



## Effects of Suction and Internal Heat Generation on Hydromagnetic Mixed Convective Nanofluid Flow over an Inclined Stretching Plate

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### ABSTRACT

The problem of a steady, non-linear, mixed convective two dimensional laminar boundary layer flow of a viscous, incompressible nanofluid past an inclined stretching plate in the presence of magnetic field, heat generation and suction has been considered. Two different types of nanoparticles such as copper and alumina with the base fluid water are considered. Using a similarity approach, the governing partial differential equations are transformed into ordinary differential equations and they are solved numerically using MATLAB. Numerical investigations are carried out for different values of physical parameters and the effect of all these parameters over the flow field and temperature are discussed by means of graphs. The numerical values of skin friction coefficient and rate of heat transfer for various values of physical parameters are also obtained and are tabulated. Comparison with previously published results for the regular fluid is presented and it is found to be in excellent agreement.

**Key words:** Nanofluid, MHD, suction, internal heat generation, inclined plate

### INTRODUCTION

The laminar boundary layer flow over a stretching surface has attracted the attention of many researchers in the last several decades. The analysis of such flows find applications in many areas such as the aerodynamic extrusion of plastic sheets, the boundary layer along material handling conveyors, the cooling of an infinite metallic plate in a cool bath, and the boundary layer along a liquid film in condensation processes. In special, the problems concerned with the mixed convective hydromagnetic boundary layer flow and heat transfer past a stretching surface find applications in polymer technology and metallurgy where hydromagnetic techniques have been used. Some of the industrial examples of the problem are extrusion processes, cooling of nuclear reactors, glass fibre production, hot rolling, wire drawing and crystal growing.

Sakiadis [1] was the first person to discuss the laminar boundary layer flow of a viscous and incompressible fluid caused by a continuous moving rigid surface. The flow over a linearly stretching sheet for the steady two-dimensional problem was analysed by Crane [2]. These types of flows usually occur in the drawing of plastic films and artificial fibres. The hydromagnetic mixed convective flows over a stretching surface were investigated by [3-6]. Mucoglu and Chen [7] analysed the study on mixed convection along an inclined flat plate, with the angle of inclination from the vertical and the plate is kept at a uniform temperature. Later, many investigations were proposed on hydromagnetic flow over inclined stretching surface considering various physical situations and few of them are [8-12].

Recently, Nanofluid is a new class of fluid with nanosized particles dispersed in a poor thermal conductivity base fluid, such as water and ethylene glycol, to increase its thermal conductivity. These particles, generally metal or metal oxide, increase conduction and convection coefficient, allowing for more heat transfer out of the coolant. It seems that the nanofluid was first introduced by Choi and Eastman [13] and it was adopted by many researchers. There are several industrial and engineering applications of nanofluids such as chemical production, solar and power plant cooling, cooling of transformer oil, production of microelectronics, automotive and air conditioning cooling, advanced nuclear systems, nano-drug delivery, micro fluidics, transportation, biomedicine, solid-state lighting and manufacturing.

The recent book by Das et al [14] and more recent review paper by Kakac and Pramuanjaroenkij [15] examined an excellent aggregation of the study done on nanofluids. Oztop and Abu-Nada [16] analyzed the numerical study of free convection in partially heated rectangular enclosures filled with nanofluids using different types of

nanoparticles. Khan and Pop [17] have proposed the problem of laminar fluid flow over the stretching surface in a nanofluid and they investigated it numerically. Anjali Devi and Julie Andrews [18] studied the laminar boundary layer flow of nanofluid over a flat plate and it was found out that the suspended nanoparticles enhance the heat transfer capacity of the fluids. The convective flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in the presence of a magnetic field was examined by Hamad [19]. Chamkha and Aly [20] focused on the numerical solution of steady natural convection boundary-layer flow of a nanofluid consisting of a pure fluid with nanoparticles along a permeable vertical plate in the presence of magnetic field, heat generation or absorption, and suction or injection effects. Rana et al [21] investigated the steady mixed convection boundary layer flow of an incompressible nanofluid along an inclined plate embedded in a porous medium. The laminar hydromagnetic mixed convection flow of copper-water and alumina-water nanofluids over an inclined plate was considered by Anjali Devi and Suriyakumar [22]. Aly et al [23] analysed the analytical and numerical solutions for mixed convection boundary-layer nanofluid flow along an inclined plate embedded in a porous medium. Yacob [24] demonstrated the mixed convection of fluid flow and heat transfer over a stretching vertical surface immersed in a nanofluid. Srikanth et al [25] investigated theoretically the MHD flow of a nanofluid past an inclined permeable plate with constant heat source and thermal radiation.

