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***Journal of Scientific and Engineering Research*, 2016, 3(5):77-84**

ISSN: 2394-2630
CODEN(USA): JSERBR

Research Article

A Conceptual Constitutive Model for Structured Soils

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Abstract A conceptual constitutive model is presented for consolidation and yield of structured clays. The effect of the soil structure is introduced in the model by variable parameters for slope of isotropic compression curve, pre-compression pressure and the slope of the critical state line. Generalized thermodynamic principals with the specific form of free energy and dissipative energy functions are used to derive the yield function and elastic-plastic response of the model. Free energy and dissipative energy functions are introduced by an additional term related to the work of the additional volumetric plastic strain due to the soil structure and the disturbed state concept is employed to derive the incremental response of the model.

Keywords Constitutive Model, Structured Soils, Cam-Clay, Thermodynamics, DSC

Introduction

Mechanical behavior of natural soils is different from their behavior in remolded state in the laboratory. Natural soils usually possess structures depending on how they are formed, loading history and possible cementation. They always show anisotropic response under loading and cohesion [1, 2]. The term "destructuration" is introduced to explain the plastic behavior of natural clay due to disruption of its natural structure [3]. The difference between structured and destructured soil can be due to the fabric of the soil mass, consisting of the spatial arrangement of soil particles and inter-particle contacts which may causes anisotropic response, and; bonding between particles, which can be progressively destroyed during plastic straining [4]. In this paper, the effect of the second character of natural soils in constitutive modeling is considered. The presence of bonding between soil particles provides an additional strength against of failure in the soil. Experimental results show that the compression curves for natural clay always are above the curves of same clay in reconstructed state in odometer tests. Increasing the load causes the compression curve of the natural clay to converge with the compression curve of the destructured clay. This means that the bonding between soil particles is progressively destroyed during plastic straining. There are many different physical causes of bonding in soils and rocks related to mineralogy and arrangement of its particles and cementation, but the effects on mechanical behavior is remarkably similar. There are several elastic/plastic constitutive models presented for structured soils based on classical theory of plasticity and experimental results such as the reported works by Gens and Nova (1993), Rouainia and Muir Wood (2000), Kavvadas and Amorosi (2000), Baudet and Stallebrass (2001), Gajo and Muir Wood (2001). Their works differs in the precise form of destructuration law applied and in the form of the underlying reference model used for the remolded material [5-9].

Gens & Nova (1993) presented a general framework for incorporating bonding and destructuration within elasto-plastic constitutive models. In addition to the real yield surface for the natural material a notional "intrinsic yield surface" is introduced, to represent the size yield surface would be if there were no structure effect [6]. The difference in size of the real yield surface and the yield surface for remolded soil is a measure of the bonding effect. Increase in size of the intrinsic yield surface is related to plastic strain increments by a conventional hardening law for reconstituted soil, while the reduction of bonding effect is related to plastic strain increments by a destructuration law.

Liu and Carter (2002) presented a constitutive model called “Structured Cam-Clay” to predict the elastic/plastic behavior of structured or natural clays. Their constitutive model is a family of Modified Cam-Clay models incorporating the effects of the soil structure in hardening/softening and flow rules. They introduced three additional parameters to MCC model [10].

In this paper a different approach is used to establish constitutive relationships for yield criteria and flow rule for structured or natural clays based on generalized thermodynamics. The thermodynamics principles are using in constitution of elastic/plastic models for geomaterials. Houlsby (1981 and 2000), Maugin (1992), Puzrin and Houlsby (2000, 2001), Collins (2002) and, Collins and Hilder (2003) developed this new approach to established new sophisticated constitutive models for clays and sands [11-19].This method is based on two main functions, Helmholtz or Gibbs free energy function and Dissipative energy function. Free energy function is related to elastic plastic response of material under loading and Dissipation function is related to plastic behavior of material. When these two functions specified, then the yield locus and flow rule can be determine based on these two function.

Materials and Methods

Compression of strcutured soils

As explained in the previous section almost all of the natural soils possess a structure or bonding due to its particles arrangement, chemical reactions, cementation and, other causes. Failure of these soils begins with braking bonds or destructing their structure. Difference between isotropic compression behaviors of remolded and intact samples of a same soil indicates the effect of its structure. Slope of the isotropic consolidation line for structured clay is more than once for a remolded sample of the same clay in low stress but it converges to the destructured one as the pressure increases; this is shown is Fig. 1.

