Available online www.jsaer.com

Journal of Scientific and Engineering Research, **2016, 3(6):222-227**

ISSN: 2394-2630
CODEN(USA): JSERBR

Research Article

Synthesis of adequate mathematical description for dynamic systems with the inexactly defined mathematical model

Yuri L Menshikov*, Margarita S Barannik

National University of Dnepro, Ukraine, Gagarina av.,72, 49010, Dnepro, Ukraine

Abstract The problem of construction of the adequate mathematical description of real dynamic process by a method of identification is considered. Two basic approaches for the solution of this problem are offered. On the basis of real measurements the synthesis of the adequate mathematical description is suggested for dynamic systems with the inexactly defined mathematical model. The example was given.

Keywords dynamic systems, adequate description, ill-posed problems, regularization

Introduction

The main problem of mathematical modeling of open dynamic system is the construction (synthesis) of mathematical model (MM) of motion which in aggregate with model of external load (MEL) gives the adequate to experimental observations results of mathematical modeling. The pair of MM and MEL will be name as mathematical description (MD) of dynamic system. If the motion of MM of dynamic system coincides with experimental measurements with experiment accuracy under action of MEL then such AMD is understood as adequate mathematical description (AMD) of dynamic system.

The linear dynamic system with concentrated parameters is consideredfor simplicity.

The paper has five parts: statement of synthesis problem, algorithm of a synthesis of AMD, method of special mathematical models, conclusion,references.

Statement of synthesis problem

Let us assume that dynamic system has only one unknown external load z. Some variables $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \dots \tilde{x}_m)^T$ of state variables $x = (x_1, x_2, \dots x_n)^T$ of dynamic system (outputs) ($(\cdot)^T$ is a mark of transposition) were obtained by experimental way [1,2]. The connection between external load *z* and known variables of state $\tilde{x}$ can been written in many cases in form [3,4,5]

$$A_p z = u_\delta = B_p \tilde{x} \,. \tag{1}$$

where u_d is scalar function from the functional space *U*; $z \in Z$, $\tilde{x} \in X$, Z, X are the functional normal spaces; A_p, B_p are linear operators of the certain structure which are carrying out the connection of MEL (*z*) and the output u_δ of MM on EL and which depend from parameter vector *p*.

The vectorfunction $\tilde{x}$ is obtained from experiment with a known error δ_0:

$$\left\| \tilde{x} - x_{ex} \right\|_X \le \delta_0 \,, \tag{2}$$

where $\| \cdot \|$ is norm in normal functional space; x_{ex} is the exact output of dynamic system on real EL.

The check of adequacy to mathematical description of dynamic system in this case is reduced to check of performance of an inequality

$$\| A_p z - B_p \tilde{u} \|_U \le d, \qquad (3)$$

where $d = \| B_p \| \cdot d_0$, d – const, $d > 0$.

Characteristic feature for problems of a considered type is the fact that the operator A_p is compact operator [6]. The value d is given a priori and it characterizes desirable quality of mathematical modelling.

It is obvious that in the case of performance of inequality (3) an operator A_p and a function z are connected. It is easy to show that with the fixed operator A_p in (3) there is an infinite set of functions z which satisfy to an inequality (3) and which are various among themselves [6]. On the contrary, at the fixed function z there are infinite set of various operators A_p for which an inequality (3) are valid. Thus there are no opportunities of a choice of good mathematical model of dynamic system separately from a choice of correct MEL.

As a rule the structure of the mathematical description is fixed at research of concrete dynamic systems. For example, at research of dynamics rolling mills [5,7] and at the solution of a problem of unbalance diagnostics [4] are being used the models with a fixed structures. However it is necessary to believe that the parameters of structure are given approximately. Thus at execution of calculations it is necessary to take into account that the operators A_p, B_p, depend on a some parameter vector p of mathematical model of motion of dynamic system.

Besides it is supposed that the vector parameters p is given inexactly. So vector p can has values in some closed domain $D: p \in D \subset R^N$. Two operators A_p, B_p correspond to each vector from D. The set of possible operators A_p has been denoted as class of operators K_A, the set of possible operators B_p has been denoted as class of operators K_B. So we have $A_p \in K_A, B_p \in K_B$. The maximal deviations of operators A_p from class K_A and operators B_p from class K_B are equal:

$$\sup_{p_a, p_b \in D} \| A_{p_a} - A_{p_b} \|_{Z \to U} \le h, \quad \sup_{p_h, p_g \in D} \| B_{p_h} - B_{p_g} \|_{X \to U} \le d. \qquad (4)$$

This error can be appreciated from above and, as a rule, it does not surpass 5–10% [5,10].

Two approaches exist to problem of construction of adequate mathematical description:

1) MM is given a priori with inexact parameters and then MEL is being determined for which the inequality (3) is valid [3,4,5];
2) Some MEL is given a priori and then MM is being chosen for which the inequality (3) is valid [7,8].

