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Research Article

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Total Cross Section for Elestic Electron Strontium Scattering

Salisu I. Kunya[1*], Sadiq G. Abdu[2], Muhammad Y. Onimisi[3]

[1]Department of Science Laboratory Technology, Jigawa State Polytechnic, Dutse-Nigeria
[2]Department of Physics, Kaduna State University, Kaduna-Nigeria
[3]Department of Physics, Nigerian Defence Academy, Kaduna-Nigeria

Abstract The result of total cross section for electron strontium collision shows the dependence on energy ranging from 100e V to 1000e V. The numerical calculations were performed with eikonal approximation and the FORTRAN Code developed by Koonin and Meredith (1989). The results obtained were significant at higher energies.

Keywords Total cross section, Scattering, Elestic Electron

1. Introduction

Strontium belongs to group two elements of the periodic table, it has an atomic number of 38, and it is most similar chemically to the heavier alkali earth elements. Strontium is a soft, silvery metallic element found in rock, soil, dust, coal and oil. Strontium found in nature is not radioactive but strontium-90 is a radioactive form of strontium [1]. Electron atom collision are always characterized by the differential cross section, the differential cross section is the main observable in quantum scattering experiments. The notion was introduced first to describe the Rayleigh scattering of sunlight and the Rutherford scattering of alpha particles. In both scattering process, the differential cross section is well established in the framework of the correspondingly dynamical equations: The Maxwell equations in the case of Rayleigh scattering and the Newton's equations in the case of Rutherford scattering. On the other hand, a satisfactory justification of the quantum scattering cross section can be completely described by the framework of the Schrödinger wave equation.

In scattering theory, the total cross section (TCS) is a measure of the probability that an interaction occurs, the larger the cross section the greater the probability that an interaction will take place when a particle is incident on a target [2]. Several processes might occur during collision, one of these processes is elastic scattering, in which the two particles are simply scattered without any change in their internal structure. Inelastic scattering is also one of the processes that can occur during collision, in this type of scattering, the two particles undergo a change in their internal quantal state. The problem of collision between an electron and an atom is of interest in quantum mechanics for which various methods have been found useful in describing the scattering processes, ranging from classical to quantal. Partial wave method is one of the methods applied to problem of scattering of electron by atom, there are various kinds of approximation methods which have been developed in collision theory, and these include Born and eikonal approximation.

2. Scattering Theory

Consider a particle of mass m and energy

$$E = \frac{\hbar^2 K^2}{2m} \rangle 0 \qquad (1)$$

Described by a plane wave

$$\Psi_{in} = e^{ikz} \quad (2)$$

Traveling in the Z- direction that satisfy Schrödinger wave equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = E\Psi \quad (3)$$

The free particle wave function becomes "distorted" in the presence of a potential $V(r)$. the distorted wave function is composed of an incident plane wave and a scattered wave.

$$\Psi_{sc} = e^{ikz} + f(\theta)\frac{e^{ikr}}{r} \quad (4)$$

Equation (4) can be calculated by solving the Schrödinger wave equation. Where $f(\theta)$ is the complex scattering amplitude embodies the observable scattering properties and is the basic function we seek to determine.

Moreover, collisions are always characterized by the differential cross section (that is, measure of the probability distribution) given by:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad (5)$$

This has the simple interpretation of the probability of finding scattered particles within a given solid angle. The total cross section can be obtained by integrating the differential cross section on the whole sphere of observation (4π steradian).

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = \int_0^{2\pi} d\phi \int_o^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega} \quad (6)$$

3. Eikonal approximation

For scattering problems where the potential V(x) is much smaller than the energy, one can make use of the Eikonal approximation in order to solve the problem. This approximation covers a situation in which the potential varies very little over distances of the order of Compton wavelength. This approximation is semi classical in nature; it is essence is that each ray of the incident plane wave suffers a phase shift as it passes through the potential on a straight line trajectory as shown in Fig. 1.were, $r = (b^2+z^2)^{1/2}$.