Failure of these soils starts with destruction of its structure and when its structure removed, it becomes same as remolded samples. The compression behavior of structured clays can be introduced by an additional volumetric plastic strain in v-lnp plain [8]. Also the distributed stated concept (DSC) presented by Desai (2001) is used to predict the compression behavior of structured clays [8, 20].

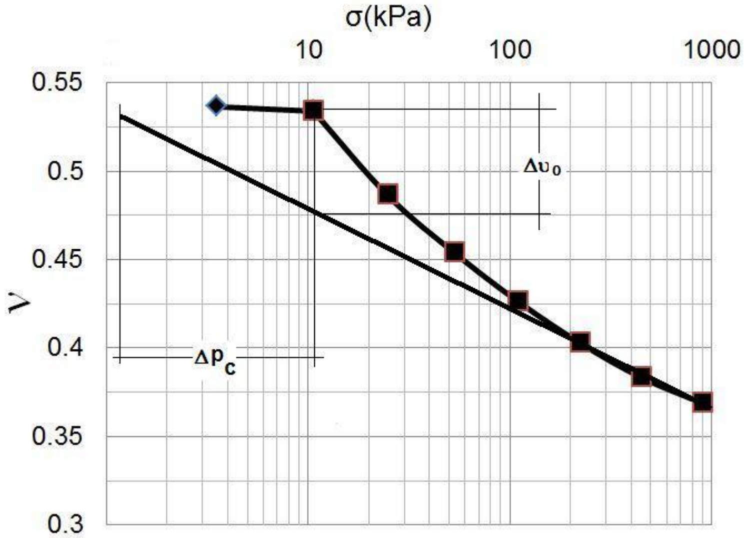

Figure 1: Stress-strain behavior of a structured soil in semi-logarithmic scale

Liu and Carter (1999, 2001) proposed the flowing relation for the additional plastic compression of structured clays [8].

$$\Delta \bar{v} = \Delta \bar{v}_0 \left(\frac{p}{p_y} \right)^{-b} \quad (1)$$

This volumetric strain is totally plastic and should be superimposed to the strains calculated with remolded soil parameters. $\Delta\upsilon_0$, and b are additional material parameters that can be determined by two isotropic consolidation tests on remolded and virgin samples.

In traditional critical state soil mechanics, the main assumption is that there is a semi-logarithmic relationship between confining pressure and volumetric strain as follow:

$$\Delta v = v_0 + \lambda \ln(\frac{p}{p_r}) \tag{2}$$

Butterflied proposed a fully logarithmic relationship between confining pressure and volumetric strain [11]. Therefore, the equation (2) can be rewritten as follow:

$$\ln(v) = c + \lambda^* \ln(\frac{p}{p_r}) \tag{3}$$

in which:

$$\lambda^* \approx \frac{\lambda}{v} \tag{4}$$

Comparing Equations (1) and (3) show that the fully logarithmic relationship for structured soils is in accordance with Butterflied proposed model.

Implementing the additional volumetric strain due to soil structure in equation (3) results:

$$\ln(v) = \ln(v_0) + \lambda^* \ln(\frac{p}{p_r}) + \ln(\Delta v_0) + B\ln(p) - B\ln(p_y), \quad \mathrm{B} = b\,\bar{\Delta} v_0 \tag{5}$$

Rewriting the above equation and defining a new reference pressure results:

$$\ln(v) = A + \lambda^{**} \ln(\frac{p}{p_r^*}) \tag{6}$$

In the above equation parameter lambda is variable and changes as the stress level changes as follow:

$$\lambda^{**} = \lambda^* + \alpha(\frac{p}{p_r^*})^{-b} \tag{7}$$

Application of variable λ in free energy functions results in complicated integral equation that is required in derivation of elastic-plastic response of the model. In order to avoid complicated mathematical procedure, disturbed state concept (DSC) can be used to find a simple solution. In the DSC framework, the response of material is considered a mixture of the two interacting material parts in the RI and FA states. The RI and FA states are termed as reference states. Then the observed or actual response of the material is expressed in terms of the responses of material parts in the reference states. The disturbance, D, denotes the deviation of the observed response from those of the reference states. Figure 3 shows a symbolic and schematic representation of disturbance in the DSC. The observed or average response (denoted by a) is then expressed in terms of the RI response (denoted by i) and the FA response (denoted by c) by using the disturbance function, D, as an interpolation and coupling mechanism. The behavior of the RI and FA materials, as well as the disturbance function, needs to be defined from laboratory tests or appropriate methods [20, 21].