In the present study, our main objective is to investigate the effects of suction and internal heat generation over the mixed convective hydromagnetic flow over an inclined stretching plate with two different types of nanofluids namely, copper-water and alumina-water nanofluids. Through an appropriate similarity transformation, the governing partial differential equations are reduced into ordinary differential equations, which are then solved numerically using MATLAB. The effects of the various non-dimensional parameters namely, Magnetic parameter, volume fraction, angle of inclination, Suction parameter, heat generation parameter, mixed convection parameter and Prandtl number over the flow field and temperature distribution are discussed with the aid of graphs. The numerical values of skin friction coefficient and Nusselt number are also analysed with the help of tables.

### FORMULATION OF THE PROBLEM

Consider the steady two-dimensional, incompressible, laminar, hydromagnetic mixed convective nanofluid flow over an inclined stretching plate with the effects of suction and internal heat generation. The plate is inclined at an angle of inclination  $\alpha$  with the vertical direction. The gravitational acceleration  $\mathbf{g}$  is acting downward. The physical coordinates  $(x, y)$  are chosen such that  $x$  - axis is chosen along the plate and the  $y$  - axis is measured normal to the surface of the plate. The stretching velocity of the plate is linear, which is taken as  $u = U_w(x) = ax$  where  $a$  is a dimensional constant. The temperature of the stretching surface is  $T_w(x) = T_\infty + bx$  and the ambient temperature is  $T_\infty$ . The fluid is water-based nanofluid containing two different types of nanoparticles namely Cu (copper) and Alumina ( $\text{Al}_2\text{O}_3$ ). It is assumed that the constant suction velocity  $v_w$  normal to the stretching plate. A uniform magnetic field of strength  $\mathbf{B}_0$  is applied normal to the plate. The magnetic Reynolds number is assumed to be small and therefore the induced magnetic field is assumed to be negligible in comparison to that of the applied magnetic field. Further, since the flow is steady,  $\text{curl } \mathbf{E} = 0$ . Also  $\text{div } \mathbf{E} = 0$  in the absence of surface charge density. Hence  $\mathbf{E} = 0$  is assumed. Moreover, it is assumed that the viscous and joule's dissipation are considered to be negligible. The thermophysical properties of the nanofluids are given in Table 1.

Table - 1 Thermo Physical Properties of Base Fluid Water, Copper and Alumina at 25°C (Oztop and Abu Nada [16])

	$\rho$ (Kg/m <sup>3</sup> )	$C_p$ (J/Kg.K)	$K$ (W/m.K)	$\beta \times 10^5 \text{ K}^{-1}$
Water	997.1	4179	0.613	21
Copper	8933	385	400	1.67
Alumina	3970	765	40	0.85

Under the above assumptions, the equations of MHD boundary layer flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{nf} \frac{\partial^2 u}{\partial y^2} + \frac{g(\rho\beta)_{nf}(T-T_\infty)\cos\alpha}{\rho_{nf}} - \frac{\sigma B_0^2 u}{\rho_{nf}} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0(T-T_\infty)}{(\rho C_p)_{nf}} \quad (3)$$

The boundary conditions for the velocity and temperature of this problem are given by

$$\text{At } y = 0, \quad u = U_w(x), \quad v = -v_w, \quad T = T_w(x) = T_\infty + bx \quad (4)$$

$$\text{As } y \rightarrow \infty, \quad u \rightarrow 0, \quad T \rightarrow T_\infty$$

Here  $u$  and  $v$  are velocity components in  $x$  and  $y$  directions respectively,  $U_w(x) = ax$  is the stretching velocity,  $v_w$  is the suction ( $v_w > 0$ ),  $B_0$  is the strength of the magnetic field and  $Q_0$  is the internal heat generation ( $Q_0 > 0$ ).

