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# Rayleigh waves in elastic medium with double porosity

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**ABSTRACT**

The present paper deals with the propagation of Rayleigh waves in isotropic homogeneous elastic half-space with double porosity whose surface is subjected to stress-free boundary conditions. The compact secular equations for elastic solid half-space with voids are deduced as special cases from the present analysis. In order to illustrate the analytical developments, the secular equations have been solved numerically. The computer simulated results for copper materials in respect of Rayleigh wave velocity and attenuation coefficient have been presented graphically.



## 1. Introduction

Rayleigh waves are always generated when a free surface exists in a continuous body. Rayleigh firstly introduced them as solution of the free vibration problem for an elastic half-space (on waves propagated along the plane surface of an elastic solid). Rayleigh wave play an important role in the study of earthquakes, seismology, geo-physics and geodynamics. During earthquake, Rayleigh waves play more drastic role than other seismic waves because these waves are responsible for destruction of buildings, plants and loss of human lives etc.

Porous media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Representation of a fluid saturated porous medium as a single phase material has been virtually discarded. The material with the pore spaces such as concrete can be treated easily because all concrete ingredients have the same motion if the

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concrete body is deformed. However, the situation is more complicated if the pores are filled with liquid and in that case the solid and liquid phases have different motions. Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behavior of a fluid saturated porous thermoelastic medium becomes very difficult. So researchers from time to time, have tried to overcome this difficulty and we see many porous media in the literature. A brief historical background of these theories is given by de Boer [1,2].

Biot [3] proposed a general theory of three-dimensional deformation of fluid saturated porous salts. Biot theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. Another interesting theory is given by Bowen [4], de Boer and Ehlers [5] in which all the constituents of a porous medium are assumed to be incompressible. The fluid saturated porous material is modeled as a two phase system composed of an incompressible solid phase and incompressible fluid phase, thus meeting the many problems in engineering practice, e.g. in soil mechanics. One important generalization of Biot's theory of poroelasticity that has been studied extensively started with the works by Barenblatt et al. [6], where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifiers.

The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well-known that, under super- saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system.

Wilson and Aifanits [7] presented the theory of consolidation with the double porosity. Khaled et al. [8] employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Aifantis [7]. Wilson and Aifantis [9] discussed the propagation of acoustics waves in a fluid saturated porous medium. The propagation of acoustic waves in a fluid-saturated porous medium containing a continuously distributed system of fractures is discussed. The porous medium is assumed to consist of two degrees of porosity and the resulting model thus yields three types of longitudinal waves, one associated with the elastic properties of the matrix material and one each for the fluids in the pore space and the fracture space.

Beskos and Aifantis [10] presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Khalili and Valliappan [11] studied the unified theory of flow and deformation in double porous media. Aifantis [12-15] introduced a multi-porous system and studied the mechanics of diffusion in solids. Moutsopoulos et al. [16] obtained the numerical simulation of transport phenomena by using the double porosity/diffusivity continuum model. Khalili and Selvadurai [17] presented a fully coupled constitutive model for thermo-hydro-mechanical analysis in elastic media with double porosity structure. Pride and Berryman [18] studied the linear dynamics of double-porosity dual-permeability materials. Straughan [19] studied the stability and uniqueness in double porous elastic media. Svanadze [20-24] investigated some problems on elastic solids, viscoelastic solids and thermoelastic solids with double porosity.

L. Rayleigh [25] investigated the propagation of waves along the plane surface of an elastic solid. Lockett [26] studied the effect of thermal properties on Rayleigh waves velocity. Propagation of Rayleigh waves along with isothermal and insulated boundaries discussed by Chadwick and Windle [27]. Kumar and Kansal [28, 29] presented the problem of Rayleigh waves in an isotropic generalized thermoelastic with diffusive half-space medium. Sharma and Kaur [29] presented the problem of Rayleigh waves in rotating thermoelastic with voids. Kumar et.al. [30] discussed the problem of Rayleigh waves in isotropic microstretch thermoelastic diffusion solid half-space. Kumar and Gupta [30] discussed the problem of Rayleigh waves in generalized thermmoelastic medium with mass diffusion. Abd-Alla et al. [33-40] investigated the propagation of Rayleigh waves in different theories.