Because the loading causes the structure of the soil to remove, then the behavior of the structured material tends to the behavior of the remolded material as the stress level increases. Therefore; the initial state of the material can be considered as intact state and the final state of the material can be considered as fully disturbed state and a state function can be used to define the response of the material between these two known states. Using equation (7) to calculate the value of λ at reference pressure results:

$$\lambda^{**} = \lambda^* + \alpha(\frac{p}{p_r})^{-b}\bigg|_{p=p_r} = \lambda^* + \alpha = \lambda^{**}{}_{RI} \tag{8}$$

In which λ** RI is the initial value of λ** in fully logarithmic scale. Increasing the pressure causes to remove the additional amount of the lambda and results:

$$\lambda^{**} = \lambda^{*} + \alpha(\frac{p}{p_r})^{-b}\Big|_{p>>p_r} = \lambda^{*} = \lambda^{**}{}_{FA} \qquad (9)$$

Therefore, at each stress level the value of lambda for structured soil can be introduced based on disturbed state concept.

Referring to Figure 1, there is an additional amount of pressure required to produce the same void ratio or volumetric strain including plastic strain. This additional force causes the elastic region of structured soil to increase in compression with remolded state of the same material. Therefore, an additional parameter is required to apply the bigger size of the structured soil. Since the precompressions pressure is a state function related to plastic volume change. Since parameter λ includes the effect of plastic volumetric straining, therefore; similar state function can be used to describe the larger size of the yield locus. The additional force due to soil structure vanishes as the stress increase; therefore in the fully disturbed state the precompression pressure will be identical to the remolded soil;

$$p_c{}^{*} = p_c + \Delta p_c(\frac{p}{p_r})^{-b}\Big|_{p=p_r} = p_c + \Delta p_c = p_{cRI} \qquad (10)$$

$$p_c{}^{*} = p_c + \Delta p_c(\frac{p}{p_r})^{-b}\Big|_{p>>p_r} = p_c = p_{cFA} \qquad (11)$$

A new parameter is defined as:

$$\Delta p^{*}{}_{c} = \left(\frac{p}{p_r}\right)^{-b} \Delta p_c \qquad (12)$$

Using equation (10), (11) and (12), the stress strain behavior and failure of structured soils can be applied in calculations by interpolating the actual behavior using the fully remolded and intact structured behaviors of the same soil.

In this paper the main aim is to obtain an elastic/plastic constitutive model for structured soils based on the observed stress-strain behavior of these soils employing the generalized thermomechanical principals. In order to preserve the simplicity, the anisotropic behavior of structured soils is not investigated.

Constructing Constitutive Model based on the Thermodynamics of plasticity

In the recent years, application of the generalized thermodynamics offered a powerful method to study the behavior of elastic/plastic solids. This new approach is based on the first and second laws of the thermomechanics. Constitutive models based on this framework obey the lows of the thermomechanics while many of other models based on classical theory of plasticity violate these principles. The main advantage of this approach is that only two functions a) Free energy function and b) Dissipative energy functions are require to derive all of the constitutive relations need to describe the yield locus in stress space, flow rule, hardening/softening rule and incremental response of the material. Another advantage is that the models formulated in the thermomechanical framework don't need to check for the Drukers' and Ilyushin's normality and stability postulates; because these are local forms of the second low of the thermomechanics [17]. Recently it has been demonstrated that the convexity of the yield locus of the constitutive models based on the thermomechanical framework is not an essential requirement [19].

