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Research Article

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Through the Analysis of the Advantages and Disadvantages of the Several Methods of Design of Infinite Impulse Response Filters

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Abstract Several methods for designing of FIR filters have been considered. The advantages and disadvantages of each method have been estimated.

Keywords iterative, impulse invariance, bilinear transformation, matched z-transform, Newton algorithm method, minimax algorithm method

1. Introduction

Any filter design may be achieved by the *direct* or *indirect* methods. In *direct* methods the discrete-time transfer function is generated directly in the z domain whereas in *indirect* methods it is derived from a continuous-time transfer function. The other way of the classification of design of filters is dividing into *iterative* or *noniterative* methods. Iterative methods are on basis of the optimization algorithms whereas noniterative method usually entails a set of formulas and transformations. *Infinite Impulse Response* (IIR) filters are designed by using *indirect noniterative* methods or *direct iterative* methods. It means that IIR filters can be designed from continuous-time transfer functions using a set of formulas and transformations or the discrete-time transfer function can be achieved directly in z domain using optimization algorithms.

The impulse response of IIR filter is infinite because there is feedback in the filter it means if one put in an impulse (a single "1" sample followed by many "0" samples), an infinite number of non-zero values will come out.

Approximation methods for the design of IIR filters are differ from the design of *Finite Impulse Response* (FIR) filters. It's due to their transfer function – the transfer function of IIR filters is a ratio of polynomials in z whereas the transfer function of FIR filters is a polynomial in z^{-1} [1].

The simple block diagram of FIR (Finite Impulse Response) and IIR filters is shown below (see Fig.1)

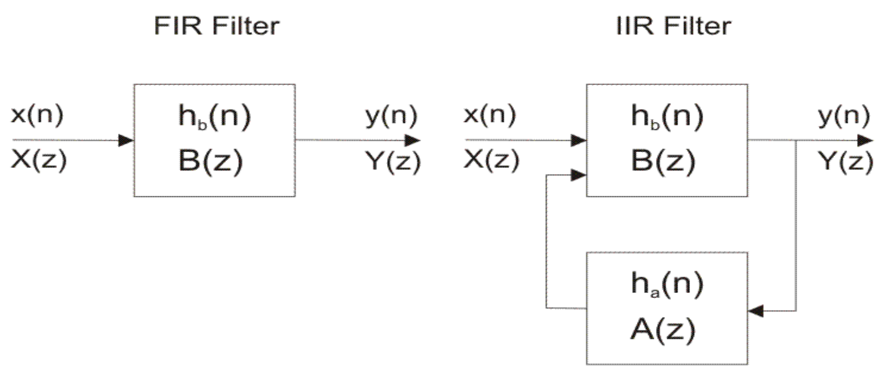

Figure 1: Block diagrams of FIR and IIR filters

2. The Comparison of the FIR and IIR Filters

The comparison of the FIR and IIR filters was comprehensively analyzed in [2].

Advantages of FIR:

1. Possibility to have exact linear phase;
2. Always stable with finite-duration transients;
3. Design methods generally linear in filter parameters;
4. Flexibility in the choice of the frequency response.
5. Can be adaptive.

Advantages of IIR:

1. Much smaller filter order for fixed specifications;
2. Typically much smaller delay introduced;
3. Noniterative design methods no need for computer-aided techniques;
4. Can be exactly all pass.

3. Transfer Function of IIR Filters

FIR filters are better suited for applications that require a linear phase response while IIR filters are well suited for applications that require no phase information, for example, for monitoring the signal amplitudes.

The transfer function of IIR filter must have some constraints:

1. It must be a rational function of z with real coefficients;
2. For ensuring of the stability of the filter its poles must lie within the unit circle of z-plane;
3. For ensuring of the causality of the filter the degree of the numerator of the transfer function must be equal to or less than its denominator.

IIR filters are designed by using indirect noniterative methods or direct iterative methods.

4. Analysis of the Methods of IIR Filters

Scientific articles [3]-[13] were concerned to the properties of IIR filters. IIR filters, as was mentioned above, can be designed with indirect noniterative or direct iterative methods.

The advantages of indirect noniterative methods:

1. These methods lead to a complete description of the transfer function in closed form, either in terms of its zeros and poles or its coefficients;
2. Theyarevery efficient;
3. They lead to very precise designs.

The advantages of direct iterative methods are:

1. The direct iterative method is applicable for the design of all types of filters.

4.1. Indirect Noniterative Method of the Design of IIR Filters

In indirect noniterative method first a continuous-time transfer function that satisfies certain specifications is obtained using one of the standard analog-filter approximations. Then a corresponding discrete-time transfer function is obtained using one of the following methods:

1. Invariant impulse-response method;
2. Bilinear transformation method;
3. Matched-z transformation method;
4. Modified invariant impulse-response method.

