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INVENTORY MODELS SOLVED WITH HYBRID APPROACHES

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ABSTRACT

This paper is a further discussion of models with ramp type or trapezoidal type demand. In a previously published paper in European Journal of Operational Research that their solution procedure is accurate, but tedious that will be substituted by our more compact approach to imply a board setting to solve a more general problem. We expand their inventory model from a ramp type demand to an arbitrary demand to reveal a new discovery that the optimal solution is not related to the demand type. Our paper will provide a new trend in inventory models to study new models and solution procedures.

KEYWORDS: *Ordering Cost, Inventory Model, Deteriorating Item*

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INTRODUCTION

Since Hill (1995) developed the inventory system with a ramp type demand there has been a trend towards examining this kind of an inventory model both in depth and breadth. Some examples include Mandal and Pal (1998) with deterioration items; Wu et al. (1999) assuming that the backlogging rate is proportional to the waiting time; Wu and Ouyang (2000) with two different strategies beginning with stock or shortage; Wu (2001) with the Weibull distributed deterioration; Giri et al. (2003) with a more generalized Weibull distributed deterioration; Deng (2005) to revise the work of Wu et al. (1999); Manna and Chaudhuri (2006) to extend the inventory model a with time dependent deterioration rate; Deng et al. (2007) to modify Mandal, and Pal (1998), Wu and Ouyang (2000).

However, their work is always restricted to ramp type demands. This paper points out that there is a general property that is shared for every demand without being limited to the ramp type or any type of demand. Our findings show that the lengthy discussion and the derivation of Deng et al. (2007) in the consideration of two different expressions of ramp type demand are in fact unnecessary. There are several related papers that deserved to pay attention to them, for examples, Chu et al. (1998), Chung et al. (1998), Lan et al. (1999), Chu and Chen (2001), Chu et al. (2004), Ronald et al. (2004), Yang et al. (2005), Deng et al. (2006), Lin and Chu (2006), Chou et al. (2008), Deng et al. (2008), Liang et al. (2008), Wu et al. (2008), Yang et al. (2011), Lin et al. (2012), Lin et al. (2013b). This paper attempts to investigate an inventory model with stock in the first phase and then shortages in the second phase. In a multiple item inventory model, if holding cost, deterioration cost and shortage cost are the same for all items, owing to the optimal solution is not related to demand then different items with different demand still imply the same replenishment policy. Of particular interest is that when all targeted stock items reach zero simultaneously, and all subsequent demands are intentionally backlogged until the end of the inventory cycle, then one-shot replenishment takes place. In so setting, the total inventory cost, including holding cost, deteriorating cost, and shortage cost will achieve its minimum. As a matter of fact, this kind of design of an inventory system has been prevalent in various industries such as the electronic industry, food industry, as well as the financial industry. For instance, three major passive components: resistance, capacitance, and inductance in the IC industry, the memory chip sets or IO (Input-Output) control device in IC sub-industries all refer to common materials. For years, most common materials have been standardized in the specifications so as to be widely used in manufacturing electronic components,semi-products or finished goods. In other words, under planned production or flexible manufacturing systems, the average consumption rate for each of them can be accurately tracked, mastered, and modulated respectively. In this way, the stock levels for these well-scheduled common materials will reduce to zero at the same time and hereupon the reordering process and replenishment activities are automatically initiated.

ASSUMPTIONS AND NOTATION

We try to generalize the inventory model of Mandal and Pal (1998), Wu and Ouyang (2000) and Deng et al. (2007) with the following assumptions and notations for the deterministic inventory replenishment policy with a general demand.

- The replenishment rate is infinite; thus, replenishments are instantaneous.
- The lead time is zero.
- T is the finite time horizon under consideration. Thus, it is a constant.
- C_h is the inventory holding cost per unit per unit of time.
- C_s is the shortage cost per unit per unit of time.
- C_d is the cost of each deteriorated item.
- θ is the constant fraction of the on-hand inventory deterioration per unit of time.
- $I(t)$ is the on-hand inventory level at time t over the ordering cycle $[0,T]$.
- The Shortage is allowed and fully backordered.
- The demand rate $R(t)$ is assumed to be any positive function for $t>0$.
- t_1 is the time when the inventory level reaches zero.
- $f(t_1)$ is an auxiliary function defined as $\left(C_d+\frac{C_h}{\theta}\right)\left(e^{\theta t_1}-1\right)-C_s(T-t_1)$.
- $C(t_1)$ is the total cost that consists of holding costs, deterioration cost, and shortage cost.

