. There are many transport processes that are governed by the combined action of buoyancy forces due to both thermal and mass diffusion in the presence of chemical reaction effect. These processes are observed in the nuclear reactor safety and combustion systems, solar collectors, as well as metallurgical and chemical engineering. In nature, the presence of pure air or water is impossible. Some foreign mass may be present either naturally or mixed with the air or water.

In recent years, MHD flow with heat and mass transfer in presence of chemical reaction are of importance in any practical processes such as distribution of temperature and moisture over agricultural field, energy transfer in a wet cooling tower, in method of generating and extracting power from a moving fluid. So many researchers have taken interest in study the above effects; Rajeswari et al [1] studied chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through vertical porous surface in the presence of suction. Chemical reaction and heat generation or absorption on double diffusive convection flow from vertical truncated cone in a porous media with variable viscosity is studied by Mahdy [2]. Pal and Talukdar [3] have studied perturbation analysis of unsteady MHD convective heat and mass transfer in boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Further the effect of thermal radiation and heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate in presence of transverse magnetic field was investigated by Makinde and Ogulu [4]. Aziz [5] theoretically examined that a similarity solution for a laminar thermal boundary layer over a flat plate with a convective surface boundary condition. The effects of transversely applied magnetic field on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate is studied by Soundalgekar et al [6]. Radiation and chemical reaction of convective fluids in presence of heat source within a porous medium are of great practical importance in geophysics and energy related problem such as recovery of petroleum resources, cooling of underground electric cable, ground water pollution, fiber and granular insulation, chemical catalytic reactors and solidification of casting. The above applications attracts authors like; Bagai [7] studied the effect of variable viscosity on free convective over a non-isothermal axisymmetric body in a porous medium with internal heat generation. Malla et al [8] presented natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Hady et al [9] studied the heat generation or absorption effect on MHD free convection flow along a vertical wavy surface. Singh and Kumar [10] established the free convection effects on flow past an exponentially accelerated vertical plate. Muthucumaraswamy et al [11] presented the mass transfer effects on exponentially accelerated isothermal vertical plate. Rajput and Kumar [12] studied MHD flow past an impulsively vertical plate with variable temperature and mass diffusion.

Recently, Rout et al. [13] studied the influence of chemical reaction and radiation on MHD heat and mass transfer fluid flow over a moving vertical plate in presence of heat source with convective boundary condition. Unsteady rotating MHD free and forced convection flow in a channel has been studied by Barik [14]. Barik et al [15] have studied Hall effects on unsteady MHD flow between two rotating disks with Non-coincident parallel axes.

The objective of the present study is to consider the heat and mass transfer effect on unsteady rotating MHD flow through an accelerated isothermal vertical plate embedded in porous medium in the presence of heat source and chemical reaction. We have extended the work of Muthucumarswamy et al [16] by incorporating heat source, chemical reaction and permeability of the medium on the flow, heat and mass transfer phenomena.

MATHEMATICAL FORMULATION

Consider a transient, laminar, unsteady free convection flow of a viscous incompressible electrically conducting fluid past a uniformly accelerated motion of an isothermal vertical infinite plate embedded in a porous medium. The fluid and plate are considered to rotate as a rigid body with a uniform angular velocity Ω' about z'-axis in the presence of an imposed uniform magnetic field B_0 normal to the plate. Initially it is assumed that the temperature of the plate and concentration near the plate are T'_∞ and C'_∞. At time $t' > 0$, the plate starts moving with a velocity $u = u_0 t'$ in its own plane and the temperature of the plate and concentration of the fluid near the plate is raised to T'_w and C'_w. An external uniform magnetic field is applied normal to the plate i.e., parallel to the z'-axis and has a constant magnetic flux density B_0 with a small magnetic Reynolds number so that the induced magnetic field is small in comparison with the applied magnetic field $\bar{B} = (0, 0, B_0)$. Under the above assumptions, the governing boundary layer equations for a laminar, unsteady free convection and mass transfer of a viscous incompressible electrically conducting and heat absorbing fluid in a rotating system in a porous medium with usual Boussinesq's approximation in the presence of thermal diffusion are as follows:

