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INFLUENCE OF AT WINGLETS WINGTIPS ON THE INDUCTIVE REACTANCE OF THE WING

ВЛИЯНИЕ ЗАКОНЦОВОК ТИПА “AT WINGLETS” НА ИНДУКТИВНОЕ СОПРОТИВЛЕНИЕ КРЫЛА

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Abstract. In this article by the method of equations of steady horizontal flight the impact of AT winglets wingtips on the inductive reactance of the wing is explored. The total aerodynamic force, created by all four parts of the wingtips is determined. The vector of the total aerodynamic force of the wingtips is represented in the form of components of a linked coordinate system. Equilibrium equations for the steady motion of an aircraft with AT winglets wingtips in a horizontal flight are recorded. From these equations it is obtained that, in the direction of motion, the longitudinal component of the vector of the total aerodynamic force of the tips reduces the inductive drag of the wing, the vertical component is added to the lifting force of the wing, and the lateral component, due to symmetry, is zero. It is shown that the lifting force created by the AT winglets wingtips is larger compared to the same ones created by the upper tips. An important consequence is that the distribution of the lifting force over the wing span with double tips is more even in comparison with the wing without a tip or with a wing with the upper wingtip. The expression for the effective aspect ratio of the wing with the tips is determined, which is greater than the inductive drag of the wing without the tip. An interesting result is obtained: for identical aircraft weights, the product of effective aspect ratio of the wings with double and only upper wingtips on their coefficients of inductive resistance is equal to the product of the wing aspect ratio without a tip on its coefficient of inductive resistance. This law can also be formulated as follows: at the constant weight of the aircraft, the effective aspect ratio of the wing with the tips by its coefficient of inductive resistance is a constant value, independent of the form of wingtips.

Аннотация. В статье методом системы уравнений установившегося горизонтального полета исследуется влияние законцовок “AT winglets” на индуктивное сопротивление крыла. Определяется полная аэродинамическая сила, созданная всеми четырьмя частями законцовок. Вектор полной аэродинамической силы законцовок представляется в виде компонент в связанной системе координат. Приведены уравнения равновесия установившегося движения самолета с законцовками крыла типа “AT winglet” в горизонтальном полете. В итоге, из этих уравнений получено, что, направленная в сторону движения, продольная компонента вектора полной аэродинамической силы законцовок уменьшает индуктивное сопротивление крыла, вертикальная компонента увеличивает подъемную силу крыла, а боковая компонента, за счет симметрии, равна нулю. Показано, что подъемная сила, созданная законцовками типа “AT winglets”, больше по сравнению с теми же, созданными верхними законцовками. Сделан важный вывод о том, что распределение подъемной силы по размаху крыла с двойными законцовками более равномерное по сравнению с крылом без законцовок или же с крылом с верхними законцовками. Определяется выражение эффективного удлинения крыла с законцовками, значение которого больше значения индуктивного сопротивления крыла без законцовок. Получен интересный вывод: при одинаковом весе самолетов произведение эффективных удлинений крыльев с двойными и только верхними законцовками и коэффициентов индуктивного сопротивления равны произведению удлинения крыла без законцовок и его коэффициента индуктивного сопротивления. Этот закон можно сформулировать еще следующим образом: при условии постоянства веса самолета, произведение эффективного удлинения крыла с законцовками и его коэффициента индуктивного сопротивления есть величина постоянная, не зависящая от вида законцовок.

Keywords: AT winglet, mathematical model, effective extension, inductive reactance, steady motion.

Ключевые слова: AT winglet, математическая модель, эффективное удлинение, индуктивное сопротивление, установившееся движение.

Inductive resistance of the wing is associated with the finiteness of the wing span and arises from the flow of air through the end sections, perpendicular to the main one — in front of the incoming flow. Being inversely proportional to wing span, it decreases with increasing span. When designing the constructors in every way try to reduce the drag of the aircraft, which leads to an increase in the aerodynamic quality of the device. In this case, a decrease in the inductive drag of the wing is also one of the first to occur. This, at least, can be achieved by increasing the wing span, or by preventing the flow of air through the end sections of the wing. Thus, the idea of using wingtips (winglets) arose. The wingtips have been used in aviation since the seventies of the last century, and bring tremendous fuel savings for the year. However, theoretical studies in this field began in 2012 in Azerbaijan in the National Aviation Academy [1–3]. In the mentioned studies, the influence of vertical wingtips (IWT) on the inductive resistance is investigated. Various mathematical models have been constructed that reflect the influence of the IWT on aerodynamic characteristics. In the present study, similar questions are solved for a wing with wingtips such as Advantage Technology winglets [4].

