JOURNAL OF BALTIC SCIENCE EDUCATION

ISSN 1648-3898 /Print/
ISSN 2538-7138 /Online/

TEACHING ABOUT ROLLING MOTION: EXPLORING THE EFFECTIVENESS OF AN EXTREME CASE REASONING APPROACH

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Abstract. *Earlier research has shown that students have tremendous difficulties with understanding certain aspects of rolling without slipping, such as the zero-velocity at the contact point and plausibility of application of the law of conservation of mechanical energy despite action of the friction force. The aim of this research was to explore whether using analogies and reasoning about extreme cases can facilitate conceptualization of the above-mentioned phenomena. A pre-test – post-test quasi-experiment has been conducted, with 93 students in the control group (CG) and 91 students in the experimental group (EG). Whereas control group students received conventional teaching, in the experimental group rolling of a cylinder has been considered as a special case of a tumbling prism for which the number of prism surfaces tended to infinity. The results of analysis of covariance showed that students from the experimental group significantly outperformed their peers from the control group on the Rolling Motion Concept Test (RMCT). Between-group differences were greater on test items that required higher level of cognitive transfer. This research suggests that using analogies and extreme case reasoning can facilitate comprehension of certain seemingly counterintuitive aspects of rolling motion.*

Keywords: *analogy-based teaching, energy conservation, extreme case reasoning, misconceptions, rolling motion.*

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Introduction

Teaching and learning about rolling motion often requires combining of concepts of translational and rotational motion. Consequently, this relatively complex topic provides a natural context for synthesis of conceptual knowledge from various areas of mechanics. However, earlier research consistently shows that most science and engineering students struggle with developing a satisfactory understanding of rolling motion (De Ambrosis, Malgieri, Mascheretti, & Onorato, 2015; Duman, Demirci, & Sekercioglu, 2015; Lopez, 2003; Mashood & Singh, 2012; Rimoldini & Singh, 2005). Rimoldini and Singh (2005) found that students in introductory and physics junior courses very often do not understand the role of friction force in rolling motion, as well as the distribution of linear velocities across a rolling wheel. In their research none of the 16 interviewed students was able to explain the velocities of the points at top and bottom of the wheel, relative to the ground. It seems that for these students it was hard to understand that, in some instant, a certain point of a moving object can be at rest if we know that the corresponding object as a whole is continually moving (Hasović, Mešić, & Erceg, 2017). According to Lopez (2003) this difficulty could be at least partly associated with the fact that many students do not understand the concept of relative velocity. In addition, by overgeneralizing their experience with translational motion, students often come to the wrong conclusion that acting of frictional forces always is associated with losses of mechanical energy.

Being aware of the many difficulties that students have with developing understanding about rolling motion, Rimoldini and Singh (2005) called for designing a conceptual approach to teaching this topic.

A possible method for identifying an approach that overcomes the limits of human intuition would be to refer to corresponding excerpts from history of physics. Examples from history of physics show that, in order to discover deeper truths about the physical reality, scientists often had to resort to using analogical and extreme case reasoning (Halloun, 2004; Nersessian, 2008). Stephens and Clement (2007) consider that "*an analogy has been proposed*

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

when, in order to facilitate reasoning about a situation A (the target), a situation B (the base) is suggested, which differs in some significant way from A, and an implicit or explicit suggestion is given to apply findings from B to A". According to Clement (1993) often the analogy between the source and target domain is not obvious to many students, which can be handled by introducing intermediate cases that share features with both the source and the target domain. Sometimes these intermediate cases can be obtained by maximizing or minimizing some feature of the target, i.e. by using extreme case reasoning (Clement, 1991; Stephens & Clement, 2007). Generally, extensive earlier research shows that using analogies and extreme cases facilitates developing understanding about the physical world (Clement, 1988; Clement, 1993; Nersessian, 2008). As a matter of fact, such an approach has proven to be equally effective through history of physics and earlier physics education research. Some of the well-known examples are related to Galileo's discovery of the inertia law which required him to think about gradually minimizing the frictional force between object and the ground, as well as Maxwell's use of analogies and extreme case reasoning in his endeavor to describe electromagnetic phenomena (Einstein & Infeld, 1938; Nersessian, 2008). In the research by Zietsman and Clement (1997) it has been shown that using extreme cases in the context of teaching about levers resulted in improvement of students' ability to construct imageable, intuitively, grounded, explanatory models as well as in an improved students' ability to create new causal variables.

Hasović, Mešić and Erceg (2017) presented the idea that understanding about certain aspects of rolling motion of a cylinder can be potentially improved by externally representing its analogy to a tumbling cube whose number of sides gradually tends to infinity. Thereby, the tumbling motion of the cube can be considered to represent the source domain, whereas the rolling motion of the cylinder represents the target domain. In order to help the students to understand the similarity between the source and the target, polygonal prisms are introduced as bridging cases. Here, extreme case reasoning is used for purposes of showing how the cube transforms to a cylinder when the number of prism edges tends to infinity, as well as for showing how a "finite time interval axis of rotation" of a prism transforms to an "instantaneous axis of rotation" of a cylinder. As a matter of fact, when a prism rotates around one of its edges, it is intuitively clear that the corresponding edge rests with respect to the ground during a finite time interval Δt. As the number of edges of the prism increases, this time interval becomes shorter and in the limiting case, when the number of edges tends to infinity, the mentioned time interval tends to dt. In all these cases, the edge that is in contact with the ground is at rest during the mentioned time interval. Consequently, for a cylinder the contact points with the ground have zero-velocity, because the traveled distances of the contact points during the interval dt amount to zero (Hasović, Mešić, & Erceg, 2017). In addition, within the model which treats a cylinder as an extreme case of a prism, losses of mechanical energy in the rolling motion context can be also explained in an intuitive manner by drawing an analogy to collisions between prism surfaces and the contact surface. As a matter of fact, unlike for a tumbling cube, for a rolling cylinder the collisions with the ground cannot be directly observed by the students. Consequently, the "cube-cylinder" analogy may help the students to more readily accept the idea of micro-collisions which are at the mere heart of rolling friction. Thereby, students' everyday experiences with observing collisions between bodies made of different materials can help them to establish a qualitative relationship between mechanical energy losses and characteristics of the material of which the rolling body and contact surface are made of.

