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***Journal of Scientific and Engineering Research*, 2016, 3(5):101-104**

ISSN: 2394-2630
CODEN(USA): JSERBR

Research Article

On The Linear Transformation of Division Matrices

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Abstract In this study, we deal with functions from the square matrices to square matrices, which the same order. Such a function will be called a linear transformation, defined as follows:

Let $M_n(R)$ be a set of square matrices of order n, $n \in S$, and A be regular matrix in $M_n(R)$, then the special function

$$T_A: M_n(R) \to M_n(R)$$

$$X \to T_A(X) = \frac{X}{A}$$

is called a linear transformation of $M_n(R)$ to $M_n(R)$ the following two properties are true for all $X,Y \in M_n(R)$, and scalars $\alpha \in R$:

i. $T_A(X+Y) = T_A(X) + T_A(Y)$. (We say that T_A preserves additivity)

ii. $T_A(\alpha X) = \alpha T_A(X)$ (We say that T_A preserves scalar multiplication)

In this case the matrix A is called the standard matrix of the function T_A.

Here, we transfer some well known properties of linear transformations to the above defined elements in the set all { T_A: A regular in $M_n(R)$} [1].

Keywords Linear, transformation, division, matrices, operations.

1. Introduction

We investigate the properties of some special square matrices by using the following division operation;

$$\frac{A}{B} = \frac{1}{|B|}\left[\left({}^{A}_{B}i_j\right)_{ji}\right]_n \text{, if } A, B \in M_n(R) \text{ and } \det(B) \neq 0.$$

Here ${}^{A}_{B}i_j$ is real number obtained by the determinant of writing i^{th} column of A matrix in j^{th} column of B.

Let A be $n \times n$ an regular matrix. The function defined T_A by

$$T_A(X) = \frac{X}{A} = \frac{1}{|A|}\left[\left(\begin{vmatrix} a_{11} & \cdots & a_{1(j-1)} & x_{1i} & a_{1(j+1)} & \cdots & a_{1n} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{n1} & \cdots & a_{n(j-1)} & x_{ni} & a_{n(j+1)} & \cdots & a_{1n} \end{vmatrix}\right)_{ji}\right]_n$$

Two simple linear transformations are the zero transformation and the identity transformation, which are defined as follows.

i. $T_A(0) = 0$, *for* $0 \in M_n(R)$ Zero transformation.

ii. $T_I(X) = X$, *for* $X \in M_n(R)$ Identity transformation.

iii. $T_A(A) = I$, *for* $A \in M_n(R)$.

$$0,\ T_I,\ T_A^{-1} \in \{T_A: A \text{ regular in } M_n(R)\}.$$

2. Properties of Linear Transformations The Linear Transformation of Division Matrices

This property is true for all linear transformations, as stated in the first property of the following theorem.

Theorem 2.1. Let T_A be a linear transformations from $M_n(R)$ into $M_n(R)$, where X and Y are in $M_n(R)$. Then the following properties are true.

$$T_A: M_n(R) \rightarrow M_n(R)$$
$$X \rightarrow TA\ (X) = \frac{X}{A}$$

i. $T_A\,(0) = 0.$

ii. $T_A\,(X\text{-}Y) = T_A(X) - T_A(Y).$

iii. If $S = \sum_{i=1}^{n} \alpha_i X_i$, then
$T_A(S) = T_A(\sum_{i=1}^{n} \alpha_i X_i) = \sum_{i=1}^{n} \alpha_i T_A(X_i).$

iv. $T^{-1}{}_A(X) = \frac{A}{X}.$

Proof:

i. To prove the first property, note that $0A = [0]$. Then it follows that

$$T_A([0]) = T_A([0.X]) = 0T_A(X) = [0].$$

ii. The second property follows from $-X = (-1)X$, which implies that

$$T_A\,(-X) = T_A((-1)X) = (-1)T_A(X) = -T_A(X)$$

iii. The third property follows from $X\text{-}Y = X + (-1)Y$, which implies that

$$T_{T_A}(X-Y) = T_A(X+(-1)Y) = \frac{X}{A} + \frac{(-1)Y}{A} = T_A(X) - T_A(Y)$$

iv. It is clearly

$$T^{-1}{}_A(X) = \left(\frac{X}{A}\right)^{-1} = \frac{A}{X}.$$

Example 1: Show that the linear transformation.

$$T_A: M_n(R) \rightarrow M_n(R)$$

represented if the matrices

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \text{ and } X = \begin{bmatrix} a & c \\ \mathrm{b} & d \end{bmatrix}$$

Then

$$T_A(X) = \begin{bmatrix} a\cos\theta + b\sin\theta & c\cos\theta + d\sin\theta \\ b\cos\theta - a\sin\theta & d\cos\theta - c\sin\theta \end{bmatrix}$$

An operator that maps respectively each point (a, b) and (c, d) on the plane onto its image point