In the present paper, we investigate the propagation of Rayleigh waves in homogeneous isotropic elastic material with double porosity structure. Secular equations are derived mathematically for the boundary conditions. The values of phase velocity and attenuation coefficient with respect to wave number are computed numerically and depicted graphically.

## 2. Basic equations

Following Iesan and Quintanilla [41], the constitutive relations and field equations for homogeneous elastic material with double porosity structure without body forces and extrinsic equilibrated body forces can be written as :

**Constitutive relations**

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\varphi + d\delta_{ij}\psi \tag{1}$$

$$\sigma_i = \alpha\varphi_{,i} + b_1\psi_{,i} \tag{2}$$

$$\tau_i = b_1\varphi_{,i} + \gamma\psi_{,i} \tag{3}$$

**Equation of motion**

$$\mu\nabla^2 u_i + (\lambda+\mu)\, u_{j,ji} + b\varphi_{,i} + d\psi_{,i} = \rho\ddot{u}_i \tag{4}$$

**Equilibrated stress equations of motion**

$$\alpha\nabla^2\varphi + b_1\nabla^2\psi - bu_{r,r} - \alpha_1\varphi - \alpha_3\psi = \kappa_1\ddot{\varphi} \tag{5}$$

$$b_1\nabla^2\varphi + \gamma\nabla^2\psi - du_{r,r} - \alpha_3\varphi - \alpha_2\psi = \kappa_2\ddot{\psi} \tag{6}$$

where $\lambda$, and $\mu$ are Lame's constants, $\rho$ is the mass density; $u_i$ is the displacement components; $t_{ij}$ is the stress tensor; $\kappa_1$ and $\kappa_2$ are coefficients of equilibrated inertia; $\nu_1$ is the volume fraction field corresponding to pores and $\nu_2$ is the volume fraction field corresponding to fissures; $\varphi$ and $\psi$ are the volume fraction fields corresponding to $\nu_1$ and $\nu_2$ respectively; $\sigma_1$ is the equilibrated stress corresponding to $\nu_1$; $\tau_1$ is the equilibrated stress corresponding to $\nu_2$; and $b$, $d$, $b_1$, $\gamma$, $\gamma_1$, $\gamma_2$ are constitutive coefficients; $\delta_{ij}$ is the Kronecker's delta; a superposed dot represents differentiation with respect to time variable $t$.

$\nabla = \hat{\imath}\frac{\partial}{\partial x_1} + \hat{\jmath}\frac{\partial}{\partial x_2} + \hat{k}\frac{\partial}{\partial x_3}$, $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$
are the gradient and Laplacian operators, respectively.

## 3. Formulation of the problem

We consider homogeneous isotropic elastic with double porous half space.We take the origin of the coordinate system $(x_1, x_2, x_3)$ at any point plane on the horizontal surface and $x_1-axis$ in the direction of the wave propagation and $x_3-axis$ pointing vertically downward to the half-space so that all particles on line parallel to $x_2-axis$ are equally displaced. Therefore, all the field quantities will be independent of $x_2-coordinate.$

For the two-dimensional problem, we take

$$\begin{aligned} x_1^{'} &= \frac{\omega_1}{c_1}x_1, \quad x_3^{'} = \frac{\omega_1}{c_1}x_3,\ u_1^{'} = \frac{\omega_1}{c_1}u_1\ , u_3^{'} = \frac{\omega_1}{c_1}u_3, t_{ij}^{'} = \frac{t_{ij}}{\lambda} \\ \varphi^{'} &= \frac{\kappa_1{\omega_1}^2}{\alpha_1}\varphi,\ \ \psi' = \frac{\kappa_1{\omega_1}^2}{\alpha_1},\ t' = \omega_1 t, \sigma_1^{'} = \left(\frac{c_1}{\alpha\omega_1}\right)\sigma_1,\ \tau_1^{'} = \left(\frac{c_1}{\alpha\omega_1}\right)\tau_1 \end{aligned} \tag{7}$$

where $c_1^2 = \frac{\lambda+2\mu}{\rho}, \omega_1 = \frac{\lambda}{\kappa_1}$

Here $\omega_1$ and $c_1$ are the constants having the dimension of frequency and velocity in the medium respectively.