However, Hasović, Mešić and Erceg (2017) did not offer any empirical evidence for the effectiveness of the suggested approach.

Aim of the Present Research

Although the ideas presented by Hasović, Mešić and Erceg (2017) seem to be intuitively appealing, it is important to note that analogy-based teaching sometimes results in cognitive overload (Dagher, 1995; Sturges, 2017). According to Lin and Chiu (2017) the effectiveness of analogy-based teaching depends on whether or not the students perceive the analogy as personally familiar, as well as on the level of students' knowledge about the source domain. In well-designed analogy-based teaching, learning is typically optimized by using of external visualizations and step-by-step guidance, as well as by selecting an intuitively comprehensible source domain, highlighting of most important information and providing explicit cues to the students (Lin & Chiu, 2017; Mayer & Moreno, 2003; Richland & Begolli, 2016; Richland & Hansen, 2013).

The aim of this research was to explore whether enriching the conventional teaching with the cube-cylinder analogy (see Hasović, Mešić, & Erceg, 2017) can improve the university students' conceptual understanding about rolling motion. Thereby "understanding" is defined as "*constructing meaning from instructional messages*" and it

encompasses the processes of interpreting, classifying, comparing, summarizing, exemplifying, inferring and explaining (Anderson & Krathwohl, 2001, p. 70). It is important to note that "understanding" in certain contexts can be more complex than "applying" (Anderson & Krathwohl, 2001, p. 267).

The significance of this research is that it provides empirical evidence on effectiveness of analogy-based approaches to overcoming some of the most frequent misconceptions about rolling motion. Besides that, the results of this research offer additional insight into students' conceptions (pre- and post-treatment) about different aspects of rolling.

Methodology of Research

Research Design

A pre-test - post-test quasi-experiment with three subgroups receiving the control treatment (conventional recitation sessions), and another three subgroups receiving the experimental treatment (recitation sessions that were enriched with elements of analogical and extreme case reasoning) has been conducted. In each of the six subgroups the treatment lasted for two teaching hours (90 minutes). One week before the teaching treatment the students were asked to solve the Rolling Motion Concept Test (RMCT). The same test has been again administered to the students immediately following the teaching treatment. In both occasions students were given 20 minutes to solve the test. Students from all subgroups received the teaching treatment in their natural learning environment. One week before administration of the pre-test, the students from all subgroups had received conventional lectures about rolling motion.

Participants and Curriculum

The target population for this research consists of first year university students who are enrolled in typical introductory physics courses for scientists and engineers in Croatia. For purposes of obtaining a student sample, convenience sampling was used. The sample included all 184 students (mostly 19- year-olds) who were enrolled in the introductory physics course at the Faculty of Chemical Engineering and Technology at the University of Zagreb (Croatia). Three subgroups consisting of altogether 93 students received the conventional treatment, and another three subgroups consisting of altogether 91 students received the experimental treatment. In the sample 73% of students were female and the gender distribution was approximately the same across the sampled subgroups.

The educational intervention has been situated within recitation sessions of the introductory physics course that is delivered in the first year of a five-year study program. The quoted study program is following a four-year high school, and on its completion, students acquire an engineering degree which qualifies them to work in industry or to continue their education at PhD level. Generally, the introductory physics course at Faculty of Chemical Engineering and Technology Zagreb could be characterized as a course whose curriculum is well aligned with typical introductory physics textbooks, such as Physics for Scientists and Engineers by Serway and Faughn (2006). In the first and second semester of the first year, students receive 2 hours of lectures per week, as well as 2 hours of recitations. In recitation sessions, student groups are typically not bigger than 35 students. Usually, in recitations the teaching assistant devotes most of the time to modeling problem solving related to physics topics that had been earlier delivered in lectures.

Treatment

This research was situated within the context of the regular curriculum. All subgroups received lectures on rolling motion by the same lecturer, within the same large-class environment. Thereby a conventional lecturing approach has been implemented.

Similarly, for all subgroups the recitation sessions were led by the regular teaching assistant of the introductory physics course, i.e. by the first author of this article. At the time of the implementation of the teaching treatments the experimenter had 5 years of teaching experience at the university level.

For all subgroups the recitation sessions lasted the same time (90 minutes) and covered the same concepts. However, the approaches to implementing recitation sessions were different for the experimental and control subgroups. Concretely, the control subgroups received the conventional treatment which was mainly characterized by

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

summarizing and application of most important principles that had been earlier covered in the lectures. Thereby, most of the time, the teaching assistant used the blackboard to model solving of problems related to rolling motion. The problems were primarily chosen to cover the concepts that were in the focus of this research - velocity at top, center and bottom of the wheel, role of friction and conservation of mechanical energy in the rolling motion context (see Table 1). This modeling of problem solving was accompanied by corresponding classroom discussions. In the control group the students were also occasionally required to finish a problem on their own (in notebooks), after which a randomly chosen student was expected to demonstrate the problem-solving process at blackboard.