Governing Equations

$$\frac{\partial u}{\partial t} - 2\Omega' v = g\beta\left(T - T_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{K_p^*}u \tag{1}$$

$$\frac{\partial v}{\partial t'} - 2\Omega' u = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2 v}{\rho} - \frac{\nu}{K_p^*} v \tag{2}$$

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial z^2} - S'\left(T - T_\infty\right) \tag{3}$$

$$\frac{\partial C}{\partial t'} = D \frac{\partial^2 c'}{\partial z^2} - K_c'\left(C - C_\infty\right) \tag{4}$$

On introducing the following non-dimensional quantities

$$U = \frac{u}{\left(\nu u_0\right)^{1/3}}, V = \frac{v}{\left(\nu u_0\right)^{1/3}}, t = t'\left(\frac{u_0^2}{\nu}\right)^{1/3}, Z = z\left(\frac{u_0}{\nu^2}\right)^{1/3}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, Gr = \frac{g\beta\left(T_w - T_\infty\right)}{u_0}$$

$$C = \frac{C - C_\infty}{C_w - C_\infty}, Gc = \frac{g\beta^*\left(C_w - C_\infty\right)}{u_0}, M = \frac{\sigma B_0^2}{\rho}\left(\frac{\nu}{u_0^2}\right)^{1/3}, \Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}, \Omega = \Omega'\left(\frac{\nu}{u_0^2}\right)^{1/3}$$

The dimensionless governing equations are

$$\frac{\partial U}{\partial t} - 2\Omega V = Gr\theta + GcC + \frac{\partial^2 U}{\partial z^2} - \left(M + \frac{1}{k_p}\right)U \tag{5}$$

$$\frac{\partial V}{\partial t} - 2\Omega U = \frac{\partial^2 V}{\partial z^2} - \left(M + \frac{1}{k_p}\right)V \tag{6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 T}{\partial z^2} - S_1 T \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial z^2} - K_c C \tag{8}$$

With the following initial and boundary conditions:

$$\begin{aligned} & u = 0, T = T_\infty, C' = C_\infty' \text{ for all } y, t' \le 0 \\ & t' > 0 : u = u_{0t'}, T = T_w, C' = C_w' \text{ at } y = 0 \\ & u \to 0, T \to T_\infty, C' \to C_\infty' \text{ as } y \to \infty \end{aligned} \tag{9}$$

To solve equations (5) and (6), we introduce a complex velocity q = U + iV, equations (5) and (6) can be combined into a single equation as

$$\frac{\partial q}{\partial t} = GrT + GcC + \frac{\partial^2 q}{\partial z^2} - mq \tag{10}$$

where $$m = M + \frac{1}{k_p} + 2i\Omega \tag{11}$$

with the initial and boundary conditions in non-dimensional form are

$$\begin{aligned} & q = 0, T = 0, C = 0 \text{ for all } Z, t \le 0 \\ & t > 0 : q = t, T = 1, C = 1 \; at \; Z = 0 \\ & q \to 0, T \to 0, C \to 0 \text{ as } Z \to \infty \end{aligned} \tag{12}$$

Method of Solution

The dimensionless governing equations (2), (3) and (7) subject to the initial and boundary conditions (9) are solved by Laplace transform technique. The solutions are derived as follows:

$$q = \frac{1}{2}\left[\left(t + \frac{z}{2\sqrt{m}}\right)e^{z\sqrt{m}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{mt}\right) + \left(t - \frac{z}{2\sqrt{m}}\right)e^{-z\sqrt{m} erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{mt}\right)\right]$$

$$-\frac{1}{2}\alpha_5 e^{\alpha_2 t}\left[e^{z\sqrt{m+\alpha_2}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{(m+\alpha_2)t}\right)+e^{-z\sqrt{m+\alpha_2}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{(m+\alpha_2)t}\right)\right]$$