Using (7) in Eqs. (4)-(6) and with the aid of (8), after suppressing the primes, we obtain

$$\left(\frac{\lambda+\mu}{\rho c_1^2}\right)\frac{\partial e}{\partial x_1} + \frac{\mu}{\rho c_1^2}\nabla^2 u_1 + a_1\frac{\partial\phi}{\partial x_1} + a_2\frac{\partial\psi}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2} \tag{8}$$

$$\left(\frac{\lambda+\mu}{\rho c_1^2}\right)\frac{\partial e}{\partial x_3} + \frac{\mu}{\rho c_1^2}\nabla^2 u_3 + a_1\frac{\partial\phi}{\partial x_3} + a_2\frac{\partial\psi}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2} \tag{9}$$

$$a_3\nabla^2\phi + a_4\nabla^2\psi - a_5 e - a_6\phi - a_7\psi = \frac{\partial^2\varphi}{\partial t^2} \tag{10}$$

$$a_8\nabla^2\phi + a_9\nabla^2\psi - a_{10} e - a_{11}\phi - a_{12}\psi = \frac{\partial^2\psi}{\partial t^2} \tag{11}$$

where

$a_1 = \frac{b\alpha_1}{\rho c_1^2\kappa_1^2\omega_1^2}, a_2 = \frac{d\alpha_1}{\rho c_1^2\kappa_1^2\omega_1^2}, a_3 = \frac{\alpha}{\kappa_1 c_1^2}, a_4 = \frac{b_1}{\kappa_1 c_1^2}, a_5 = \frac{b}{\alpha_1}, a_6 = \frac{\alpha_1}{\kappa_1\omega_1^2},$
$a_7 = \frac{\alpha_3}{\kappa_1\omega_1^2}, a_8 = \frac{b_1}{\kappa_2 c_1^2}, a_9 = \frac{\gamma}{\kappa_2 c_1^2}, a_{10} = \frac{d\kappa_1}{\kappa_2\alpha_1}, a_{11} = \frac{\alpha_3}{\kappa_2\omega_1^2}, a_{12} = \frac{\alpha_2}{\kappa_2\omega_1^2}$

Here $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}, \quad e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}$

The displacement components $u_1$ and $u_3$ are related by potential functions $\varphi_1$ and $\psi_1$ as

$$u_1 = \frac{\partial \varphi_1}{\partial x_1} - \frac{\partial \psi_1}{\partial x_3},\ u_3 = \frac{\partial \varphi_1}{\partial x_3} + \frac{\partial \psi_1}{\partial x_1} \tag{12}$$

Making use of (13) in equations (9)-(12), we obtain

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\varphi_1 + a_1\varphi + a_2\psi = 0 \tag{13}$$

$$-a_5\nabla^2\varphi_1 + \left(a_3\nabla^2 - a_6 - \frac{\partial^2}{\partial t^2}\right)\varphi + \left(a_4\nabla^2 - a_7\right)\psi = 0 \tag{14}$$

$$-a_{10}\nabla^2\varphi_1 + \left(a_8\nabla^2 - a_{11}\right)\varphi + \left(a_9\nabla^2 - a_{11} - \frac{\partial^2}{\partial t^2}\right)\psi = 0 \tag{15}$$

and

$$\left(a_{12}\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\psi_1 = 0 \tag{16}$$

## 4. Solution of the problem

We assume the solution of the form

$$(\varphi_1, \varphi, \psi, \psi_1) = (\varphi_1^*, \varphi^*, \psi^*, \psi_1^*)e^{i\xi(x_1 - ct)} \tag{17}$$

where $\xi$ is the wave number, $\omega = \xi c$ is the angular frequency and $c$ is the phase velocity of the wave.