Table 1. Short description of open-ended problems that were solved in the recitation sessions.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
Calculating the moment of inertia of a given cylinder	Calculating the velocity and acceleration of a cylinder that is rolling down an incline	Determining velocity vectors across a wheel (top, bottom, center) that is rolling on a horizontal surface	Calculating the stopping time of a cylinder that is initially rolling up an incline	Calculating coefficient of friction for a cylinder rolling up an incline

In the experimental subgroups the same physics problems have been solved as in the control subgroups. However, in experimental subgroups the problem-solving session has been preceded by a presentation of carefully selected analogies and extreme cases. Concretely, the students have been presented with the analogy between tumbling prisms and a rolling cylinder as it has been proposed by Hasović, Mešić and Erceg (2017). First, the students were shown the figure of a tumbling cube (Figure 1).

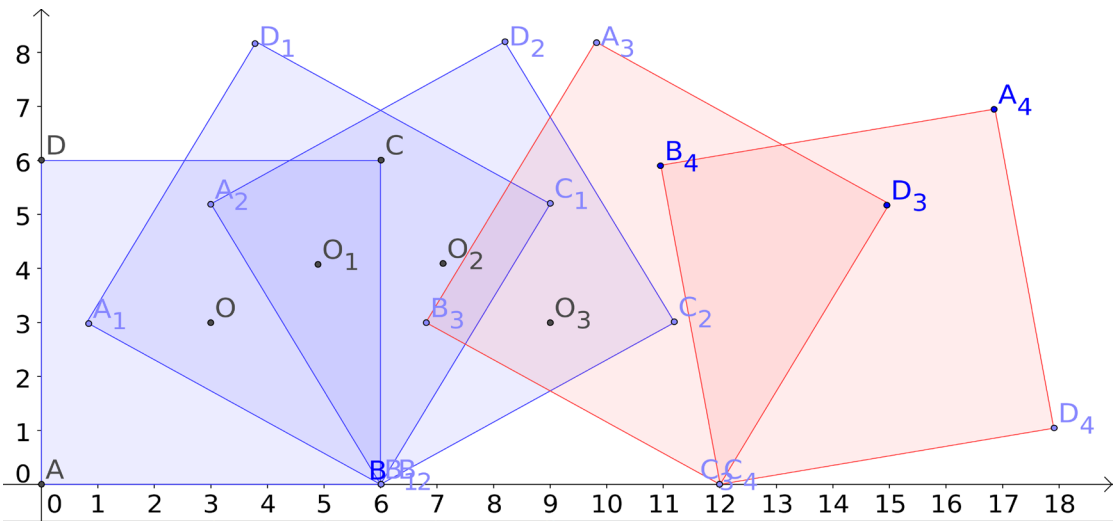

Figure 1: Tumbling of a cube; edge B is at rest until the moment when edge C comes into a contact with surface.

From Figure 1 it could be seen that the cube was initially rotating around an axis passing through its edge B whereby edge B's velocity was zero until the very moment when edge C came into a contact with the surface. Thereafter, the rotation axis was passing through C for another time interval of Δt_1. The same reasoning could be applied to the remaining edges of the cube which led to the conclusion that between two subsequent changes of the rotation axis, a cube's edge is at rest within the time interval Δt_1. After that, the students were showed what happens as a result of increasing the number of the prism edges. Specifically, the tumbling of an octagonal prism has been analyzed (Figure 2).

Thereby, again the edges of the prism stayed at rest within a certain time interval Δt_2, which was shorter than the corresponding time interval for the tumbling cube. After that it has been concluded that if the number of edges increases to infinity a polygonal prism transforms to a cylinder. Similarly, as for the prisms, the contact "edge" of the cylinder also remains at rest, but only during an infinitesimal interval *dt*.