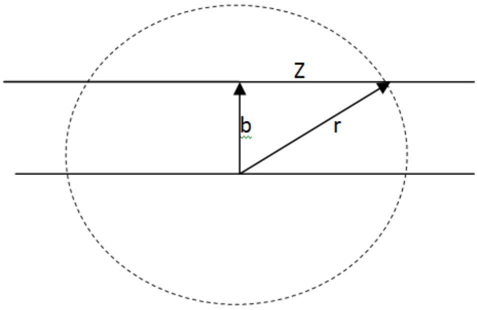

Figure 1: Geometry of Eikonal approximation

The approximation can be derived by using the semi classical wave function

$$\Psi(r) = \phi(r)e^{ik_i.r} \quad (7)$$

Where, ϕ(r) is a slowly-varying function, describing the distortion of the incident wave. The dynamic of the motion can be described by Schrödinger wave equation

$$\frac{-\hbar^2}{2m}\nabla^2\Psi(r) + V(r)\Psi(r) = E\Psi(r) \quad (8)$$

Putting equation (7) in equation (8) give

$$\frac{-\hbar^2}{2m}(2ik_i\nabla + \nabla^2)\phi(r) + V\phi(r) = 0 \quad (9)$$

If we now assume that ϕ(r) varies slowly enough so that the $\nabla^2\phi$ term can be ignored (i.e. k is very large), we have

$$\frac{ik\hbar^2}{m}\frac{\partial}{\partial z}\phi(b,z) = V(b,z)\phi(b,z) \quad (10)$$

Here, we have introduced the coordinate b in the plane transverse to the incident beam, so that;

$$V(b,z) = V(r) \quad (11)$$

From, Fig.1

$r = (b^2 + z^{2)^{\frac{1}{2}}}$ (12)

From symmetry considerations, we expect that Ψ will be azimuthally symmetric and so independent of b. equation (10) can be integrated immediately and using the boundary condition that $\Psi \rightarrow 1$ as $Z \rightarrow \infty$ since there is no distortion of the wave before the particle reaches the potential, we have

$\phi(b,z) = e^{2i\chi(b,z)}$ (13)

$\chi(b,z) = -\frac{m}{2\hbar^2 k}\int_{-\infty}^{\infty} V(b,z')dz$ (14)

Having obtained the eikonal approximation to the scattering wave function, we can now obtain the eikonal scattering amplitude $f(\theta)$, inserting equation (8) in to an exact integral expression for the scattering amplitude.

$f(\theta) = -\frac{m}{2\pi\hbar^2}\int e^{-ik_f.r}V(r)\Psi(r)d^3 r$ (15)

We have,

$f_e = \frac{-m}{2\pi\hbar^2}\int d^2 b\int_{-\infty}^{\infty} dz\, e^{-iq.r}V(b,z)\phi(b,z)$ (16)

Using eqn. (9), we can relate V(r)ϕ(r) directly to $\frac{\partial\phi}{\partial z}$.

Furthermore, if we restrict our consideration to relatively small scattering angles, so that$q_z = 0$, then the Z integral in equation (17) can be done immediately and using eqn. (15) for φ(r), we obtain.

$f_e = -\frac{ik}{2\pi}\int d^2\ be^{-iq.b}$ (17)

With the profile function

$\chi(b) = \chi(b,z=\infty) = -\frac{m}{2\hbar^2 k}\int_{-\infty}^{\infty} V(b,z)dz$ (18)

Since χ is azimuthally symmetric, we can perform the azimuthally integration in equation (17) and obtain our final expression for the eikonal scattering amplitude.

$f_e = -ik\int_0^{\infty} bdbJ_o\,(qb)\left(e^{2i\chi(b)} - 1\right)$ (19)

In deriving this expression, we have used the identity of Bessel function.

$J_o(qb) = \frac{1}{2\pi}\int_0^{2\pi} e^{-iqbcos\phi}\ d\phi$ (20)

Hence, f_e depend upon both E (through K) and q.

An important property of the exact scattering amplitude is the optical theorem, which relates the total cross-section to the imaginary part of the forward scattering amplitude. After a bit of algebra, one can show that f_e satisfied this relation in the limit that the incident momentum becomes large compared to the length scale over which the potential varies.

$\delta = \frac{4\pi}{k}Imf(q=0) = 8\pi\int_0^{\infty} bdbsin^2\chi(b)$ (21)

4. Central Potential

A three dimensional physical systems have a central potential i.e. a potential energy that depends only on the distance r from the origin $V(r) = V(r)$. If we use spherical coordinates to parameterize our three dimensional space, a central potential does not depend on the angular variable θ and Φ. Therefore, in a scattering experiment it is easier to work in the Centre of mass frame, where a spherically symmetric potential has the form V(r) with $r = |\overrightarrow{x}|$, due to the quantum mechanical uncertainty (i.e. we can only predict the probability of scattering in a certain direction).

In Born and eikonal approximation calculations of the scattering of electrons from atoms, in general it is a complicated multi- channel scattering problem, since there are reactions leading to final states in which the atom is excited. However, as the reaction probabilities are small in comparison to elastic scattering, for many purposes the problem can be modeled by the scattering of an electron from a central potential [3]. This potential represents the combined influence of the attraction of the central nuclear charge (Z) and the screening of this attraction by the Z atomic electrons. For a target atom, the potential vanishes at large distances faster than r^{-1}. A very accurate approximation to this potential can be solved for the self-consistent Hartree Fock potential of the neutral atom. However a much simpler estimate can be obtained using an approximation to the Thomas Fermi model of the atom given by Lenz and Jensen (Blister and Hautala, 1979).