The inductive velocity V_i is directed along the normal to the vector velocity of the oncoming undisturbed flow V_∞. This motion is superimposed on the forward flow, which comes from the front, and causes the current lines on the upper surface of the wing to deviate toward the center of the wing by the angle of the bevel [5–7]:

$$\Delta\alpha = \arg tg V_i / V_\infty .$$

In the case of a subsonic flight regime with a non-breaking flow around the wing, the coefficient of inductive resistance is expressed by the formula [5–7]:

$$C_{xi} = \frac{C_y^2}{\pi\lambda}(1+\delta)$$

where $\delta = 20{,}41\delta_1(m)\delta_2(\eta_{kp})$ is small value, depending on the shape of the wing in the plan. For a rectangular wing. $\delta_{cp} \approx 0{,}05$. For other wings this value is even smaller. In the particular case of an elliptical wing in plan $\delta = 0$.

Statement of the Problem

We take the following coordination system. We place the beginning of the coordinates in the middle of the wing, direct the axis of OZ along the span to the right, the axis OY directed upwards and OX axis on the undisturbed flow. We define all the forces influencing on the aircraft with Advantage Technology winglets at a steady level flight. Due to equilibrium of these forces we determine the influence of aerodynamic forces of wingtip on the inductive reactance of the wing.

The solution of the problem

The following forces effect the plane in level flight [4, 6–7]:

–The power of the weight G — always directed vertically down to the center of the earth;

–Lift of the aircraft Y — is perpendicular to the direction of the undisturbed flow;

–Drag force of the aircraft Q — aimed in the direction opposite to the movement of aircraft;

–Thrust P – is generally directed towards the aircraft movement motion, along the axis;

–Full aerodynamic force created by the upper wingtips. The force created by the upper left wingtip is symbolized $\vec{R}^{в}_{л}$, but the force created by the right wingtip is symbolized $\vec{R}^{в}_{n}$;

–Complete aerodynamic forces created by the lower wingtips. They are symbolized respectively, the left $\vec{R}^{н}_{л}$ and the right force $\vec{R}^{в}_{n}$.

The angle between the true velocity and the free–flow speed for a wing with wingtips α_z equals to the angle between the vector of the total aerodynamic force generated by the upper left wingtip $\vec{R}^{в}_{л}$ and the longitudinal axis of the wing, as the sides of these angles are perpendicular to each other. Then the projection of the full aerodynamic force of the left wingtip will have the form:

$$\vec{R}^{в}_{л} = \{R^{в}_{лx}, R^{в}_{лy}, R^{в}_{лz}\},$$

where $R^{в}_{lx} = R^{в}_{l}\cos\varphi\sin\alpha_z$ — longitudinal force created by the upper left wingtip, φ–angle of wingtip camber (Angle between the aircraft vertical plane of symmetry and the wingtip plane at its point of pressure center); $R^{в}_{ly} = R^{в}_{l}\sin\varphi$ lift force created by the upper left wingtip; $R^{в}_{lz} = R^{в}_{l}\cos\varphi\cos\alpha_z$ — the lateral force generated by the upper left wingtip.

Here, $R^{в}_{l}$ is the vector unit of $\vec{R}^{в}_{l}$,

$$R^{в}_{l} = \sqrt{R^{в2}_{lx} + R^{в2}_{ly} + R^{в2}_{lz}}\ .$$

This force is applied to the center of the wingtip pressure. The total aerodynamic force of right upper wingtip differs from it only with the mark of the third component, so it can be written as:

$$\vec{R}^{в}_{n} = \{R^{в}_{n}\cos\varphi\sin\alpha_z, R^{в}_{n}\sin\varphi, -R^{в}_{n}\cos\varphi\cos\alpha_z\}.$$

Obviously, $R_п^в = \sqrt{R_{пx}^{в2} + R_{пy}^{в2} + R_{пz}^{в2}} = R_л^в$ therefore, the lower indices that indicate the left and right wingtips will be removed in the future.