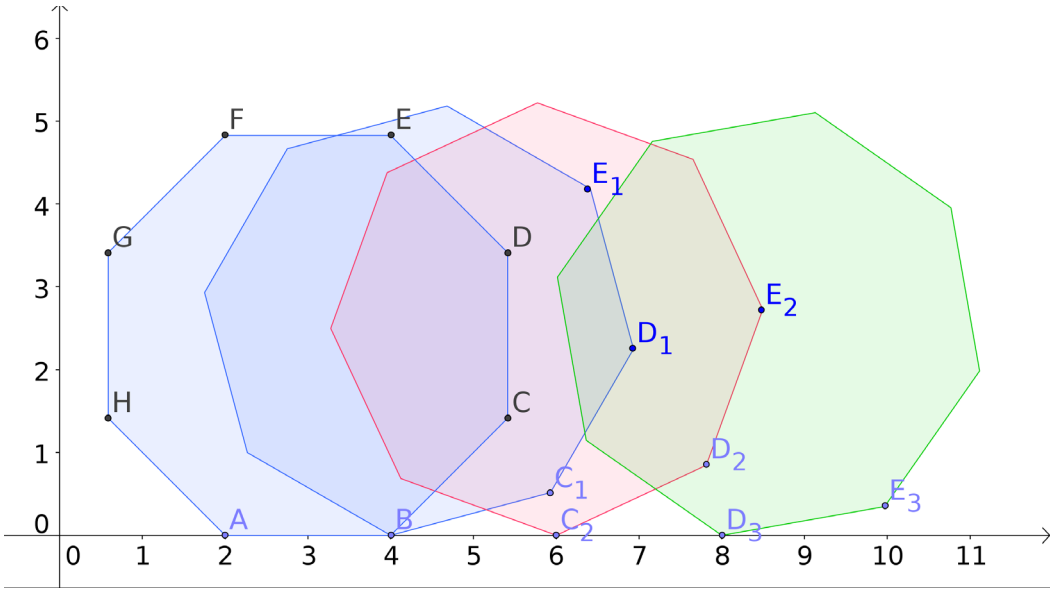

Figure 2: Tumbling of an octagonal prism; edge B is at rest until the moment when edge C comes into a contact with surface.

In line with the recommendations by Hasović, Mešić and Erceg (2017) the analogy with the tumbling cube (see Figure 3) has been used to explain why the velocity of the cylinder's center is twice smaller than the velocity at the top of the cylinder.

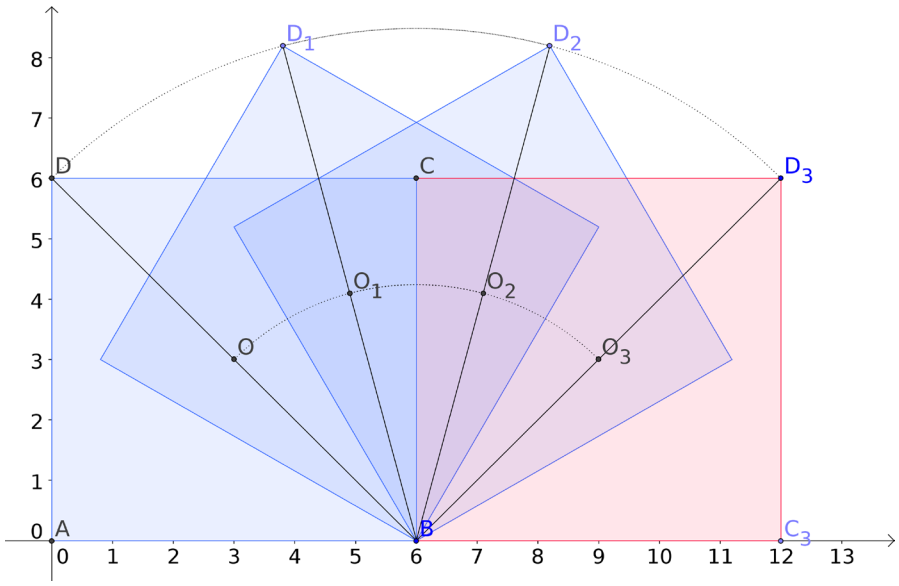

Figure 3: Tumbling of a cube; the center of a cube O moves around the arc of circle whose radius is twice lower then radius of the circle point D moves along.

Next part of the lecture was dedicated to discussing the possibility of using the law of conservation of mechanical energy in the rolling motion context. To that end the students from the experimental subgroups have been guided to the conclusion that collisions between prism surfaces and the ground surface do not cease to

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

exist when the number of edges tends to infinity. Thereby, for perfectly rigid bodies the collisions are elastic and there is no loss of mechanical energy, whereas for real bodies a part of the mechanical energy is being lost due to deformation of the body and surface (Hasović, Mešić, & Erceg, 2017). Students were led to note that this holds in a similar way for cylinders, as it holds for the easier imaginable tumbling of prisms. They were also directed to conclude that the static friction does not displace the edge around which a cube rotates. Similarly, if static friction acts on a rolling cylinder it does not displace its "edge".

After introducing the students with analogies and extreme cases (which lasted 30 minutes) the teaching assistant started the problem-solving session. Thereby, he attempted to follow a minds-on approach. As a matter of fact, besides modeling the problem-solving process, the teaching assistant frequently asked the students for feedback. Concretely, the students were asked to use the earlier introduced analogies in order to answer questions about certain aspects of the given phenomena. Thus, modeling of problem solving was combined with classroom discussions.

Instruments

Students' pre- and post-treatment conceptions about rolling motion were assessed by the Rolling Motion Concept Test (RMCT) which consisted of nine test items (see Table 2). Taking into account that cognitive transfer is one of the main determinants of conceptual understanding, the test items were designed to describe physical situations that are more or less different from the situations that had been explicitly discussed within lectures and recitation sessions (except test item 1) (Mayer, 2002).

The focus of interest was on students' understanding of the velocity at different points of the wheel, as well as in their understanding of the role of static friction and factors that influence mechanical energy losses in rolling motion.

Table 2. A brief description of RMCT items.

Item 1	Item 2	Item 3	Item 4	Item 5
Velocity vectors across a wheel (top, bottom, center) that is rotating around a stationary axle	Velocity vectors across a wheel (top, bottom, center) that is rolling down an incline	Influence of slipping on the velocity vectors	Drawing the trajectory of a point on cylinder's rim	Traveled distances of different points across a cylinder (top, bottom, center) in an interval dt
Multiple-choice	Multiple-choice	Multiple-choice	Open-ended	Multiple-choice
Item 6	**Item 7**	**Item 8**	**Item 9**	
Velocity direction of arbitrary point on a wheel's rim	Equation for linear velocity of the point which is exactly between center and top of the wheel	Motion of a cylinder when static friction is zero	Law of conservation of energy-action of static friction and negligible mechanical energy losses due to micro collisions	
Multiple-choice	Open-ended	Multiple-choice	Multiple-choice	

It should be noted that some of these test items are adapted versions of conceptual questions that are widely used in the university physics literature (Halliday, Resnick, & Walker, 2013) (items: 1, 2, 4). Test item 6 has been adapted from Knight (2014) and test item 8 has been adapted from Rimoldini and Singh (2005), whereas the remaining test items are mostly original. The test as a whole is available on the following web address: http://marie.fkit.hr/~avidak/BURMS.pdf.