Making use of (18) in Eqs. (14)-(16), we obtain three homogeneous equations in three unknowns and these equations have non-trivial solutions if the determinant of the coefficient $\varphi_1^*$, $\varphi^*$ and $\psi^*$ vanishes, which yield to the following characteristics equation :

$$E_1\frac{d^6}{dz^6} + E_2\frac{d^4}{dz^4} + E_3\frac{d^2}{dz^2} + E_4 = 0 \tag{18}$$

where

$E_1 = a_3a_9 - a_4a_8$

$E_2 = a_3g_7 + a_9g_4 - a_4g_5 - a_8a_6 + a_1(a_5a_9 - a_4a_{10}) + a_2(a_3a_{10} - a_5a_8) + g_1(a_3a_9 - a_4a_8)$

$E_3 = g_4g_7 - g_5g_6 + g_1(a_3g_7 + a_9g_4 - a_4g_5 - a_8g_6) - a_1(a_9g_2 - a_5g_7 + a_{10}g_6 - a_4g_3)$
$\quad +a_2(a_8g_2 + a_{10}g_4 - a_5g_5 - a_3g_3)$

$E_4 = a_1(g_6g_3 - g_2g_7) + a_2(g_2g_5 - g_3g_4) + g_1(g_4g_7 - g_5g_6)$

Here

$g_1 = \xi^2(c^2 - 1)$, $g_2 = a_5\xi^2$, $g_3 = a_{10}\xi^2$, $g_4 = \xi^2(c^2 - a_3) - a_6$,
$g_5 = -a_8\xi^2 - a_{11}$, $g_6 = -a_4\xi^2 - a_7$, $g_7 = \xi^2(c^2 - a_4) - a_{12}$

and

$$\left(\frac{d^2}{dx_3^2} - \zeta_4^2\right)\psi_1^* = 0 \tag{19}$$

where
$\zeta_4^2 = \xi^2\left(1 + \frac{c^2}{a_{12}}\right)$
Since we are interested in surface waves only, it is essential that the motion is confined to the free surface $x_3 = 0$ of the half-space. Therefore, to satisfy the radiation conditions, $\varphi_1, \varphi, \psi, \psi_1 \to 0$ as $x_3 \to \infty$ are given by

$$(\varphi_1, \varphi, \psi) = \sum_{i=1}^{3}(1, r_i, s_i)B_i e^{-m_i x_3} e^{i\xi(x_1 - ct)} \tag{20}$$

and

$$\psi_1 = B_4 e^{-m_4 x_3} e^{i\xi(x_1 - ct)} \tag{21}$$

Here, $m_4 = \sqrt{\zeta_4}$
and $B_i$ $(i = 1, 2, 3, 4)$ are arbitrary constants in equations (21)and (22) , the coupling constants $r_i$, $s_i$ are given as

$$\begin{aligned} r_i &= -\frac{(a_4a_{10}-a_5a_9)m_i^4+(a_9g_2-a_5g_7+a_{10}g_6-a_4g_3)m_i^2+(g_2g_7-g_3g_6)}{(a_3a_9-a_4a_8)m_i^4+(a_3g_7+a_9g_4-a_4g_5-a_8g_6)m_i^2+(g_4g_7-g_5g_6)}, \\ s_i &= \frac{(a_3a_{10}-a_5a_8)m_i^4+(a_8g_2+a_{10}g_4-a_5g_5-a_3g_3)m_i^2+(g_2g_5-g_3g_4)}{(a_3a_9-a_4a_8)m_i^4+(a_3g_7+a_9g_4-a_4g_5-a_8g_6)m_i^2+(g_4g_7-g_5g_6)}, \\ & i = 1, 2, 3 \end{aligned} \tag{22}$$

## 5. Boundary conditions

The boundary conditions are the vanishing of the stress components at the free surface $x_3 = 0$. Mathematically these can be written as

$$t_{33} = 0 \tag{23}$$

$$t_{31} = 0 \tag{24}$$

$$\sigma_3 = 0 \tag{25}$$

$$\tau_3 = 0 \tag{26}$$

The expressions for normal stress $t_{33}$, tangential stress $t_{31}$ and equilibrated stress $\sigma_3$, $\tau_3$ in non-dimensional form are

$$t_{33} = p_1\frac{\partial u_3}{\partial x_3} + \frac{\partial u_1}{\partial x_1} + p_2\varphi + p_3\psi \tag{27}$$

$$t_{31} = p_4\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right) \tag{28}$$