4.1.1. Invariant Impulse-Response Method

The Invariant Impulse-Response Method of IIR filter design is based upon the notion that one can design a discrete filter whose time-domain impulse response is a sampled version of the impulse response of a continuous analog filter. If that analog filter (often called the prototype filter) has some desired frequency response, then IIR filter will yield a discrete approximation of that desired response.

Advantages of this method are:

1. The phase response as well as the loss characteristic of the analog filter.
2. Preserves the order and the stability of the analog filters

Disadvantages of this method are:

1. Can be seen only as an approximation of the continuous transfer function;
2. Very sensitive to the aliasing errors;
3. Non applicable to the all filter types (high-pass, band-stop);
4. There exists the distortion of the shape of frequency response due to aliasing.

$$\hat{H}_A(j\omega) = H_D(e^{j\omega T}) = \frac{h_A(+0)}{2} + 1/T \sum_{k=-\infty}^{+\infty} H_A(j\omega + jk\omega_s) \quad (1)$$

Where H_A-continuous-time transfer function of the analog filter; $\hat{H}_A$- continuous-time transfer function of the impulse modulated filter; H_D-discrete-time transfer function; h_A-impulse response of the analog filter.

4.1.2. Bilinear Transformation Method

The digital filter with a discrete-time transfer function can be obtained from an analog filter with a continuous-time transfer function (from s-plane to z-plane) by applying the equation (2).

$$H_D(z) = H_A(s)|_{s=\frac{2}{T}(\frac{z-1}{z+1})} \quad (2)$$

This method is called as bilinear transformation method.

The advantages of the bilinear transformation are:

1. A stable analog filter gives a stable digital filter;
2. The maxima and minima of the amplitude response, a pass band ripple and minimum stopband attenuation in the analog filter are preserved in the digital filter.
3. The method requires very little computation and leads to very precise optimal designs.

The disadvantages are:

1. For higher frequencies the relationship between frequency variables of the analog and derived digital filters becomes nonlinear and distortion is introduced in the frequency scale of the digital filter relative to their analog filter. This is called as warping effect.

4.1.3. Matched Z-Transform Method

This method is based on the converting of all poles and zeros of the s-plane (*H(s)*) design into poles and zeros in z-plane locations.

If the transfer function of the analog filter in s-plane is expressed in the form

$$H(s) = \prod_{k=1}^{M}(s - z_k) / \prod_{k=1}^{N}(s - p_k) \quad (3)$$

Where z_k – zeros and p_k – poles of the filter, then the transfer function for the digital filter is

$$H(z) = \prod_{k=1}^{M}(1 - e^{z_k T} z^{-1}) / \prod_{k=1}^{N}(1 - e^{p_k T} z^{-1}) \quad (4)$$

Where T is the sampling interval.

Advantages of this method are:

1. The method is fairly simple to apply;

2. The method gives reasonable results provided that a sufficiently large sampling frequency is used.

Disadvantages of this method are:

1. This method suffers from aliasing problems.
2. It introduces a relatively large error in the pass band loss.

4.1.4. Modified Invariant Impulse Response Method

Invariant Impulse-Response Method is restrictedly applicable for the design of all pole filters due to aliasing errors. In this case, the modified version of the Invariant Impulse-Response Method will come to help. In this method, any poles of $H_D(z)$ located outside the unit circle can be replaced by their reciprocals without changing the shape of the loss characteristic. This will introduce a constant vertical shift in the loss characteristic but the problem can be easily eliminated by adjusting H_0, the multiplier constant of the transfer function.

The advantages of this method are:

1. This method has all advantages of Invariant Impulse-Response Method;
2. The method is especially good for elliptic filters;

Disadvantages of the method are:

1. The main problem with the modified invariant impulse-response method has to do with the order of the filter obtained.

4.2. Direct Iterative Methods of the Design of IIR Filters

In these methods a discrete-time transfer function will be assumed and on the basis of some desired amplitude and/or phase response an error function will be formulated. Then a norm of error function will be minimized with respect to the transfer function coefficients. When the value of the norm approaches to zero, the resulting amplitude or phase response approaches to the desired amplitude or phase response.

The direct iterative methods divided into:

1. Newton algorithm method;
2. Quasi-Newton algorithm method;
3. Minimax algorithms method.

4.2.1. Newton Algorithm Method

This method is based on using the quadratic approximation of the Taylor Series. Then, assuming the initial point of nonquadratic convex problem a series of the correction to the initial point will be computed and the objective function will be minimized along a Newton direction in each case.