It should be noted that test item 1 does not require reasoning about rolling, but only reasoning about pure rotation. As a matter of fact, the purpose of including test item 1 into RMCT was to make possible exploration of students' ability to differentiate between rotation and rolling.

The Cronbach's alpha for RMCT amounts to 0.354 which increases to 0.395 if test item 1 (that has a negative item-total correlation) is dropped out. Certainly, this value is low, but comparable to some widely known conceptual surveys, such as Quantum Mechanics Conceptual Survey by McKagan, Perkins and Wieman (2010). The obtained alpha value could be probably accounted for by the fact that RMCT items were primarily designed to identify misconceptions that are relatively independent from each other, as well as by the fact that the scale consists of a relatively low number of test items. Finally, it should be noted that for all items except test item 1 the difficulty indices are inside the range between 0.2 and 0.8 with a mean of 0.40 which can be considered satisfactory (Kline, 2015).

Data Analysis

Students' answers to each of the RMCT items at pre- and post-test were entered into a database. For multiple-choice test items the information about the chosen response option has been entered. On the other hand, for constructed-response test items firstly the students' answers were thoroughly analyzed with the purpose of identifying categories of students' answers. Then these categories were coded and entered into the database. This database has been used for identifying the students' misconceptions about rolling motion. Next, the original database has been recoded, whereby for each correctly answered RMCT item students were awarded one point. The recoded database has been used for calculating between-group differences in mean scores and class averaged normalized gains, as well as for analysis of covariance.

Results of Research

Pre- and Post-test Scores across Groups

In Table 3 the pre-test and post-test scores across the individual subgroups are reported.

Table 3. Mean scores on pre-test and post-test across the individual experimental groups (EG) and control groups (CG) are provided.

	EG1	EG2	EG3	CG1	CG2	CG3
Pre-test	2.45 (0.97)	2.06 (1.27)	2.12 (0.88)	2.18 (1.31)	1.80 (1.13)	2.46 (1.46)
Post-test	4.79 (1.54)	3.88 (1.69)	4.28 (1.72)	3.18 (1.36)	3.13 (1.63)	1.39)

Note: Standard deviations are given in parentheses. Theoretically, the scale ranges from 0 to 9.

Taking into account the fact that experimental subgroups consistently outperformed the control subgroups, as well as our attempt to keep the analyses as concise and plain as possible, the decision to collapse the data for individual subgroups has been made. Consequently, the results below are reported only for the two broad groups - control and experimental.

Exploring the Significance of the Observed Between-group Differences

For purposes of exploring the significance of the observed between-group differences on the post-test an analysis of covariance (ANCOVA) (Field, 2009) has been conducted. Firstly, the assumptions of ANCOVA had been explored. A visual inspection of the Q-Q plots led to the conclusion that the distributions of post-test scores in the control and experimental group did not significantly depart from the normal distribution. Further, an independent samples t-test for the pre-test scores, showed that the assumption of independence of the covariate (pre-test score) and treatment ("group" as teaching treatment variable) was met ($t(182) = -0.33$, $p = .74$). In addition, the Leven's statistic proved to be non-significant ($F(1, 182) = 0.87$, $p = .35$), as well as the interaction between the covariate and treatment variable ($F(1, 180) = 2.03$, $p = .16$). In other words, the homogeneity of variance and homogeneity of regression slopes assumptions were also met. The results of ANCOVA showed that there was a significant effect of the teaching treatments on students' post-test scores after controlling for pre-test scores, $F(1,181) = 35.88$, $p<.001$, partial $\eta^2 = .17$. As a matter of fact, a planned contrast revealed that students from the experimental group significantly outperformed their peers from the control group, $t(181) = 5.99$, $p < .001$, $r = .41$.

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

Finally, it could be found that the class average normalized gain amounted to .32 in the EG group and to .11 in the CG group.

Item-level Analyses

Next, the between-group differences on each of the RMCT items were explored (see Table 4).

Table 4. Proportion of correct answers on individual RMCT items on the post-test.

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8	Item 9
EG	.13 (.34)	.64 (.48)	.63 (.49)	.45 (.50)	.69 (.46)	.50 (.50)	.30 (.46)	.56 (.49)	.42 (.49)
CG	.17 (0.38)	.45 (.50)	.48 (.50)	.03 (.18)	.24 (.43)	.42 (.49)	.31 (.46)	.51 (.50)	.32 (.47)

Note: Standard deviations are given in parentheses.

The most frequently made errors at pre-test and post-test are reported in Table 5.

Table 5. Most frequent errors for RMCT items.