For the present study, water has been considered as the base fluid with  $Pr = 6.2$  at  $25^\circ\text{C}$ . The nanofluid considered is water mixed with solid spherical copper and aluminium nanoparticles. The effective density, heat capacity, dynamic viscosity, thermal expansion coefficient, thermal diffusivity and the thermal conductivity of the nanofluids are given by

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \quad (\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$$

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$$

### METHOD OF SOLUTION

In order to seek the solution of the problem, the following dimensionless variables are introduced:

$$\psi(x, y) = x\sqrt{av_f} F(\eta), \quad \eta = y\sqrt{\frac{a}{v_f}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where  $\psi(x, y)$  is the stream function such that it satisfies Eq.(1) with  $u = \frac{\partial\psi}{\partial y}$ ,  $v = -\frac{\partial\psi}{\partial x}$  and  $\theta$  is the dimensionless temperature. It is obtained that

$$u = U_w(x) F'(\eta), \quad v = -\sqrt{av_f} F(\eta) \tag{6}$$

The momentum and energy equations together with the boundary conditions can be written as

$$F''' + (1 - \phi)^{2.5} \left\{ (FF'' - F'^2) \left[ (1 - \phi) + \phi \left( \frac{\rho_s}{\rho_f} \right) \right] - M^2 F' + \lambda \left[ (1 - \phi) + \phi \left( \frac{(\rho\beta)_s}{(\rho\beta)_f} \right) \right] \theta \cos\alpha \right\} = 0 \tag{7}$$

$$\frac{1}{Pr} \frac{k_{nf}}{k_f} \theta'' + \left[ (1 - \phi) + \phi \left( \frac{(\rho C_p)_s}{(\rho C_p)_f} \right) \right] (F\theta' - F'\theta) + H_s \theta = 0 \tag{8}$$

with the boundary conditions as follows:

$$\text{At } \eta = 0, \quad F(\eta) = S, \quad F'(\eta) = 1, \quad \theta(\eta) = 1 \tag{9}$$

$$\text{As } \eta \rightarrow \infty, \quad F'(\eta) = 0, \quad \theta(\eta) = 0$$

Here the primes denote differentiations with respect to  $\eta$ . The corresponding dimensionless group that appears in the governing equations are defined by:

$$(Re_x)_f = \frac{U_w(x)x}{v_f}, \quad (Gr_x)_f = \frac{g\beta(T_w - T_\infty)x^3}{v_f^2}, \quad \lambda = \frac{(Gr_x)_f}{Re_x^2} = \frac{gb\beta_f}{a^2}$$

$$M^2 = \frac{\sigma B_0^2}{a\rho_f}, \quad S = \frac{v_w}{\sqrt{av_f}}, \quad H_s = \frac{Q_0}{a(\rho C_p)_f}, \quad Pr = \frac{v_f}{\alpha_f}$$

Where  $(Re_x)_f$  is the local Reynolds number,  $(Gr_x)_f$  is the local Grashof number,  $\lambda$  is the mixed convection parameter,  $M^2$  is the magnetic interaction parameter,  $S$  is the Suction parameter,  $H_s$  is the Heat generation parameter and  $Pr$  is the Prandtl number.

### NUMERICAL SOLUTION

The set of nonlinear coupled differential equations (7) and (8) along with the boundary conditions (9) constitute a boundary value problem. This boundary value problem cannot be solved analytically. The system of coupled nonlinear differential equations with the boundary conditions are solved numerically in the symbolic computation software MATLAB for various values of the governing parameters such as Magnetic interaction parameter, angle of inclination, volume fraction, suction parameter and heat generation parameter with fixed values of Prandtl number and mixed convection parameter. The asymptotic boundary conditions given by equation (9) were replaced by using a value of 15 for the similarity variable  $\eta_{max}$  as follows

$$\eta_{max} = 15, \quad F'(15) = 0, \quad \theta(15) = 0$$

The choice of  $\eta_{max} = 15$  ensured that all numerical solutions approached the asymptotic values correctly. The absolute error tolerance for this method is  $10^{-6}$ . The numerical values for skin friction coefficient and the Nusselt number are also obtained and are tabulated for different values of  $M^2$ ,  $\phi$ ,  $\alpha$ ,  $S$  and  $H_s$ .