REVIEW OF PREVIOUS RESULTS

In Deng et al. (2007), they considered the ramp type demand where demand $R(t)$ is assumed as follows:

$$R(t)=\begin{cases} D_0 t, & t<\mu \\ D_0\mu, & t\ge\mu \end{cases}, \tag{1}$$

so that μ is the changing point of linear demand to constant demand. Depending on the relation between μ and t_1, they developed two inventory models: when $\mu \ge t_1$, the total cost, $TC_0(t_1)$, is defined for $0 \le t_1 \le \mu$ and for $\mu < t_1$, $TC_1(t_1)$ is defined for $\mu \le t_1 \le T$. They studied properties of $TC_1(t_1)$ in Theorem 1, and the properties of $TC_0(t_1)$ in Theorem 2. After which they found from Theorem 3, for the ramp type demand, that if $\Delta_1 < 0$, the minimum occurs at $TC_1(t_1^{\#})$. On the other hand, if $\Delta_1 \ge 0$, then the minimum occurs at $TC_0(t_1^{\#})$ where $t_1^{\#}$ is the unique solution of $f(t_1)=0$ for $0 \le t_1 \le T$ with

$$f(t_1)=\left(C_d+\frac{C_h}{\theta}\right)\left(e^{\theta t_1}-1\right)-C_s\left(T-t_1\right), \tag{2}$$

and $\Delta_1 = f(\mu)$.

OUR PROPOSED INVENTORY MODEL

We consider an inventory model that starts with a stock. This model was first proposed by Hill (1995), and then further investigated by Mandal and Pal (1998), Wu and Ouyang (2000) and Deng et al. (2007). Replenishment occurs at a time $t=0$ when the inventory level attains its maximum, S. From $t=0$ to t_1, the inventory level reduces due to both demand, $R(t)$, and deterioration. At, t_1, the inventory level achieves zero, after which shortages are allowedduring the time interval (t_1, T), and all of the demand during the shortage period (t_1, T) is completely backlogged. The inventory levels $I(t)$ of the model are described by the following equations:

$$\frac{d}{dt}I(t)+\theta I(t)=-R(t),\ 0<t<t_1, \tag{3}$$

and

$$\frac{d}{dt}I(t)=-R(t),\ t_1<t<T. \tag{4}$$

We directly solve Equations (3) and (4) to imply that

$$I(t)=e^{-\theta t}\int_t^{t_1} R(x)e^{\theta x}dx, \text{ for } 0\le t\le t_1, \tag{5}$$

and

$$I(t)=\int_t^{t_1} R(x)\,dx, \text{ for } t_1\le t\le T. \tag{6}$$

The amount of deteriorated items during $\left[0,t_1\right]$ is evaluated

$$I\left(0\right)-\int_0^{t_1} R\left(x\right)dx=\int_0^{t_1} R\left(x\right)\left(e^{\theta x}-1\right)dx. \tag{7}$$

Using integration by part, the holding cost during $\left[0,t_1\right]$ is evaluated

$$C_h\int_0^{t_1} I\left(t\right)dt=C_h\int_0^{t_1} R\left(x\right)\frac{e^{\theta x}-1}{\theta}dx. \tag{8}$$

The shortage cost during $\left[t_1,T\right]$ is evaluated through integration by part

$$C_s\int_{t_1}^{T} -I\left(t\right)dt=C_s\int_{t_1}^{T}\left(T-x\right)R\left(x\right)dx. \tag{9}$$

Therefore, the total cost is expressed as

$$C\left(t_1\right)=C_d\int_0^{t_1} R\left(x\right)\left(e^{\theta x}-1\right)dx+C_h\int_0^{t_1} R\left(x\right)\frac{e^{\theta x}-1}{\theta}dx+C_s\int_{t_1}^{T}\left(T-x\right)R\left(x\right)dx. \tag{10}$$

From Equation (10), it follows that

$$C'\left(t_1\right)=R\left(t_1\right)\left[\left(C_d+\frac{C_h}{\theta}\right)\left(e^{\theta t_1}-1\right)-C_s\left(T-t_1\right)\right]. \tag{11}$$