$$\sigma_3 = p_5\frac{\partial \varphi}{\partial x_3} + p_6\frac{\partial \psi}{\partial x_3} \tag{29}$$

$$\tau_3 = p_6\frac{\partial \varphi}{\partial x_3} + p_7\frac{\partial \psi}{\partial x_3} \tag{30}$$

where
$p_1 = \frac{\lambda+2\mu}{\lambda}, \quad p_2 = \frac{b\alpha_1}{\lambda\kappa_1\omega_1^2}, \quad p_3 = \frac{d\alpha_1}{\lambda\kappa_1\omega_1^2},$
$p_4 = \frac{\mu}{\lambda}, \quad p_5 = \frac{\alpha_1}{\kappa_1\omega_1^2}, \quad p_6 = \frac{b_1\alpha_1}{\alpha\kappa_1\omega_1^2}, p_7 = \frac{\gamma\alpha_1}{\alpha\kappa_1\omega_1^2}$

## 6. Derivation of the secular Equations

Making use of equations (21) and (22) in the boundary conditions (24)-(27) and with the aid of (28)-(31), we obtain a system of four simultaneous homogeneous linear equations

$$\sum_{j=1}^{4} Q_{ij}B_j = 0 \qquad \text{for } i = 1,2,3,4 \tag{31}$$

where

$$Q_{1j} = \begin{cases} p_1 m_j^2 - \xi^2 + p_3 r_j + p_4 s_j, & \text{for } j = 1,2,3 \\ (1-p_1)i\xi m_j, & \text{for } j = 4 \end{cases} \tag{32}$$

$$Q_{2j} = \begin{cases} -2i\xi m_j p_4\,, & \text{for } j = 1,2,3 \\ (m_j^2 - \xi^2), & \text{for } j = 4 \end{cases} \tag{33}$$

$$Q_{3j} = \begin{cases} -m_j(p_5 r_j + p_6 s_j), & \text{for } j = 1,2,3 \\ 0, & \text{for } j = 4 \end{cases} \tag{34}$$

$$Q_{4j} = \begin{cases} -m_j(p_6 r_j + p_7 s_j), & \text{for } j = 1,2,3 \\ 0, & \text{for } j = 4 \end{cases} \tag{35}$$

The system of Eq. (32) has a non-trivial solution if the determinant of the unknowns $B_j$ $(j = 1,2,3,4)$ vanishes i.e.

$$|Q_{ij}|_{4\times4} = 0 \tag{36}$$

**Particular case**
If $b_1 = \gamma = \alpha_3 = \alpha_2 = d \to 0$, the Eqs. (37) yield the expressions for elastic material with voids.

## 7. Numerical results and discussion

$\lambda = 7.76 \times 10^{10} Nm^{-2}, \mu = 3.86 \times 10^{10} Nm^{-2}, \omega_1 = 1 \times 10^{11} s^{-1}, \alpha = 1.3 \times 10^{-5} N,$
$t = 0.1s, \rho = 8.954 \times 10^{3} Kgm^{-3}, \alpha_1 = 1.65 \times 10^{10} Nm^{-2}, \alpha_2 = 1.96 \times 10^{10} Nm^{-2},$
$\alpha_3 = 1.86 \times 10^{10} Nm^{-2}, \gamma = 0.19 \times 10^{-5} N, b_1 = 0.12 \times 10^{-5} N, d = 0.49 \times 10^{10} Nm^{-2},$
$\kappa_1 = 0.1456 \times 10^{-12} Nm^{-2} s^2, \kappa_2 = 0.1546 \times 10^{-12} Nm^{-2} s^2, b = 0.4 \times 10^{10} Nm^{-2}$