Concerning this study, the physical quantities of practical interest are the local Skin friction coefficient  $C_f$  and the local Nusselt number  $Nu_x$  and are defined as

$$C_f = \frac{\tau_w}{\rho_f(ax)^2} \quad \text{where } \tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right)_{y=0} \tag{10}$$

$$Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)} \quad \text{where } q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \tag{11}$$

Using Eq.(5) into Eqs.(10) and (11), we get

$$C_f(Re_x)_f^{1/2} = \frac{1}{(1-\phi)^{2.5}} F''(0), \quad Nu_x(Re_x)_f^{-1/2} = -\frac{k_{nf}}{k_f} \theta'(0) \tag{12}$$

RESULTS AND DISCUSSION

The mixed convection problem associated with steady, non-linear, two-dimensional laminar flow of nanofluids over an inclined stretching plate in the presence of magnetic field, suction and internal heat generation is thoroughly studied and numerical results are obtained. Numerical solutions of the problem are obtained for various values of physical parameters involved in the study such as  $M^2$ ,  $\alpha$ ,  $\phi$ ,  $S$ ,  $H_s$ ,  $Pr$  and  $\lambda$ . The Prandtl number is kept constant at  $Pr = 6.2$  and the mixed convection parameter is fixed at  $\lambda = 1.5$  for different values of physical parameters such as  $M^2 = 0, 1, 2, 4$ ,  $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ ,  $\phi = 0.01, 0.03, 0.05, 0.1$ ,  $S = 0.1, 0.3, 0.5, 0.7$  and  $H_s = 0.0, 0.5, 0.7, 1.0$ . Numerical computations of results are demonstrated through graphs over the flow field and temperature. Further, Skin friction coefficient and the non-dimensional rate of heat transfer are found out and are presented by means of tables.

In order to check the accuracy of the computational method, the numerical values of the skin friction coefficient and the reduced nusselt number for a regular fluid and in the absence of volume fraction, suction parameter and Heat generation parameter with those reported by Ishak et al. [4] and they are presented in Table 2. It is observed from table 2, that the present results are found to be in excellent agreement between the results exists which justifies our numerical scheme in the case of  $m = 1$  and  $n = 1$  to that of Ishak et al. [4]. The effect of magnetic interaction parameter over the dimensionless velocity for both the copper-water and alumina-water nanofluids is shown in Fig.1. The presence of transverse magnetic field sets in Lorentz force effect, which results in the retarding effect on the velocity field. Increasing values of magnetic field, the retarding force increases and consequently the velocity gets decelerated. Thus, the presence of the magnetic field reduces the momentum boundary layer thickness due to the Lorentz force effect for both the types of nanofluids.

Fig.2 reveals the influence of Magnetic field on temperature distribution for both copper-water and alumina-water nanofluids. It is observed that for increasing values of  $M^2$ , the temperature increases which physically conveys the fact that the effect of Magnetic field is to enhance the temperature. Also it is interesting to see that the thermal boundary layer thickness increases due to increase in Magnetic field for both the types of nanofluids. For both the copper-water and alumina-water nanofluids, the dimensionless velocity for different values of  $\alpha$  is depicted in Fig. 3. It is observed that the increase in  $\alpha$ , the velocity of the fluid gets decelerated. It is also elucidated that the thickness of the momentum boundary layer decreases for increase in  $\alpha$ . Fig. 4 displays the effect of inclination angle  $\alpha$  on temperature distribution for specified parameters for both the copper-water and alumina-water nanofluids. When  $\alpha$  increases, temperature also increases. However, the change is not significant.