The advantages of this method are:

1. This method is very effective for the design of digital filters;
2. As all optimization methods yields to very precise design of the filters.

The disadvantages of this method are:

1. In each iteration the Hessian [14] must be checked for positive definiteness and, if it is found to be nonpositive definite, it must be forced to become positive definite;
2. Both the first and second partial derivatives of the $f(\mathbf{x})$ must be computed in each iteration in order to construct the gradient and Hessian;
3. Matrix inversion is required in each iteration.

4.2.2. Quasi-Newton Algorithm Method

Newton algorithm method has the significant disadvantages, what were described above. In this case Quasi-Newton algorithm method will come for help. The essence of this method that the direction of search is based on an $n \times n$ matrix S that serves the same purpose as the inverse Hessian in the Newton algorithm.

The advantages of this method are:

1. The method has all advantages of Newton algorithm method;
2. This method is very robust;
3. The method can be used with inexact line searches which lead to improved efficiency;
4. There is no need to invert and check for positive definiteness the $n \times n$ matrix;
5. This method offer fast convergence and do not require the second derivatives of the function.

The disadvantages of this method are:

1. The lack of precision in the Hessian calculation leads to slower convergence in terms of steps;
2. This method needs to store the inverse Hessian approximation. It means that it is required a large amount of memory and could thus be detrimental in the cases of large complicated systems.

4.2.3. Minimax Algorithms

We know that a Nth-order recursive filter with N even can be expressed by the transfer function as follow:

$$H(z) = H_0 \prod_{j=1}^{J} (a_{0j} + a_{1j} z + z^2)/(b_{0j} + b_{1j} z + z^2) \quad (5)$$

Where a_{ij}, b_{ij} are real coefficients, H_0 is a positive multiplier coefficient and $J=N/2$.

Then an amplitude response is

$$A(\boldsymbol{x}, \omega) = |H(e^{j\omega T})| \quad (6)$$

If $A(\omega_0)$the specified amplitude response, an objective function$e(x, \omega)$is the value which depends on the difference between the actual and specified amplitude response:

$$e(\boldsymbol{x}, \omega) = A(\boldsymbol{x}, \omega) - A(\omega_0) \quad (7)$$

And sampling $e(x, \omega)$ at frequencies $\omega_1, \omega_2, \ldots, \omega_K$, the column vector E(x) can be formed:

$$E(x) = [e_1(x) e_2(x) \ldots e_K(x)]^T \quad (8)$$

An objective function must satisfy some requirements such as should be a *scalar* quantity, its minimization with respect to **x** should lead to the minimization of all the elements of *E(x)* in some sense and it is desirable to be *differentiable*. An objective function which satisfies these requirements can be defined in terms of the L_P norm of *E(x)*:

$$L_P = ||E(x)||_P = [\sum_{i=1}^{K} |e_i(x)|^P]^{1/P} \qquad (9)$$

where P is an integer and depending on the values of Pmay be the special cases of the norms – L_1, $L_2$and so on. The algorithms for minimizing of L_P are called as the *minimax* algorithms.

The advantages of this method are:

1. This method inherits all advantages of the Direct Iterative Method;
2. This method is very efficient due to unnecessary of derivative in the minimization process.

The disadvantages of this method are:

1. The computation process is very complicated;
2. Sampling of the objective function with respect to ω must be dens the i.e., error function may develop spikes in the intervals between sampling points during the minimization ;

A technique which is used to suppress spikes in the error function without using a large value of K is called as improved minimax algorithm method and it neglects the 2st disadvantage of this method.

Conclusion

Despite of the more popularity of the FIR filters the last in the same condition IIR filters have smaller order transfer function than FIR filters. They also require less processing power and less work to set up. The transfer function of IIR filters has some constraints. It is possible to use the feedback coefficients in order to introduce some instability and create oscillations basically using an unstable IIR filter for a practical use. Unlike the indirect noniterative methods, direct iterative methods are not constrained to the standard low pass, high pass, band pass, or band stop configurations.

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Biography

Abdulazizjon Abdullaev was born on 07.01.1978. In 2002 he graduated with (M.Sc.) at the Tashkent University of Information Technologies, Uzbekistan. Since 2015 he is the PhD student of the department of Telecommunication Engineering of the Faculty of Electrical Engineering of the Czech Technical University in Prague, Czech Republic. His scientific researchers are focused on digital signal processing and optical communication.