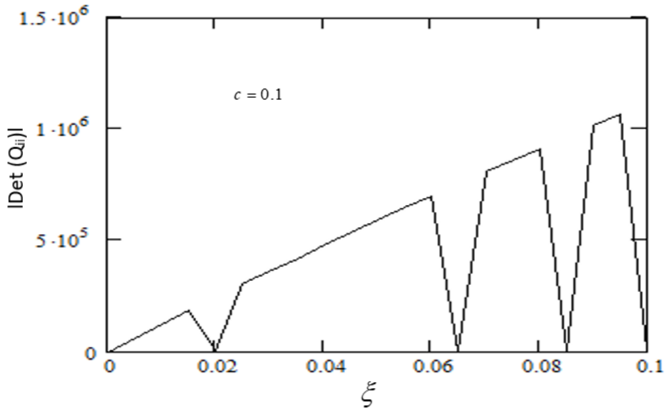


**Fig.** 1 – Determinant of Rayleigh waves secular equation with varies values of $\xi$ when $c = 0.1$.

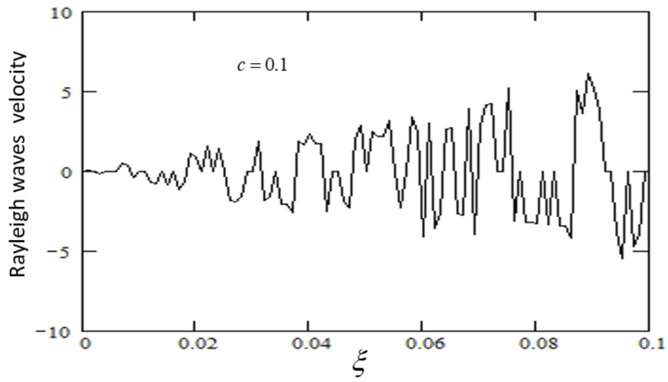


**Fig.** 2 – Rayleigh waves velocity with varies values of $\xi$ when $c = 0.1$.

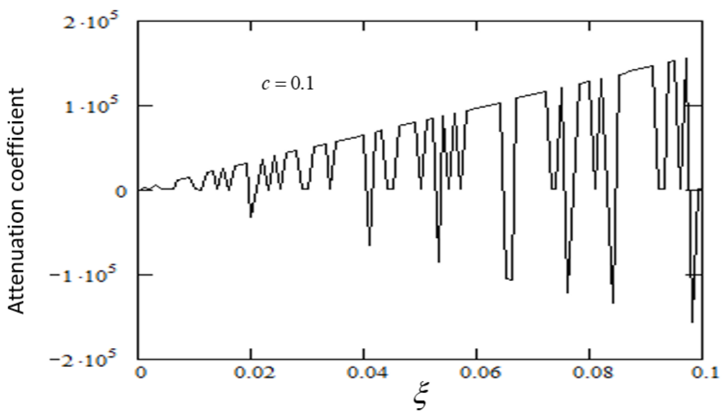


**Fig.** 3 – Attenuation coefficient with varies values of $\xi$ when $c = 0.1$.

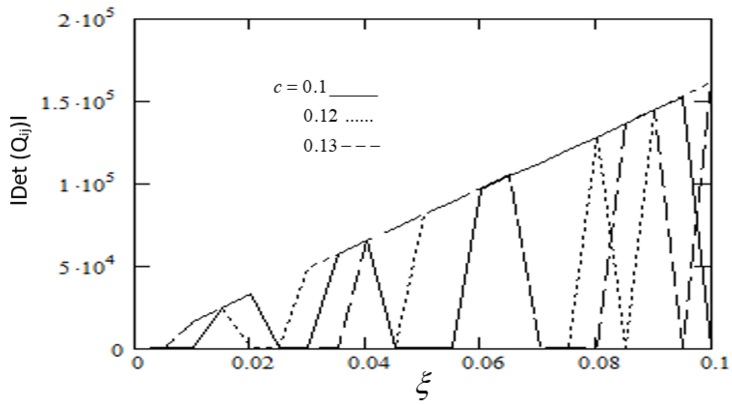


**Fig.** 4 – Determinant of Rayleigh waves secular equation with varies values of $c$ with respect to $\xi$.

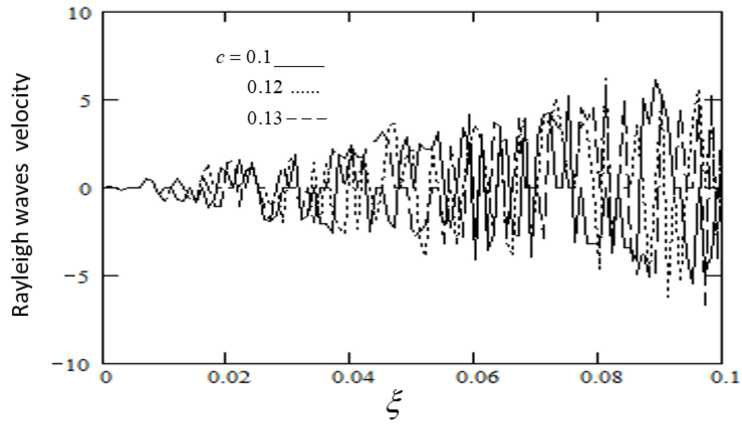


**Fig.** 5 – Rayleigh waves velocity with varies values of $c$ with respect to $\xi$.

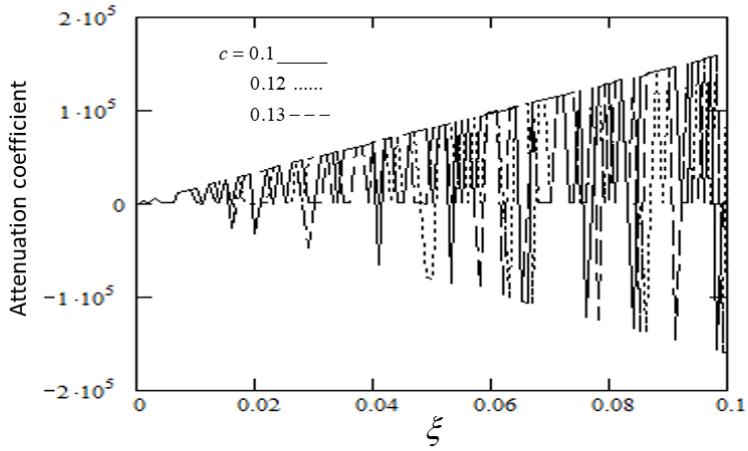


**Fig.** 6 – Attenuation coefficient with varies values of $c$ with respect to $\xi$.

Figs. 1- 3 depict the variation of determinant of Rayleigh wave secular equation, Rayleigh wave velocity and Attenuation coefficient w.r.t $\xi$ for $c = 0.1$.

Fig. 1 shows that variation of determinant of Rayleigh wave secular equation with $\xi$ for $c = 0.1$. It is found that the variation is harmonic in nature. Also, the magnitude of the determinant of Rayleigh wave secular equation increases with the increasing value of $\xi$.

Fig.2 and 3 represent that variation of Rayleigh wave velocity and attenuation coefficient with $\xi$ for $c = 0.1$. An oscillatory trend of variation is noticed for both Rayleigh wave velocity and attenuation coefficient .Also, it is evident that the amplitude of oscillations increases as value of $\xi$ increases.