$$+\frac{1}{2}\alpha_5\left[e^{z\sqrt{m}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{mt}\right)+e^{-z\sqrt{m}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{mt}\right)\right]$$

$$-\frac{1}{2}\alpha_6 e^{\alpha_4 t}\left[e^{z\sqrt{m+\alpha_4}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{(m+\alpha_4)t}\right)+e^{-z\sqrt{m+\alpha_4}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{(m+\alpha_4)t}\right)\right]$$

$$+\frac{1}{2}\alpha_6\left[e^{z\sqrt{m}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{mt}\right)+e^{-z\sqrt{m}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{mt}\right)\right]$$

$$+\frac{1}{2}\alpha_5 e^{\alpha_2 t}\left[e^{z\sqrt{\Pr}\sqrt{s+\alpha_2}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{(s+\alpha_2)t}\right)+e^{-z\sqrt{\Pr}\sqrt{s+\alpha_2}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{(s+\alpha_2)t}\right)\right]$$

$$-\frac{1}{2}\alpha_5\left[e^{z\sqrt{\Pr}\sqrt{s}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{st}\right)+e^{-z\sqrt{\Pr}\sqrt{s}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{st}\right)\right]$$

$$+\frac{1}{2}\alpha_6 e^{\alpha_4 t}\left[e^{z\sqrt{Sc}\sqrt{Kc+\alpha_4}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(Kc+\alpha_4)t}\right)+e^{-z\sqrt{Sc}\sqrt{Kc+\alpha_4}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(Kc+\alpha_4)t}\right)\right]$$

$$-\frac{1}{2}\alpha_6\left[e^{z\sqrt{\Pr}\sqrt{s}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Kct}\right)+e^{-z\sqrt{\Pr}\sqrt{s}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Kct}\right)\right] \quad (13)$$

$$T=\frac{1}{2}\left[e^{z\sqrt{\Pr}\sqrt{s}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{st}\right)+e^{-z\sqrt{\Pr}\sqrt{s}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{st}\right)\right] \quad (14)$$

$$C=\frac{1}{2}\left[e^{z\sqrt{Kc}\sqrt{Sc}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Kct}\right)+e^{-z\sqrt{Kc}\sqrt{Sc}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Kct}\right)\right] \quad (15)$$

Where $\alpha_1=\frac{Gr}{\Pr-1},\alpha_2=\frac{m-\Pr S}{\Pr-1},\alpha_3=\frac{Gc}{Sc-1},\alpha_4=\frac{m-ScKc}{Sc-1},\alpha_5=\frac{\alpha_1}{\alpha_2},\alpha_6=\frac{\alpha_3}{\alpha_4}$

RESULTS AND DISCUSSION

In order to get a clear insight into the physical problem, numerical values of velocity , temperature and concentration fields are computed for different physical parameters like Gr. Ge, Sc, Kp, S, Kc, Pr, M. Fig.1 exhibits the effects of different pertinent parameters such as porosity parameter (Kp), chemical reaction parameter (Kc) and heat absorption coefficient (S) on velocity profiles. It is observed that porosity parameter enhances the velocity of flow field at all points. Further, it is observed that velocity decreases with increase in heat absorbing coefficient and chemical reaction parameter. The decrease may be attributed to the absorption of heat energy due to endothermic reaction (Kc>0). The presence of heavier diffusing species causes decreases in velocity which leads to thinning of boundary layer thickness. Further, it is seen that when Kp=100, S=0 and Kc=0 (curve V), the curve coincides with the result reported by Muthucumaraswamy et al [16].

Fig.2 demonstrates the effect of Pr, Sc, M on the velocity profile. It is observed that increase in Pr causes a decrease in velocity of fluid. It is because increase in Pr is due to increase in the viscosity of fluid which makes the fluid thick and hence causes a decrease in velocity of fluid. It is found that increase in magnetic field parameter slow down the velocity of flow field to a considerable amount due to Lorentz force acting on the flow field. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow. It is seen that increase in Schmidt number reveals that the presence of heavier diffusion species has a retarding effect on velocity of flow field.