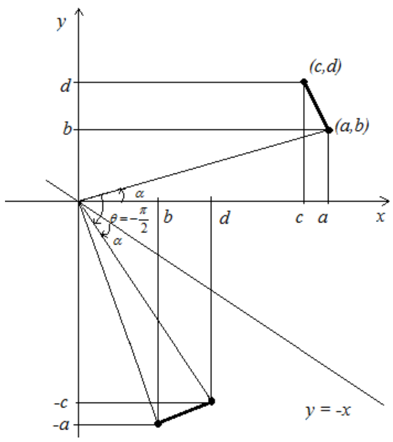
$$a\cos\theta + b\sin\theta\,, b\cos\theta - a\sin\theta$$

and

$$c\cos\theta + d\sin\theta, d\cos\theta - c\sin\theta.$$

If $\theta = \frac{\pi}{2}$ then $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and

$$T_A(X) = \begin{bmatrix} b & d \\ -a & -c \end{bmatrix}$$



These matrices map all points P (x, y), x, $y \in$ R in the plane to points Q (y, $-x$), and they are one-to-one mappings of plane onto itself that are called inverse reflections of the plane. All points on the plane is mapped on to its "mirror image" with respect to one of the coordinate axes bye these inverse reflection matrices. The line segment from A (a,b) to B(c,d) reflects line segment from A'(b,$-a$) to B'(d, $-c$).

Lemma 2.1. Let $T_{A^T}: M_n(R) \rightarrow M_n(R)$ be the function that maps an matrix to its transpose. That is

$$T_{A^T}(X) = \frac{X}{A^T}.$$

Consider a inverse rotation of the plane about the origin through an $-\theta$. The A rotation matrix of this transformation is

$$\begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$

And $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ then

$$T_{A^T}(X) = \frac{X}{A^T} = \begin{bmatrix} a\cos\theta - b\sin\theta & c\cos\theta - d\sin\theta \\ b\cos\theta + a\sin\theta & d\cos\theta + c\sin\theta \end{bmatrix} = AX.$$

Then, since

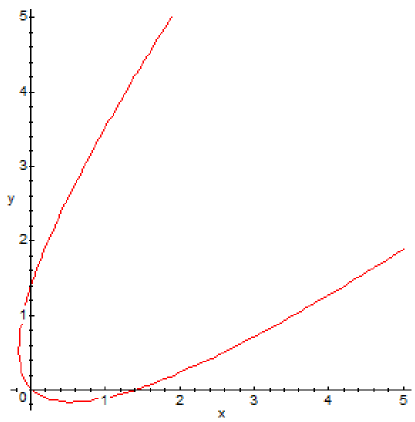
$$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2,$$

And,

$$T_{AA^T}(X) = X$$

Example 1.1. Determine the equation satisfied by the set of image points of the locus of

$$x^2 - 2xy + y^2 - \sqrt{2x} - \sqrt{2y} = 0$$

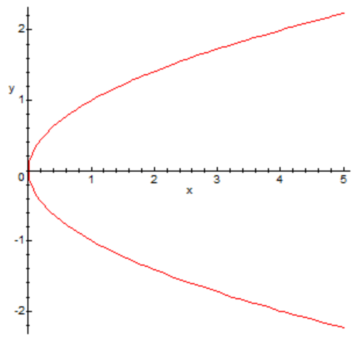


under a rotation of the plane about the origin through an angle

$$\theta = -\frac{\pi}{4} \Leftrightarrow A = \begin{bmatrix} a\frac{\sqrt{2}}{2} - b\frac{\sqrt{2}}{2} & c\frac{\sqrt{2}}{2} - d\frac{\sqrt{2}}{2} \\ b\frac{\sqrt{2}}{2} + a\frac{\sqrt{2}}{2} & d\frac{\sqrt{2}}{2} + c\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\left(a\frac{\sqrt{2}}{2} - b\frac{\sqrt{2}}{2}\right)^2 - 2\left(a\frac{\sqrt{2}}{2} - b\frac{\sqrt{2}}{2}\right)\left(b\frac{\sqrt{2}}{2} + a\frac{\sqrt{2}}{2}\right) + \left(b\frac{\sqrt{2}}{2} + a\frac{\sqrt{2}}{2}\right)^2 - \sqrt{2}\left(a\frac{\sqrt{2}}{2} - b\frac{\sqrt{2}}{2}\right) - \sqrt{2}\left(b\frac{\sqrt{2}}{2} + a\frac{\sqrt{2}}{2}\right) = 0$$

$$2b^2 - 2a = 0 \Leftrightarrow x = y^2$$



References

[1]. H. Keleş, "The Rational matrices", 3rd Conference on Nonlinear Science and Complexity, (2014), 58-60.

[2]. L. Ron, "Elementary Linear Algebra", The Pennsylvania State University The Behrend College, 7th Edition, Printed in the United States of America, (2013).