The main relation of the thermomechanics of isothermal deformations is [18]:

$$\delta w = \delta\psi + \delta\phi \qquad (13)$$

Where $\delta w = \sigma_{ij}\varepsilon_{ij}$ is the incremental work and ψ and ϕ are Helmholtz free energy and dissipative energy increments.

In this approach first Helmholtz or Gibbs free energy function must be specified based on the stress-strain behavior of the material. If one of these functions determined, another one can be found by a Legendre transform. This function will be used to determine the elastic response of the material. Free energy is a function of state variables (such as precompressions pressure) and elastic and plastic strains as kinematic and internal variables and lambda and kappa. Dissipative energy function determines the energy dissipated by the plastic deformation. Dissipation is function of the rate of the plastic strains in addition to the elastic and plastic strains. For rate independent materials the dissipative function must be a homogenous degree one function of plastic strain rates [16]. This function can be an explicit or implicit function of stress. Free energy function for critical state based material can be:

$$\psi = \psi(v, \varepsilon, \alpha_{ij}) \quad , \quad \phi = \phi(v^p, \varepsilon^p, \alpha, \dot{\alpha}) \tag{14}$$

Where; α is internal variable (equivalent to plastic strain) tensor.

Free energy function can be expressed as the sum of a function of only elastic strains, plus a function of only plastic strains.

$$\psi = p_r \kappa^* \exp\left(\frac{v - v_p}{\kappa^*}\right) + \frac{3}{2} G(\varepsilon - \varepsilon_p)^2 + p_r(\lambda^* - \kappa^*) \exp\left(\frac{\ln(\frac{\Gamma}{V_0} + v_p)}{(\lambda^* - \kappa^*)}\right) \tag{15}$$

That can be rewritten in the following form:

$$\psi = \psi_1(v^e, \varepsilon^e) + \psi_2(v^p, \varepsilon^p) \tag{16}$$

The subject of this paper is not to prove these relations. Prefect information can be found in Houlsby, Houlsby and Puzrin, and, Houlsby and Collins [11-16, 18-19].

Houlsby (1981) derived the free energy and dissipative energy functions for MCC family of constitutive models. These equations are derived assuming that the compression curve of clay is linear in ln(v)-ln(p) plane which is a realistic assumption [11]. Also Collins and Hilder (2002) suggested a general form for free energy function [19].

For MMC models without structure in which ψ1 and ψ2 are as follows:

$$\psi_1 = \int p dv_e + \int q d\varepsilon_e \tag{17}$$

$$\psi_2 = \int \pi dv_p \tag{18}$$

Where; π is the dissipative stress that its work is dissipated during plastic volumetric straining.

And dissipative energy functions:

$$\phi = \pi \sqrt{(\dot{v}_p)^2 + (M \dot{\varepsilon}_p)^2} \tag{19}$$

In order to apply the effect of the soil structure in the free energy function we should introduce the additional work need to destroy the soil structure. Referring to figure 1, in order to attain the same plastic volumetric strain for a structured soil which is produced by pc in destructured state, an additional stress is required which has shown by Δpc in figure 1. As stated in the previous section, elastic behavior of a structured soil is same as elastic behavior of same soil in destructured state. Therefore; the effect of this additional stress should be applied in the second part of the free energy function where the plastic effect is applied.

Therefore; the free energy function for structured soil will be as follows:

$$\Psi = \int p dv_e + \int q d\varepsilon_e + \int \pi(v_{p1}) dv_{p1} + \int \pi'(v_{p2}) dv_{p2} \tag{20}$$

Where: vp1, vp2 and π, π' are volumetric plastic strains and their dual dissipative stresses for destructured soil and additional plastic strain due to soil structure respectively.

Based on the Equation (3) and relationships of the critical state soil mechanics, vp2 and π' can be expressed in the term of v and vp1. Therefore, the Equation (20) can be rewritten in the flowing form:

$$\Psi = \int p dv_e + \int q d\varepsilon_e + \int \pi(v_p) dv_p + \int \pi'(v_p) dv_p \quad (21)$$

In a same manner the dissipative energy function should be modified to include the effect of the soil structure. We assumed that the structure of the soil affects the volumetric part of the dissipative energy, therefore; the following form for dissipative energy function is proposed:

$$\phi = \pi \sqrt{\left((1+\frac{\pi'}{\pi})\dot{v}^{p}\right)^2 + (M^* \dot{\varepsilon}^{p})^2} \quad (22)$$

Consequently, we will show that the parameter M* is slightly different than once in MCC model. Now, based on equations (21) and (22), the yield locus and flow rule can be derived.