Motivated by Equation (11), we assume an auxiliary function, say $f\left(t_1\right)$,

$$f\left(t_1\right)=\left(C_d+\frac{C_h}{\theta}\right)\left(e^{\theta t_1}-1\right)-C_s\left(T-t_1\right). \tag{12}$$

As a matter of fact, it is the same one proposed by Deng et al. (2007). By taking derivative of $f\left(t_1\right)$, i.e., $f'\left(t_1\right)=\left(\theta C_d+C_h\right)e^{\theta t_1}+C_s>0$, it is easy to find that $f\left(t_1\right)$ increases from $f\left(0\right)=-C_sT<0$ to $f\left(T\right)=\left(C_d+\frac{C_h}{\theta}\right)\left(e^{\theta T}-1\right)>0$. Hence, obviously there exists a unique point, say $t_1^{\#}$, that satisfies $f\left(t_1^{\#}\right)=0$ and the following equation holds

$$\left(C_d+\frac{C_h}{\theta}\right)\left(e^{\theta t_1^{\#}}-1\right)=C_s\left(T-t_1^{\#}\right). \tag{13}$$

Since $C'\left(t_1\right)\le 0$ four $0\le t_1\le t_1^{\#}$ and $C'\left(t_1\right)\ge 0$ for $t_1^{\#}\le t_1\le T$ such that $t_1^{\#}$ is the minimum solution for $C\left(t_1\right)$. We summarize our findings in the next theorem.

Theorem 1 For the inventory model beginning with stock, the minimum solution satisfies $f\left(t_1^{\#}\right)=0$ and is not related to the demand.

DIFFERENT VIEWS OF OUR FINDINGS

We intend to provide a marginal (cost) analysis to discuss our findings. If there is an item in demand quantity $R(t)dt$ where dt is a small (infinitesimal) time interval for the demand during $[t,t+dt]$, then there are two replenishment policies: (a) fulfill the demand for the stock, or (b) satisfy the demand from backorder. If we decide to fulfill the demand for the stock, we need to store $R(t)e^{\theta t}dt$ at time $t=0$, since the solution of $\frac{d}{dt}I(t)+\theta\, I(t)=0$ is $I(t)=I(0)e^{-\theta t}$ so that the beginning stock is $I(0)=R(t)e^{\theta t}dt$. It follows that after deteriorated items are removed, the remaining stock $R(t)dt$ is just enough to meet the demand. The amount of deteriorated items is $R(t)\left(e^{\theta t}-1\right)dt$. From the inventory level $R(t)e^{\theta t}e^{-\theta x}dt$ for, $x\in[0,t]$, the holding cost can be calculated by

$$C_h\int_0^t R(t)dte^{\theta t}e^{-\theta x}dx=C_hR(t)dt\frac{e^{\theta t}-1}{\theta}. \tag{14}$$

Hence, the total cost for demand $R(t)dt$ fulfilled from the stock is $\left(\frac{C_h}{\theta}+C_d\right)R(t)dt\left(e^{\theta t}-1\right)$. On the other hand, if the policy of backlog is adopted then the shortage cost is $R(t)dtC_s(T-t)$. We find that if

$$\left(\frac{C_h}{\theta}+C_d\right)R(t)dt\left(e^{\theta t}-1\right)<R(t)dtC_s(T-t), \tag{15}$$

then the better policy is to satisfy demand from the stock. Otherwise, if

$$\left(\frac{C_h}{\theta}+C_d\right)R(t)dt\left(e^{\theta t}-1\right)>R(t)dtC_s(T-t). \tag{16}$$

Then the better policy is to hold demand as backlog until the replenishment takes place. Recall from Equation (12), we have shown that $f(t)=\left(C_d+\frac{C_h}{\theta}\right)\left(e^{\theta t}-1\right)-C_s(T-t)$ has a unique root, say $t_1^{\#}$. It means that for those demands occurring during $\left[0,t_1^{\#}\right)$, they should be fulfilled from the stock And for those demands occurring during $\left(t_1^{\#},T\right]$, they should be replenished by backorder.Our marginal (cost) analysis comes up with the same results as our previous analytical approach in Section 4.