Table – 2 Comparison of skin friction coefficient and nusselt number for various values of M when  $S = 0$ ,  $H_s = 0, \alpha = 0^\circ, \phi = 0, Pr = 1$  and  $\lambda = 1$

M	Ishak et al [4]		Present Results	
	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$
0.0	-0.5607	1.0873	-0.5608	1.0873
0.1	-0.5658	1.0863	-0.5659	1.0863
0.2	-0.5810	1.0833	-0.5810	1.0833
0.5	-0.6830	1.0630	-0.6830	1.0630
1	-1.0000	1.0000	-1.0000	1.0000
2	-1.8968	0.8311	-1.8968	0.8311
5	-4.9155	0.4702	-4.9156	0.4703

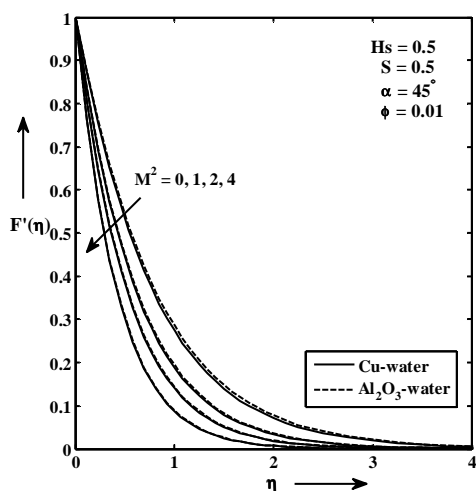


Fig. 1 Dimensionless velocity profiles for different  $M^2$

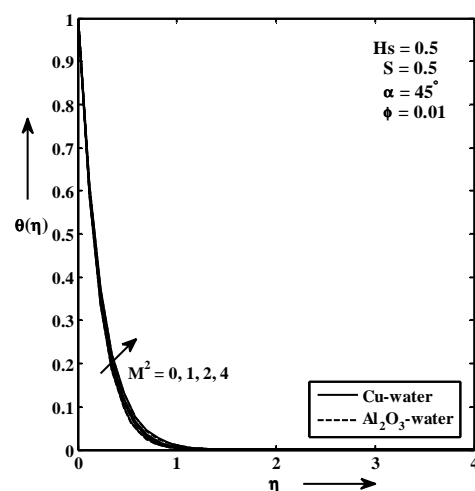


Fig.2 Temperature distribution for different  $M^2$

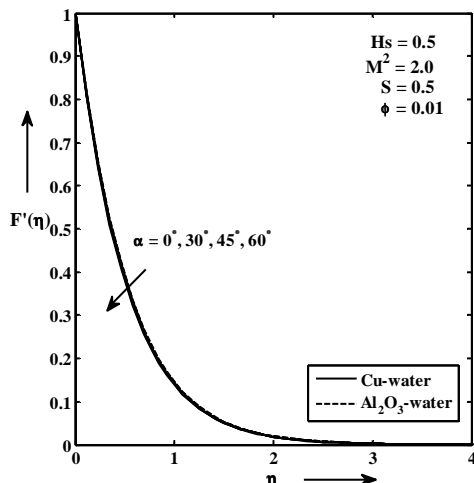


Fig.3 Dimensionless velocity profiles for different  $\alpha$

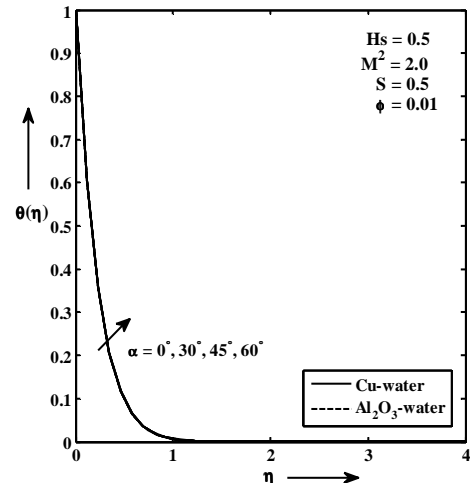


Fig.4 Temperature distribution for different values of  $\alpha$

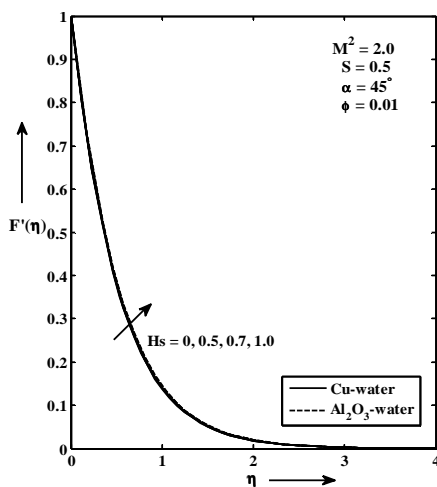


Fig.5 Dimensionless velocity profiles for different values of  $H_s$

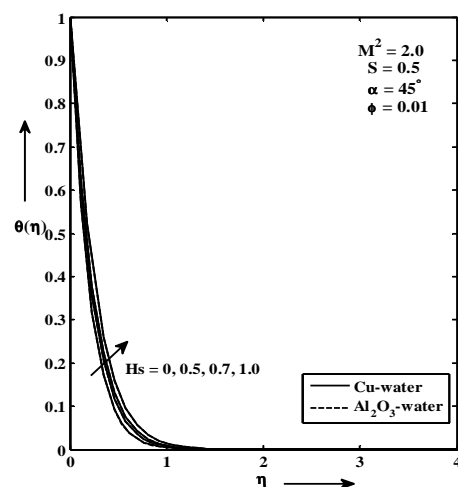


Fig.6 Temperature distribution for different values of  $H_s$