	Item 1	Item 2	Item 3	Item 4	Item 5
Pre-test (Overall)	D (24%)	C (53%)	B (23%)	Circular path (53%)	D (38%)
Post-test (CG)	A (72%)	E (32%)	B (24%)	Circular path (49%)	D (43%)
Post-test (EG)	A (82%)	E (30%)	B (22%)	Circular path (37%)	D (19%)
	Item 6	**Item 7**	**Item 8**	**Item 9**	
Pre-test (Overall)	D (69%)	(1/2)ωr (25%)	D (41%)	A (25%)	
Post-test (CG)	D (45%)	(1/2)ωr (30%)	D (26%)	A (22%)	
Post-test (EG)	D (32%)	(1/2)ωr (28%)	D (15%)	A (22%)	

Please note that even at post-test the most frequent errors were the same for both groups.

Discussion

Overall Between-group Differences

First of all, it should be noted that across all six subgroups the achievement at pre-test was very low, especially if one knows that one week before the pre-test students received conventional lectures about rolling motion. However, this finding is in line with the results of earlier research on (in)effectiveness of conventional teaching. In the study by Rimoldini and Singh (2005), none of the 16 interviewed university students was able to explain the velocities at top and bottom of the wheel relative to the ground, and tremendous difficulties with application of the energy concept have been reported, as well.

The ineffectiveness of conventional lectures could be only partly compensated through recitation sessions. Thereby, the normalized gain for the CG was low and for EG medium. The low gain of the CG is in line with the findings by Kim and Pak (2002) who showed that students often fail to overcome conceptual difficulties even after formal solving of more than 1000 conventional problems.

Furthermore, it is important to note that even the lowest achieving experimental subgroup outperformed the highest achieving control subgroup. This finding was additionally supported by results of ANCOVA whereby a highly significant, medium to large effect in favor of the experimental group was detected (Field, 2009). The results of this research support the idea that analogical and extreme case reasoning can help the students to develop more imageable and intuitively grounded mental models of physical phenomena (Clement, 1988; Clement, 1993; Stephens & Clement, 2009; Zietsman & Clement, 1997). It seems that the between-treatment differences mostly resulted from the fact that in the experimental treatment students were provided with an easily imageable anchoring intuition (e.g., images of a tumbling cube) that could be effectively used for reasoning about processes that occur in rolling motion (e.g., resting of the contact point and micro-collisions).

Most Prominent Between-group Differences on Individual Items

Firstly, it should be noted that students from the experimental group were (slightly) outperformed only on test item 1 and test item 7.

In test item 1 students were shown a purely rotating wheel and they were required to choose the diagram that best described the velocity vectors at bottom, center and top of the wheel. The purpose of test item 1 was to check whether students understand how these velocities differ in the rolling and rotation contexts. Eventually, it has been shown that this was the only item for which a negative gain has been detected, whereby that negative gain was more pronounced for the experimental group. As a matter of fact, the majority of students from the control group, and even a larger majority from the experimental group, wrongly transferred their knowledge about velocities in rolling motion to the pure rotation context that was described in test item 1. This is a vivid example of how learning of a new concept can implicitly activate restructuring of similar foreknowledge structures. If teachers are not careful enough, a negative transfer from "new knowledge" to "old knowledge" can occur which is in line with the idea that learning science is iterative in its nature (Bybee et al., 2008).

In test item 7 students were asked to write the formula for calculating velocity at the point located exactly between wheel's center and top, and it is not surprising that students from the control group performed slightly better at this item, because within the conventional approach the accent was mostly put on use of the mathematical representation. As a matter of fact, test item 7 was the only item in which students were expected to write a physics formula.

Most prominent differences in favor of the students from experimental group were found for test item 5 and test item 4. Particularly interesting is the result for test item 5 in which students were asked to compare the traveled distances of the top, center and bottom of a rolling wheel in an infinitesimal time interval. On this item students from the experimental group scored 45 % higher than their peers from the control group, although on an item that explicitly required the students to compare velocities across the wheel (test item 2) the between-group difference amounted to merely 19 %. In other words, it seems that many students from conventional courses declaratively learn to know how the velocities of certain wheel points compare, but when asked about the distances traveled by the same points of the wheel, students often refer back to their deeply rooted intuitive ideas. This is in line with the fact that knowledge is context-specific, as well as with the fact that formal physics knowledge fails to be activated in less formal contexts, and generally in contexts that are far from the explicitly taught context (Redish, 2003). The experimental approach's aim was exactly to help the students to also engage in non-formal reasoning with the goal to gain intuitive feeling of some seemingly counterintuitive aspects of rolling motion (Hasović, Mešić, & Erceg, 2017).

In test item 4 students were asked to draw the trajectory of a given point on a rolling cylinder's rim. A better achievement on item 4 may be primarily accounted for by the fact that at the mere heart of the experimental approach was to follow the motion of a certain point of a prism during a finite time interval (see Figure 2). These results suggest that the experimental treatment was more effective in helping the students to develop certain visual mental models of rolling motion, whereby mental simulation of these models facilitated solving of test item 4 (Greca & Moreira, 2000; Mešić, Hajder, Neumann, & Erceg, 2016; Nersessian, 2008).

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

Students' Misconceptions and Conceptual Change

Next, the misconceptions from Table 5 are discussed through four themes.