NUMERICAL EXAMPLES

First, we recall the numerical examples in Deng et al. (2005) with $C_d=5$, $C_h=3$, $C_s=15$, $D_0=100$,

$\theta = 0.001$, $\beta = 2$ and $T = 1$. For their example 1, $\mu = 0.12$, and for their example 2, $\mu = 0.9$. After a lengthy computation, Deng et al. (2005) derived that $t_1^* = 0.8331$ for their examples 1. By the similar complicated evaluation, Deng et al. (2005) obtained that $t_1^* = 0.8331$ for their examples 2. We can claim that Deng et al. (2005) did not aware that the optimal solution is not related to the demand type such that their example 2 should have the optimal solution as their example 1.

For the second example, we consider the numerical examples in We et al. (2008) with the following data: $C_d = 0$, $C_h = 3$, $C_s = 5$, $D_0 = 400$, $\theta = 0.05$, $T = 1$, and $\mu = 0.4$, 0.5953 and 0.6 for three different cases. Wu et al. derived that the optimal solution $t_1^* = 0.5953$, 0.5953 and 0.5953 for three different cases. Their optimal solutions are consistent with our theorem that the optimal solution is not related to the demand type such that three different values of μ will result in the same optimal solution.

For the third numerical examples, we refer to Deng et al. (2007) with the following data: $C_d = 5$, $C_h = 3$, $C_s = 15$, $D_0 = 100$, $\theta = 0.001$, $T = 1$, and $\mu = 0.12$, 0.9 and 0.8 for three different cases. Deng et al. (2007) derived that the optimal solution $t_1^* = 0.833044$, 0.833044 and 0.833044 for three different cases. From the above three examples, it indicates that Deng et al. (2007) did not realize that the optimal solution is not related to the demand such that to provide three numerical examples is unnecessary.

For the fourth example, we consider Yang et al. (2011) with $C_1 = 1$, $C_2 = 25$, $C_3 = 3$, $C_4 = 25$, $\alpha = 4$, $\delta = 4$ and $T = 1$. They constructed three examples with different demand type: (a) $R(x) = 100x$, for $0 \le x \le 0.12$ and $R(x) = 12$, for $0.12 \le x \le 1$, (b) $R(x) = 20 - 5x$, for $0 \le x \le 1$, and (3) $R(x) = 20 + 5x$, for $0 \le x \le 1$. For their three different demands, Yang et al. (2011) derived the same optimal solution $t_1^* = 0.678$. It indicates Yang et al. (2011) did not know that the optimal solution is not related to the demand type.

For the fifth example, we recall Skouri et al. (2009). They assumed that $\mu = 0.12$ for their example 1 and $\mu = 0.9$ for their example 2. They derived that $t_1^* = 0.8604 > \mu$ for their example 1 and $t_1^* = 0.8604 < \mu$ for their example 2. From their dissertation of "$t_1^* = 0.8604 > \mu$ for their example 1 and $t_1^* = 0.8604 < \mu$ for their example 2", we can conclude that they did not know that the optimal solution is not related to the demand type.

For our sixth example, we consider Tung (2013). He pointed out that Deng et al. (2007) obtained $t_1^* = 0.833044$ for three different examples with $\mu = 0.12$ for their example 1, $\mu = 0.9$ for their example 2, and $\mu = 0.8$ for their example 3. Tung (2013) mentioned that to provide a reasonable explanation for this coincide phenomenon will be an interesting research topic. This paper provide an positive solution for his question: the optimal solution is not related to the demand such that three different examples with different demand will have the same optimal solution.

DIRECTION FOR FUTURE RESEARCH AND APPLICATION FOR OUR FINDINGS

There are several papers: 0Chung et al. (2000), Chu and Chen (2002), Chu et al. (2003), Chu et al. (2005), Deng et al. (2005), Deng et al. (2007), Yang et al. (2008), Chou et al. (2009), Lin et al. (2011), Lin et al. (2013a), and Lin et al. (2014) that only used analytic methods to locate the optimal solution. Following our managerial point of view without constructing a system of differential equations may be a possible alternative solution approach for papers. It will help ordinary readers realize inventory models from a different aspect.

CONCLUSIONS

In the paper, we discover an interesting phenomenon for the finite time horizon inventory model to show that the optimal solution is not related to the demand. The generalized form of inventory model is developed and the corresponding optimal solution is derived. We believe that our findings provide an essential benchmark for those researchers who have the motive of pursuing different optimal inventory systems along with the changes of demand types. This study gives the solid evidence that the optimal solution is not related to the demand no matter ramp type, trapezoid type, fixed type or any other kinds of it are targeted.

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