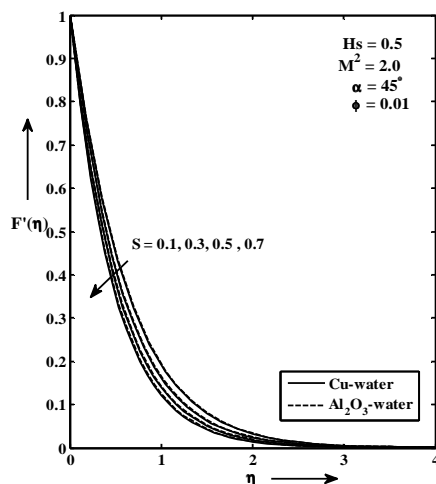


Fig.7 Dimensionless velocity profiles for different values of  $S$

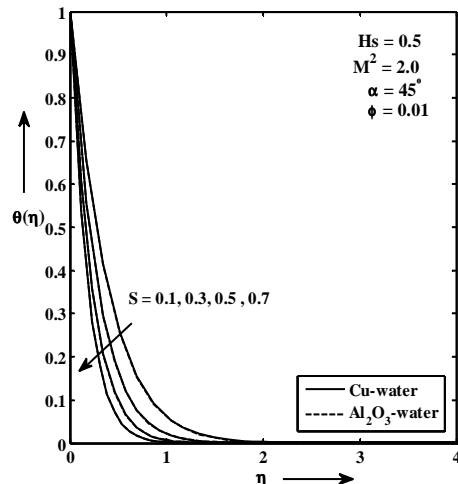


Fig.8 Temperature distribution for different  $S$

The influence of heat generation parameter on dimensionless velocity for both copper-water and alumina-water nanofluids is portrayed through Fig.5. When the heat is generated the buoyancy force increases which in turn cause the flow rate to increase giving rise to the increase in the velocity profile for both the copper –water and alumina-water nanofluids. As a consequence, the hydrodynamic boundary layer thickness of the nanofluid increases with increasing heat generation parameter. Fig.6 shows the heat generation parameter on the temperature distribution for both the copper-water and alumina-water nanofluids. Owing to the presence of heat generation ( $H_s > 0$ ), it is apparent that there is an increase in the thermal state of the fluid. Thus, the thermal boundary layer thickness increases due to increase in heat generation parameter for both the types of nanofluids.