Algorithm of a Synthesis of AMD Into First Approach

Let's consider some algorithms of synthesis of the adequate mathematical description within the framework of the first approach [3,4,5,9].

Let us consider the set of possible solution of equation (1) $Q_{d,p}$ to take into account the inaccuracy of experimental measurements only:

$$Q_{d,p} = \{ z : \| A_p z - B_p \tilde{u} \| \le \| B_p \| \cdot d_0 = d \}. \qquad (5)$$

This set is unbounded (incorrect problem) if A_p is compact operator [6].
The problem of equation solution (1) is reduced to solution following extreme problem according of regularization method [6]:

$$\Omega[z_p] = \inf_{z \in Q_{\delta,p}} \Omega[z], \qquad (6)$$

where $\Omega[z]$ is stabilizing quasi-monotonic functional [6,9].
The solution of this problem is the stable solution to small change of initial data.
The set of possible solution of equation (1) has to extend to set $Q_{h,d,\delta}$ if take into account the inaccuracy of the operators A_p, B_p [5,9]:

$$Q_{h,d,\delta} = \{z : \| A_p z - B_p \tilde{x} \| \le h \| z \| + d \| \tilde{x} \| + \delta b_0,\ b_0 = \sup_{p \in D} \| B_p \| \}. \qquad (7)$$

Any function from $Q_{h,d,\delta}$ causes the response of mathematical model conterminous to the response of investigated system to an error into which enter an error of experimental measurements and errors of a possible deviation of parameters of a vector $p \in D$. A problem of a finding $z \in Q_{h,d,\delta}$ we shall name by analogy to the previous as a *problem of synthesis of AMD for a class of models* [5,9].
Let's note that the set of the solutions of a problem of synthesis of AMD for a class of modelsat the fixed operators A_p from K_A and B_p from K_B contains elements with unlimited norm (incorrect problem) therefore the size $h \| z \|_Z$ can be infinitely large. Formally such situation is unacceptable as it means that the error of mathematical modeling is equal to infinity, if any function from $Q_{h,d,\delta}$ to use as models of external load. Hence not all functions from $Q_{h,d,\delta}$ will be as good models of external load.
The method the synthesis of AMD for a class of models,where such difficulties were overcome,was suggestedin works [5,9,10].
The models of external load z can are different in this case. They will are depend from final goals of mathematical modelling.
Let's consider the union of sets of the possible solutions $Q_{\delta,p}$ with fixed operators A_p,B_p:

$$Q_\delta^* = \bigcup_{p \in D} Q_{\delta,p} \text{ (} \cup \text{ is the sign of union).} \qquad (8)$$

In some cases as the solution of synthesis of AMD for a class of models we shall accept the stable element of set Q_δ^* instead the set $Q_{\delta,h,d}$:

$$\Omega[z^*] = \inf_{z \in Q_\delta^*} \Omega[z] \qquad (9)$$

This problem can being reduced to more simple extreme problem:

$$\Omega[z^*_{p^0}] = \inf_{p \in D} \inf_{z \in Q_{\delta,p}} \Omega[z]. \qquad (10)$$

The model of EL $z^*_{p^0}$ will be given results of mathematical modeling with operator A_{p^0} ,which coincides with given function $B_{p^0}\tilde{x}$ with inaccuracy δb_0. So the pair A_{p^0} and $z^*_{p^0}$ are the AMD of dynamic systemwhich gives more stable results of mathematical modeling to small change of initial data.
The statement of following problem of MEL by identification method is possible:

$$\Omega[z^*_{p^1}] = \sup_{p\in D} \inf_{z\in Q_{d,p}} \Omega[z]. \quad (11)$$

So the pair A_{p^1} and $z^*_{p^1}$ are the AMD of dynamic system which gives the stable results of mathematical modeling to small change of initial data with maximal value of functional $\Omega[z]$.

The function $z^*_{p^1}$ gives the evaluation from above of all possible solutions of identification problem for all operators A_p, B_p from classes K_A, K_B.

Then the stable model z_{bel} of EL which gives the evaluation from below of the selected response $B_p\tilde{x}$ of dynamic system for all possible operators A_p, B_p can been defined as result of the solution of the following extreme problem:

$$\| A_{b_{bel}} z_{bel} \|^2_U = \inf_{A_b\in K_A, B_b\in K_B} \inf_{z_p} \| A_b z_p \|^2_U, \; b, p \in D, \quad (12)$$

where z_p is the solution of extreme problem (6) on set $Q_{d,p}$. The pair $A_{p_{bel}}$ and $z_{p_{bel}}$ are the AMD of dynamic system which gives the stable results of mathematical modeling to small change of initial data with minimal value of functional $\| A_p z \|^2_U$