Figs. 4, 5 & 6 depict the variation of determinant of Rayleigh wave secular equation, Rayleigh wave velocity and Attenuation coefficient w.r.t $\xi$ for different values of $c$. In all these figs. solid line, small dashes lines and big dashes line correspond to the value of $c = 0.1$, 0.12 and 0.13 respectively.

From fig. 4, it is noticed that variation of determinant of Rayleigh wave secular equation is harmonic in nature. It is also clear from the fig. that magnitude of the determinant of Rayleigh wave secular equation increases as value of $\xi$ increases.

Fig.5 shows that Rayleigh wave velocity has oscillatory variation and the amplitude of oscillations increases with the increase in value of $\xi$ for all values of $c$.

It is found from fig. 6 that trend of variation of attenuation coefficient is oscillatory in nature. It is evident that amplitude of oscillations increases with the increase in value of $c$ for all $\xi$.

## 8. Conclusions

A problem of propagation of Rayleigh waves in elastic material with double porosity structure has been investigated. It is observed that porosity has a significant effect on the propagation of Rayleigh waves. It found that values of determinant of Rayleigh wave secular equation, Rayleigh wave velocity and Attenuation coefficient, all have similar trend of variation with respect to the wave number. It is also observed that amplitude of variation in the resulting quantities increase as the value of phase velocity increases.

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