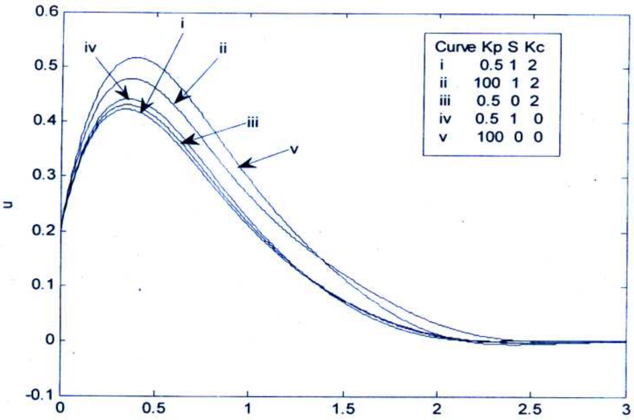

Fig.1 Velocity profiles for different Kp, S, Kc, for Gr = 5, Gc =5, Pr = 0.71, Sc =1, M =5, Ω = 0.5

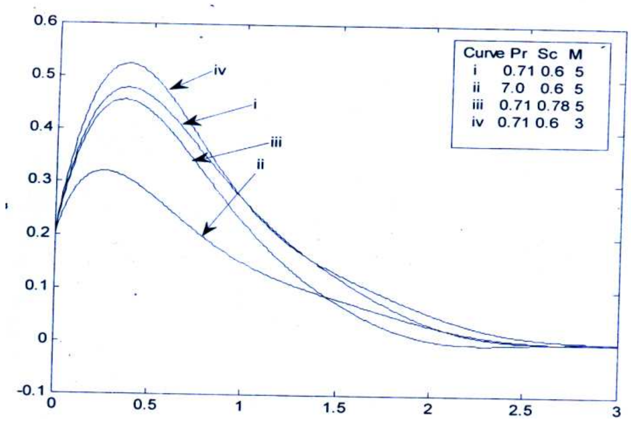

Fig.2 Velocity profiles for different Pr, Sc, M, for Gr = 5, Gc =5, S = 1,Kp = 0.5, Kc =2, Ω = 0.5

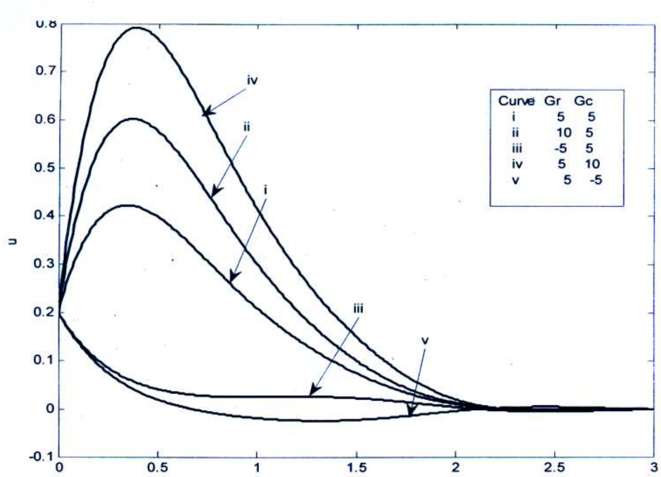

Fig.3 Velocity profiles for different Gr, Gc, for Pr = 0.71, Sc = 0.6, M = 5, S = 1,Kp = 100, Kc =2, Ω = 0.5