The stable model z_{ab} of EL which gives the evaluation from above of the selected response $B_p\tilde{x}$ of dynamic system for all possible operators A_p, B_p can been defined as result of the solution of the following extreme problem:

$$\| A_{b_{ab}} z_{ab} \|^2_U = \sup_{A_b\in K_A, B_b\in K_B} \sup_{z_p} \| A_b z_p \|^2_U, \; b, p \in D. \quad (13)$$

In some cases it is necessary to synthesize model of external load by a method of identification which to give the best results of mathematical modeling for all possible mathematical models of dynamic system motion. Actually such problem is the solution of a problem of a choice of the second component (of the model of external load) for adequacy of mathematical modeling within of the first approach. Such kind of identification problems can find applications in different areas of practice where are using the methods of mathematical modelling [9-10].

The stable model z_{un} of external load which gives the best result of motion of dynamic system with guarantee as the solution of the following extreme problem:

$$\| A_{p_{un}} z_{un} - B_{p_{un}}\tilde{x} \|^2_U = \inf_{p\in D} \sup_{c\in D} \| A_c z_p - B_c\tilde{x} \|^2_U, \; p_{un} \in D, \quad (14)$$

where z_p is the solution of extreme problem (6) on set $Q_{d,p}$ [10].

Function $z_{un} = Q^*_d$ existandisstabletosmallchangeofinitialdata (function $\tilde{x}$), ifthefunctional$\Omega[z]$is stabilizing functional and the function z_{un} is defined unique from (14).

The solution of extreme problem (14) was named as *unitary MEL*.

If the classes K_A ,$K_B$consists from the limited number of operators$K_A = \{A_1, A_2, \ldots A_N\} = \{A_i\}$, $K_B = \{B_1, B_2, \ldots B_N\} = \{B_i\}$, $i = \overline{1,N}$, then the algorithm of finding of the best unitary model of external load z_{un} has the form

$$\inf_{z\in Q_{D,d}} \sup_{p\in D} \| A_p z - B_p\tilde{x} \|_U = \| A_{p_{un}} z_{un} - B_{p_{un}}\tilde{x} \|_U = \min_j \max_i \| A_i z_j - B_i x_d \|_U, \quad (15)$$

where $Q_{D,\delta} = \{z_j : \| A_i z_j - B_i u_\delta \|_U = \delta; j, i = 1,2,\ldots,N\}$.

Method of special mathematical models

For solution of extreme problems (9) - (14) was suggested the method of special mathematical models [9,10]. Let us assumed that operators A_p, B_p are defined by help of the same vector $p \in D$.

Definition. The mathematical model ofprocesswithvectorparameters $p_0 \in D$ will becalledasspecial minimal mathematical modeliftheinequalityis valid [9,10]:

$$\Omega[A_{p_0}^{-1} B_{p_0} x] \le \Omega[A_p^{-1} B_p x]. \qquad (16)$$

for all allowablefunctions $x \in X_\delta \subset X$ $(X_\delta = \{x : \| x - x_\delta \|_X \le \delta\})$ andanyvector $p \in D$, (A_p^{-1} is a inverse operatortotheoperator A_p).

Ifspecial minimal mathematical model exists, thenthe extreme problem (10) canbereplacedbyfollowingmore simple extreme problem:

$$\Omega[z_{p^0}^*] = \inf_{z \in Q_{\delta,p_0}} \Omega[z].$$

The special maximal mathematical model is in a similar way defined also [9,10].

Definition. The mathematical model ofprocesswithvectorparameters $p^1 \in D$ will becalledasspecialmaximalmathematical modeliftheinequalityis valid [9,10]:

$$\Omega[A_{p_0}^{-1} B_{p_0} x] \le \Omega[A_p^{-1} B_p x]. \qquad (17)$$

for all allowablefunctions $x \in X_\delta \subset X$ $(X_\delta = \{x : \| x - x_\delta \|_X \le \delta\})$ andanyvector $p \in D$, (A_p^{-1} is a inverse operatortotheoperator A_p).

If special maxmal mathematical model exists, thenthe extreme problem (11) can be replaced by following more simple extreme problem:

$$\Omega[z_{p^1}^*] = \inf_{z \in Q_{\delta,p^1}} \Omega[z].$$

The examples of use the special mathematical models under synthesis of AMD are given in works [9,11].

Conclusion

In paper the problems of synthesis of adequate mathematical description of real dynamical system are considered. One of possible solution of this problem is way of choice of modelof external loads to dynamical system by identification method. The peculiarities of such approach were investigated. These problems are in correct problems by their nature and so for their solution are being used the regularization method of A.N. Tikhonov. For case when mathematical model are given approximately the different variants of choice model of external loads which are depending from final goals of mathematical modeling (modeling of given motion of system, different estimation of responses of dynamic system, modeling of best forecast of system motion, the most stable model to small change of initial data, unitary model) are considered.

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