1. *Pure rotational motion*

Students' understanding about pure rotational motion has been assessed only in test item 1, where students were expected to choose the diagram that best describes the velocity vectors at bottom, center and top of a rotating wheel. At the pre-test many students chose the incorrect option D according to which linear velocities at bottom and top of the wheel are obtained by multiplying the angular velocity with the diameter of the wheel. As a matter of fact, this option reflects a reasoning characterized by "hybrid knowledge" which is an erroneous mix of knowledge about pure rotational motion (velocity of center is zero and same magnitude of velocity at bottom and top) and rolling motion (velocity of the point at top is 2ωR) (Galili, Bendall, & Goldberg, 1993). In order to understand this finding, it is useful to remember that one week before taking the pre-test students from all groups had received conventional lectures on rolling motion. Probably, these lectures on rolling motion negatively interfered with students' foreknowledge on rotational motion. This resulted in the observed hybrid knowledge in which still the "rotational part" was predominant. However, it seems that after additional teaching about rolling motion (in recitation sessions), the rolling motion part of that hybrid model prevailed over the rotational motion part. As a matter of fact, a large share of students from both groups chose at the post-test the incorrect option A that was showing the typical distribution of velocities across the rim of a wheel that is rolling without slipping.

2. *Velocity at bottom, center and top of a rolling object*

In test items 2, 3 and 5 the students were required to reason about the velocities at bottom, center and top of a rolling object. Thereby, items 2 and 5 were situated within the context of rolling without slipping, whereas in item 3 consequences of slipping on the velocities of selected points were discussed.

Specifically, test item 2 showed a wheel rolling down an incline without slipping and students were required to reason about velocities at characteristic points of the wheel. Thereby, at the pre-test most students wrongly applied the rotational motion model to the rolling context, i.e. they chose the incorrect option C for which center of mass is stationary and points at bottom and top have velocities of equal magnitude. Mixing up of the domains of rotational and rolling models is a common error that has been already reported in earlier studies (De Ambrosis et al., 2015; Rimoldini & Singh, 2005). At the post-test the most frequently chosen distracter was E which reflects the belief that besides the given angular velocity and radius, the linear velocities of the wheel's points also depend on the angle of the incline. It seems that students who chose this option knew how the velocities at different points of the wheel compared but they thought that for rolling down an incline the formula has to be modified compared to rolling along a horizontal surface (De Ambrosis et al., 2015; Lopez, 2003; Mashood & Singh, 2012). Test item 5 thematized exactly the same physical phenomenon as test item 2, whereby the main difference was that students were now expected to reason about the traveled distances of characteristic points of the cylinder, in an infinitesimal time interval *dt*. Both at pre-test and post-test, the most common wrong answer for this item was D which again reflects a hybrid mental model according to which the points at bottom and top travel the same distances, similar as in rotational motion, but the center of the cylinder is also moving as in rolling motion. Test item 3 required the students to reason about how slipping influences the velocities of characteristic points of a wheel. In pre-test as well as in post-test the most common wrong answer was B which reflects the idea that the magnitude of linear velocities of all points of the wheel decreases when slipping occurs. Taking into account the fact that the students had no experience with numerical problems related to rolling with some slipping, they probably attempted to apply non-formal, intuitive aspects of their mental models to this situation which resulted in the described error. It seems that due to the introduction of visual analogies, in the experimental group the non-formal, visual aspects of the students' mental models were more compatible with scientifically acceptable knowledge. Generally, it is interesting to note that differences in effectiveness of conceptual change (in favor of experimental group) were more prominent on test items 5 and 3 that required a higher level of knowledge transfer.

3. Velocity of an arbitrary point of a rolling object

Similarly, to test items 2, 3 and 5, test items 4, 6 and 7 also thematized the velocity concept within the context of rolling motion. However, in items 4, 6 and 7 students were expected to reason about the velocity at arbitrary points of a rolling object. Concretely, in test item 6 students were asked to select the velocity vector that best describes the velocity at a point of the wheel's rim which is at the height of the wheel's center. Again, many students applied here the wrong model, i.e., the model of rotational motion which is evident from the fact that they chose the response option D. According to this option velocity of a point at wheel's rim has the direction of the tangent on the wheel at that point instead of the direction of the tangent to the point's trajectory. The students' difficulty to recognize that the velocity for rolling motion is obtained by combining components related to rotational and translational motion was already identified in earlier research (De Ambrosis et al., 2015; Rimoldini & Singh, 2005). It is interesting to note that within this formal context (choosing velocity vectors), the experimental group students only slightly outperformed their peers from the control group. On the other hand, when they were required to draw the trajectory of a point at cylinder's rim for a given time interval, students from the experimental group largely outperformed their peers from the control group. As a matter of fact, experimental group students scored similarly on both test items, but control group students scored extremely low in the less formal context of test item 4. Generally, in this test item many students drew circular trajectories instead of cycloids, which was the most prevalent error in both groups, at both, pre-test and post-test.

In test item 7 students were asked to write the expression for linear velocity of a point that was exactly between wheel's center and top of the rolling wheel. Here, many students wrote the expression (1/2)ωR instead (3/2) ωR and it seems as if none of the treatments proved to be effective in accomplishing conceptual change when it comes to this item.

It can be concluded that for this category of items, differences in effectiveness of conceptual change (in favor of experimental group) largely depended on whether the item stem was situated in more or less formal contexts.