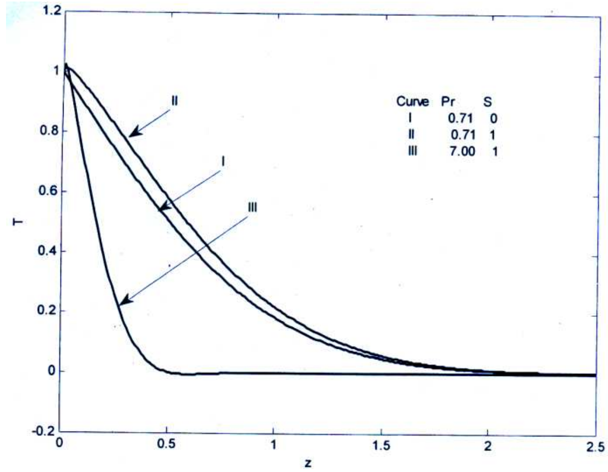

Fig. 4 Temperature profile

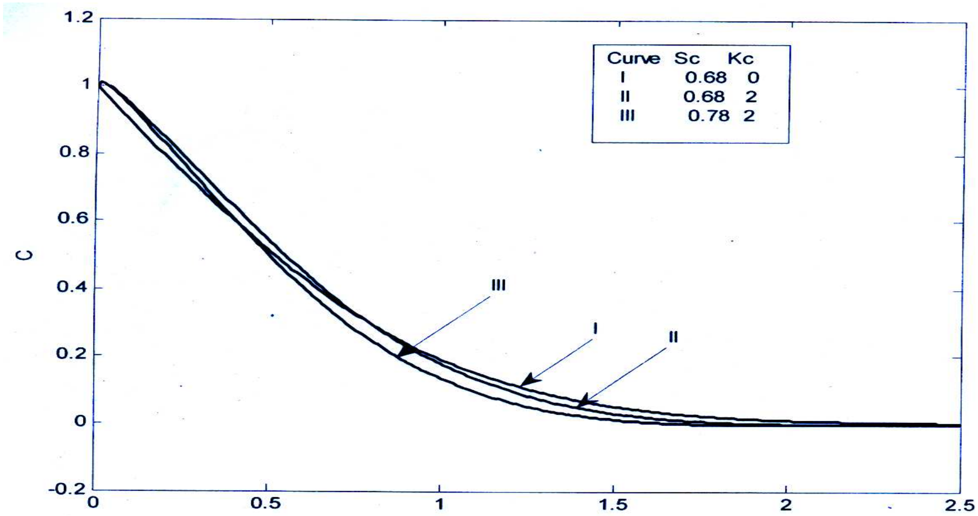

Fig. 5 Concentration profile

Fig.3 demonstrates the effect of Gr, Gc on velocity field. It is observed that increase in Gr leads to a rise in velocity due to enhancement in buoyancy force. Further, it is also found that Gc accelerates the velocity of flow field at all points.

The temperature profiles for different values of Pr and S are shown in Fig.4. Prandtl number is the ratio of kinematic viscosity and thermal diffusivity. If other things remain same, an increase in kinematic viscosity vis-a-vis momentum diffusivity leads to enhance the velocity boundary layer thickness. Similar discussion may be carried out with thermal diffusivity due to heat conduction contributing to growth of thermal boundary layer. It is seen that temperature profile decreases with an increase in Prandtl number. This is in agreement with the physical fact that at higher Prandtl number fluid has a thinner thermal boundary layer and this increases the gradient of temperature. From curves (I & II) it is observed that increase in S increases temperature. It is seen from curve I when S=0 and Pr=0.71, the curve coincides with the result reported by Muthucumaraswamy et al [16].

Fig.5 represents the effect of concentration profiles for different values of Sc and Kc. It is observed that concentration distribution decreases at all points of flow field with increase in Sc. It means that a greater delaying effect on concentration distribution is due to heavier diffusing species of flow field. It is seen that concentration of fluid reduces with increase in chemical reaction parameter (Kc).

CONCLUSION

- Velocity of the flow field decreases with increase in heat source parameter, chemical reaction parameter, Prandtl number and Schmidt number.
- The permeability of the porous medium enhances the velocity profile at all points.
- The magnetic parameter has a retarding effect on velocity profile due to Lorentz force acting on the flow field.
- The heat source parameter accelerates the temperature whereas Prandtl number decelerates it.
- The increasing effect of chemical reaction parameter and Schmidt number reduces the concentration profile.

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