The effect of Suction over dimensionless velocity is disclosed through Fig.7 for both the copper-water and alumina-water nanofluids. The velocity is found to decelerate which accompanies a rise in  $S$ . This phenomenon is expected because the suction pulls the fluid toward the wall, and the buoyancy force acts as the pulling force. It is worth mentioning that the momentum boundary layer thickness reduces for both the copper-water and alumina-water nanofluids. Fig.8 displays the temperature distribution for both the copper-water and alumina-water nanofluids for different values of Suction. While the increase of suction parameter accelerates the transverse fluid motion it has tendency to decrease the temperature. Moreover, the thermal boundary layer thickness and the surface temperature are also decreasing for both the types of nanofluids.

Fig.9 predicts the various values of volume fraction over the dimensionless velocity for both the copper-water and alumina-water nanofluids. The fluid velocity is found to decelerate with increasing number of copper nanoparticles. In controversy, the nanofluid velocity accelerated by increasing the volume fraction of aluminium nanoparticles. It is interesting to note that the nanofluid momentum boundary layer thickness decreases slightly by adding the number of copper nanoparticles while the reverse is true for the momentum boundary layer in the case of aluminium nanoparticles. The temperature distribution for different values of  $\phi$  for both the copper-water and alumina-water nanofluids is shown in Fig.10. Increasing values of volume fraction lead to both the enhancement of temperature and the thermal boundary layer thickness for both the copper-water and alumina-water nanofluids.

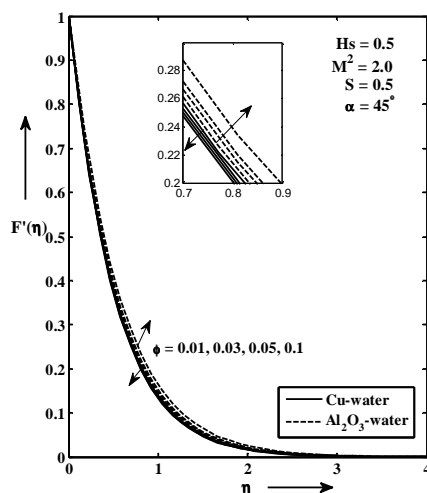


Fig.9 Dimensionless velocity profiles for different  $\phi$

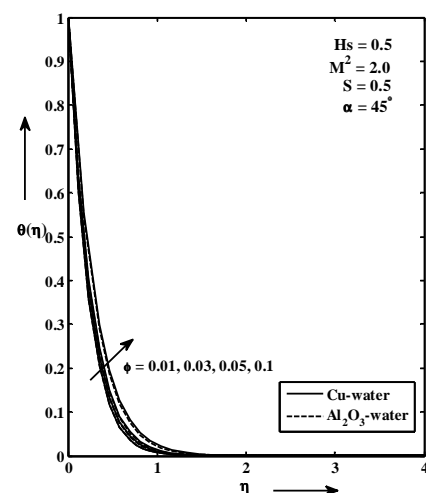


Fig.10 Temperature distribution for different  $\phi$

Table – 3

Variation in  $\frac{1}{(1-\phi)^{2.5}} F''(0)$  for copper – water and alumina-water nanofluids for different values of  $M^2, \alpha, \phi, S$  and  $H_s$  when  $\lambda = 1.5$  and  $Pr = 6.2$

$M^2$	$\alpha$	$\phi$	$S$	$H_s$	$\frac{1}{(1-\phi)^{2.5}} F''(0)$	
					Cu-water	$Al_2O_3$ - water
0	45°	0.01	0.5	0.5	-1.177528	-1.139268
1					-1.578668	-1.547770
2					-1.894297	-1.866861
4					-2.401313	-2.377426
2	0°	0.01	0.5	0.5	-1.823434	-1.796437
	30°				-1.855797	-1.828600
	45°				-1.894297	-1.866861
	60°				-1.944683	-1.916930
2	45°	0.01	0.5	0.5	-1.894297	-1.866861
		0.03			-2.024549	-1.941820
		0.05			-2.158342	-2.019739
		0.1			-2.509973	-2.228854
2	45°	0.01	0.1	0.5	-1.586887	-1.571263
			0.3		-1.742623	-1.721353
			0.5		-1.894297	-1.866861
			0.7		-2.045111	-2.011008
2	45°	0.01	0.5	0.5	-1.907772	-1.880181
			0.7		-1.894297	-1.866861
			0.9		-1.887277	-1.859938
			1.0		-1.873690	-1.846592