In the generalized thermodynamics of isothermal processes, the flowing relationships are between internal variables [11];

$$\frac{\partial \Psi}{\partial v_p} + \frac{\partial \Phi}{\partial \dot{v}_p} = 0 \quad (23)$$

$$\frac{\partial \Psi}{\partial \varepsilon_p} + \frac{\partial \Phi}{\partial \dot{\varepsilon}_p} = 0 \quad (24)$$

Differentiating the equations (23) and (24) and combination them according to equations results the yield locus and the flow rule for structured soil as follows:

$$\left(\frac{p-\pi-\pi'}{\pi+\pi'}\right)^2 + \left(\frac{q}{M(\pi+\pi')}\right)^2 = 1 \quad (25)$$

$$\frac{\dot{\varepsilon}_p}{\dot{v}_p} = \frac{q}{M^{*2}(p-\pi-\pi')} \quad (26)$$

Implementing the Equation (12) in Equations (25) and (26), results the yield locus and flow rule for structured soils in DSC framework:

$$\left(\frac{p-(Pc+\Delta p_c^*)}{(Pc+\Delta p_c^*)}\right)^2 + \left(\frac{q}{M^*(Pc+\Delta p_c^*)}\right)^2 = 1 \quad (27)$$

$$\frac{\dot{\varepsilon}_p}{\dot{v}_p} = \frac{q}{M^{*2}(p-(Pc+\Delta p_c^*))} \quad (28)$$

Elastic-plastic incremental response of the model based on Equations (7), (11), (27) and (28) can be derived as follow:

$$\begin{bmatrix} \dot{v} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} \frac{\kappa^*}{p'} & 0 \\ 0 & \frac{1}{3G} \end{bmatrix} \begin{bmatrix} \dot{p}' \\ \dot{q} \end{bmatrix} + \frac{\lambda^{**} - \kappa^*}{M^{*2} p^{2\prime} + q^2} \begin{bmatrix} \frac{M^{*2} p^{2\prime} - q^2}{p'} & 2q \\ 2q & \frac{4q^2 p^{2\prime}}{M^{*2} p^{2\prime} - q^2} \end{bmatrix} \quad (29)$$

where; λ** is a function of p, disturbance function and can be calculated by Equation (7). Other parameters are identical to those of MCC critical state model.

Figure (2) shows a typical yield locus for a structured soil in comparison with the yield locus of the same soil in remolded state.

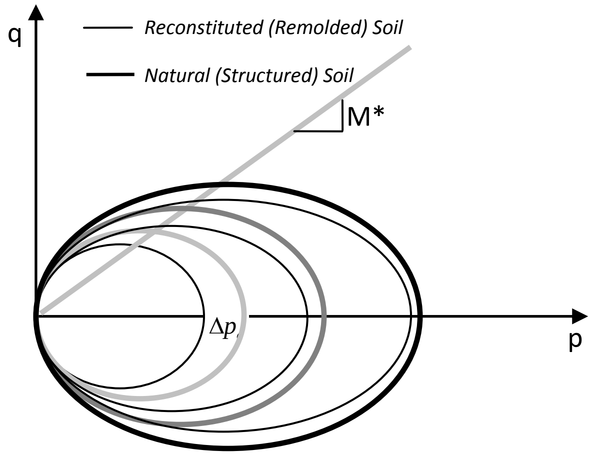

Figure 2: yield locus for structured and corresponding remolded soils

Conclusion

In this paper a conceptual constitutive model presented for simulating the elastic-plastic behavior of natural soils. This model employs the Disturbed State Concept to implement the effect of the soils' structure destruction in the calculations. Failure criteria and incremental relations of the model derived introducing three new parameters, based on DSC. DS Concept employed to simplify the mathematical formulation of the model. The model can be implemented in numerical calculations using original Modified Cam Clay model with a slight modification. The presented model can be used to simulate the behavior of natural and collapsible soils as well.

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