4. Role of static friction and conservation of mechanical energy

Static friction and conservation of mechanical energy were thematized through test items 8 and 9. Test item 8 was meant to assess students' reasoning about the role of friction for a cylinder's motion down the incline. Specifically, in item 8 students were asked to reason about a cylinder that is placed on an incline whose angle can be changed, whereby the coefficient of static friction between cylinder and incline is zero. For item 8 the most common wrong answer in the pre-test as well as in the post-test was D which reflects the misconception that cylinder would roll down the incline (without slipping) for all angles of the incline. This finding is in line with the results by De Ambrosis et al. (2015). As a matter of fact, in the study by De Ambrosis et al. (2015), at the pre-test 40% of students did not recognize that kinetic friction force produces the transition from sliding to rotational motion of a sphere and 42% of students answered that a sphere cannot simply slide along a frictionless incline. The misconception that was observed for test item 8 could be simply explained by the fact that in everyday life it is much more common to observe spheres and/or cylinders rolling down an incline than to see the same objects sliding down the incline. It is well known that many student misconceptions are rooted in students' everyday life experiences (McCloskey, 1983; Reiner, Slotta, Chi, & Resnick, 2000).

Students from the experimental group showed a slightly better performance on test item 8 than their peers from the control group. Some of them probably engaged in intuitive reasoning about tumbling a cube over its edge. Thereby, they came to the conclusion that in the extreme case of zero friction between the edge and the contact surface tumbling would become impossible and the cube could only slide across the surface.

Finally, in test item 9 students were expected to reason about the law of conservation of mechanical energy within the rolling motion context. Concretely, they were shown a steel ball that was initially at rest on a curved ramp (made of hard material) and they were asked to predict the maximal height the ball reaches after rolling down the ramp. The most common wrong answer on pre-test as well as on pre-test was A which suggests that ball's final height will be lower than its initial height because "the ball loses its energy due to the existence of static friction". In other words, at pre-test as well as at post-test nearly one in four students from both groups had the misconception that the action of static friction results in losses of mechanical energy of the rolling object. However, in the control group there were more students who believed that mechanical energy is conserved because "there is no static friction acting on the rolling ball" which is similar to the findings for test item 8. Consequently, it can be concluded

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

that the better performance of experimental group students on test item 9 is at least partly related to their better understanding of the role of static friction in rolling motion. As a matter of fact, the idea that force of static friction does not displace the edge around which a cube rotates (i.e., it does no work on the cube) is probably intuitively acceptable for most students. This idea can be also applied to a cylinder rolling down an incline as an extreme case of a cube when the number of edges tends to infinity.

Taking into account the findings for all discussed themes, it can be concluded that experimental group students mostly outperformed their peers from the control group on those items that, at the same time, required a higher level of knowledge transfer and were situated in less formal contexts. This conclusion is in line with the assertion that using teaching analogies and extreme cases promotes qualitative understanding (Clement, 1993; Nersessian, 2008). More prominent differences in favor of the experimental group were observed for the velocity concept than for the friction and energy concepts.

Limitations of Research

It should be noted that RMCT internal consistency as measured by Cronbach's alpha proved to be low which is not a surprising result for assessment instruments that are mainly intended to have a diagnostic purpose (Mc Kagan, Perkins, & Wieman, 2010). A practical consequence of an instrument characterized by relatively low reliability can be a decreased statistical power (Heo, Kim, & Faith, 2015). In addition, when drawing conclusions about between-group differences on RMCT it is safer to rely on a synthesis of item-level analyses, i.e. analyses of individual misconceptions, than on comparisons of composite scores.

When it comes to limitations of the teaching analogy itself, it should be noted that in real life tumbling prisms most often quickly come to rest due to inelastic collisions and corresponding mechanical energy losses. Similarly, prisms that "roll" down an incline reach a terminal speed (Rezaeezadeh, 2009). In addition, the threshold value of static coefficient of friction that makes combined translational and rotational motion possible approaches zero as the number of prism sides tends to infinity. Certainly, future implementations of the analogy should be directed at improving differentiation between pure rotation and rolling motion.

Conclusions

Conventional approaches to teaching about rolling motion are mainly based on use of mathematical representations. These approaches prove to be highly ineffective when it comes to developing students' understanding about rolling motion. Therefore, it is important to invest efforts in designing and evaluating alternative approaches to teaching about this complex topic.

From the results of this research it can be concluded that drawing analogies between tumbling of prisms and rolling motion of cylinders can help the students to develop deep understanding of some counterintuitive aspects of rolling motion, such as the idea of zero-velocity at contact-point. The fact that experimental group students showed a higher ability to transfer their knowledge about velocities at various points of the wheel to a context in which they are expected to reason about traveled distances of these points, indicates that analogical, visual mental models are less inert compared to mental models that are purely based on propositional representations. Furthermore, this research shows that students often have the misconception that a rolling ball loses its mechanical energy due to the work of static friction force. In addition, many students have the misconception that for a rolling wheel the points at top and bottom of the wheel travel the same distances in a given time interval, i.e. they do not sufficiently differentiate between rotational and rolling motion.

This research has important implications for the practice because it shows how teachers can use analogies and extreme cases to facilitate students' learning about rolling motion in introductory physics courses at the university level. In addition, it provides the physics education community with the RMCT which proves to be useful for identifying misconceptions that are mainly related to thinking about velocities at different points of the wheel and law of conservation of mechanical energy in the rolling motion context.

In future research about effectiveness of using an extreme case reasoning approach to teaching about rolling motion it would be useful to implement a mixed research design which would make it possible to more deeply explore the contents and structure of students' mental models.

Acknowledgments

The authors would like to thank dr. sc. Iva Movre Šapić from Faculty of Chemical Engineering and Technology, Zagreb for her contribution in data analysis.

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Received: *February 07, 2018* Accepted: *June 05, 2018*

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