Table – 4

Variation in  $-\frac{k_{nf}}{k_f} \theta'(0)$  for copper – water and alumina-water nanofluids for different values of  $M^2, \alpha, \phi, S$  and  $H_s$  when  $\lambda = 1.5$  and  $Pr = 6.2$

$M^2$	$\alpha$	$\phi$	$S$	$H_s$	$-\frac{k_{nf}}{k_f} \theta'(0)$	
					Cu-water	$Al_2O_3$ - water
0	45°	0.01	0.5	0.5	4.513825	4.517526
1					4.423501	4.425643
2					4.352213	4.353628
4					4.237745	4.238408
2	0°	0.01	0.5	0.5	4.363394	4.364693
	30°				4.358307	4.359660
	45°				4.352213	4.353628
	60°				4.344163	4.345662
2	45°	0.01	0.5	0.5	4.352213	4.353628
		0.03			4.365763	4.370552
		0.05			4.378498	4.387355
		0.1			4.407535	4.429375
2	45°	0.01	0.1	0.5	2.420316	2.424280
			0.3		3.346121	3.348833
			0.5		4.352213	4.353628
			0.7		5.413996	5.414060
2	45°	0.01	0.5	0.5	4.884654	4.884292
			0.7		4.352213	4.353628
			0.9		4.106558	4.109114
			1.0		3.681647	3.687033

Table 3 demonstrates the result of the skin friction coefficient for both the types of nanofluids for different values of  $M^2$ ,  $\alpha$ ,  $\phi$ ,  $S$  and  $H_s$  when  $Pr = 6.2$  and  $\lambda = 1.5$ . The individual effect of Magnetic interaction parameter, angle of inclination, volume fraction and Suction is to decrease the skin friction coefficient for their increasing values whereas it enhances for increasing values of heat generation parameter for both the copper-water and alumina-water nanofluids. The reduced nusselt number for various values of parameters used in our analysis is presented in Table 4. Analysis of the tabular data shows that the magnetic field, angle of inclination and heat generation parameter have the same trend so as to decrease the reduced nusselt number. However, the effect of volume fraction and the effect of Suction are similar to enhance over the reduced nusselt number.

### CONCLUSION

The problem of laminar hydromagnetic mixed convective boundary layer flow of an incompressible viscous nanofluid resulting from an inclined stretching surface in the presence of suction and internal heat generation has been investigated numerically. Numerical results for velocity and temperature distribution are obtained for various values of physical parameters. The variations in non-dimensional skin friction coefficient and reduced Nusselt number for various values of governing parameters also presented by means of tables.

The following conclusions are arrived from all the results and discussion of numerical computations:

- In the absence of volume fraction, suction and internal heat generation, the author's results are found to be in excellent agreement with that of Ishak et al. [4] in the case of  $m = 1$  and  $n = 1$ .
- The influence of Magnetic field is to reduce the dimensionless velocity, skin friction coefficient and the reduced nusselt number while its effect is to enhance the temperature for both the types of nanofluids. However, the similar effect is observed with the increase in the angle of inclination.
- Increase in volume fraction of copper-water nanofluid lead to retardation in the velocity and the opposite effect is noticed for alumina-water nanofluid. The enhancement of temperature, skin friction coefficient in magnitude, reduced Nusselt number and as well as the thickness of the thermal boundary layer with increase in volume fraction for both the types of nanofluids.
- The effect of Suction is to decrease the non-dimensional velocity, temperature and the skin friction coefficient whereas its effect is to increase the rate of heat transfer for both the copper-water and alumina-water nanofluids.
- Increasing values of heat generation parameter is to enhancement the velocity, temperature and skin friction coefficient whereas its effect over the reduced nusselt number is to suppress it.

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