$V = -\frac{ze^2}{r}e^{-x}(1 + x + b^2x^2 + b^3x^3 + b^4x^4)$ (22)

With, e^2=14.409, b_2=0.3344, b_3=0.0485, b_4=2.647$\times 10^{-3}$, and x=4.5397$Z^{1/6}$ $r^{1/2}$

The potential is singular at the origin, However, if the potential is regularized by taking it to be a constant within some small radius $r_{min,}$ (say the radius of the atom 1s shell), the calculated cross section will be unaffected except at momentum transfers large enough so that

$$Qr_{min} \gg 1 \qquad (23)$$

The incident particle is assumed to have the mass of the electron and is appropriate for atomic systems; all lengths are measured in angstrom (Å) and all energies in electron volt (eV). The potential is assumed to vanish beyond 2Å. Furthermore, the r^{-1} singularity in the potential is cut off inside the radius of the 1s shell of the atom.

5. Methodology

The computation of Eikonal approximation to the total cross section of strontium for a given central potential at specified incident energy, a FORTAN program developed by Koonin and Meredith (1989) have been used. The program is made up of four categories of file: common utility program, physics source code, data files and include files [3].

The physics sources code is the main sources code which contains the routine for the actual computation. The data files contain data to be read into the main program at run-time and have the exertion. DAT. The first thing done was the successful installation of the FORTRAN codes in the computer. This requires familiarity with the linker, editor and the graphics package to be used in plotting. The program runs interactively. It begins with a title page describing the physical problem to be investigated and the output that will be produced; next, the menu is displayed, giving the choice of entering parameter values, examining parameter values, running the program or terminating the program. When the calculation is finished, all values are zeroed (except default parameter), and the main menu is redisplayed, giving us the opportunity to redo the calculation with a new set of parameters or to end execution. Data generated from the program were saved in a file which would be imported into the graphics software for plotting [4-5].

Table 1: Table of total cross section using eikonal together with data obtained from Born and NIST SRD 64

Energy (eV)	**Approximation**		
	Eikonal	**Born**	**NIST**
10	4.494	301.70	
20	5.006	208.80	
30	7.087	161.60	
40	5.788	132.30	
50	4.707	112.30	18.933
60	3.923	97.65	17.115
70	3.305	86.48	15.535
80	3.187	77.64	14.197
90	3.155	70.47	13.071
100	3.246	64.53	12.122
200	3.469	35.20	7.472
300	3.567	24.26	5.814
400	3.374	18.39	4.927
500	3.559	15.09	4.347
600	4.057	13.14	3.925
700	3.788	11.42	3.598
800	3.704	9.997	3.334
900	3.534	8.945	3.114
1000	3.496	8.059	2.929

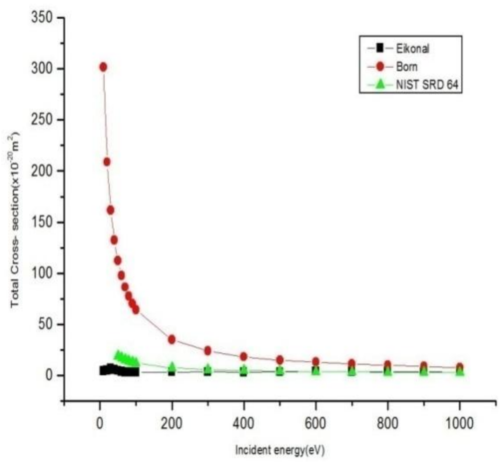

Figure 2: Graph of total cross section using eikonal together with data obtained from Born and NIST SRD 64

6. Discussion

Fig. 2 shows that, the present result and NIST SRD 64 data is much closer and converges at incidence energy above 400 eV, but in comparison with the Born approximation, the total cross section is high at lower energy, this indication shows that it valid at higher energy. Again, as we observed the curve for Born approximation is superior to the other curves. Hence the present result is in agreement to NIST SRD 64 and Born approximation at higher energy. This is because an eikonal approximation, valid at high energies and small scattering angles.

7. Conclusion

The computation of total cross section were carried out at impact energies ranging from 1.0 to 1000.0eV, using FORTRAN Source Code of Koonin and Meredith, The eikonal result and NIST SRD (64) show good agreement, the eikonal and Born approximation there is a discrepancy from 1.0eV to 400.0eV.

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