Then, the right and left upper part of the wingtips together create a force with components:

$$2\vec{R}^в = \vec{R}_п^в + \vec{R}_л^в = \left\{2R^в \cos\varphi \sin\alpha_z,\ 2R^в \sin\varphi, 0\right\} \tag{1}$$

Now we define the forces created by the lower part of wingtips (projections) (Figure). Since, under the wing the air pressure is much higher than in the environment, it can be assumed that the lower left wingtip pressure force is applied to the center of pressure of the wingtip, normal to its surface. Lower wingtip camber is indicated by the letter ϕ, and the twist angle of the center of pressure is indicated by the letter β. Then, with the same above mentioned argumentation, we can write

Figure. The AT winglet wingtip of the Airbus A319

$$2\vec{R}^н = \vec{R}_п^н + \vec{R}_л^н = \left\{2R^н \cos\phi \sin\beta,\ 2R^н \sin\phi, 0\right\}. \tag{2}$$

Thus, the longitudinal is force generated by the lower parts of wingtip, but:

$$R_{lx}^в = R_l^в \cos\varphi \sin\alpha_z$$

lift force is created by them:

$$2R_y^н = 2R^н \sin\phi$$

Because of symmetry, the lateral forces created by the left and right wingtips, balance each other.

The amount of power generated by all four parts of the Advantage Technology winglets wingtips, is indicated by the vector $\vec{R}_z$. Thus

$$\vec{R}_z = \left\{R_{zx},\ R_{zy},\ R_{zz}\right\},$$

where

$$R_{zx} = 2R^в \cos\varphi \sin\alpha_z + 2R^н \cos\phi \sin\beta. \tag{3}$$

the longitudinal component of the vector of total aerodynamic wingtip force, which obviously reduces any drag force, and, of course, is added to the force of traction motors,

$$R_{zy} = 2R^{в} \sin\varphi + 2R^{н} \sin\phi. \qquad (4)$$

the vertical component of the vector of total aerodynamic wingtip force, which is added to the lift of the wing, and the lateral component of the vector of the total aerodynamic force due to the symmetry of wingtips equals to zero $R_{zz} = 0$.

Now, considering the formulas (1–4), we can write the equilibrium equations of steady motion of the aircraft with wingtips type «AT winglets» in level flight in the form of:

$$\left.\begin{aligned} P + R_{zx} - Q = 0 \\ Y + R_{zy} - G = 0 \end{aligned}\right\} \qquad (5)$$

From the first equation of the system (5) it follows that the thrust of the engine P balances the force $Q - R_{zx}$. Since the force $Q = Q_{pr} + Q_i$ consists of the sum of the profile and inductive resistances and the profile resistance is almost unchanged, the force R_{zx} reduces the inductive resistance of the wing of the finite span without wingtips i.e.

$$\Delta Q_i = R_{zx},$$

Or, the reduced part of the inductive resistance force equals:

$$\Delta Q_i = 2R^{в} \cos\varphi \sin\alpha_z + 2R^{н} \cos\phi \sin\beta$$

Heading in the direction of flight, this force increases the thrust of the engines and reduces the drag of the wing. In the case when the angles of collapse are $\varphi = 0$ and $\phi = 0$, it has the greater value, and when $\varphi = \phi = \pi/2$ — it turns to zero. Angles α and β are usually small enough. The latter case is a consequence of the fact that when the tip turns into a wing extension with large sweep angles, in comparison with the wing itself, it also disappears. It turns out a longer wing without wingtips.

From the second equation of system (5) it follows that:

$$Y + R_{zy} = G$$

As follows from this equality, the lifting force Y is less than the weight of the aircraft, and the weight of the aircraft is balanced by the lifting forces created by the wing and the tips together. The quantities on the left side of this equation for wings without wingtips, with upper wingtips and with double wingtips, will be denoted, respectively, by the indices 0, 1 and 2 from above. Since the weight of the aircraft is constant and $R^0_{zy} = 0$, then it can be written:

$$Y^0 = Y^1 + R^1_{zy} = Y^2 + R^2_{zy} = G.$$

As follows from formula (4):

$$0 < R^1_{zy} < R^2_{zy}.$$

Then from the preceding chain of equations, it follows that:

$$Y^0 > Y^1 > Y^2.$$

Thus, provided the weight of the aircraft is constant, the lift of the wing with the tips is reduced, and the smallest is obtained for the wing with double of the AT winglets wingtips. The remaining part of the lifting force creates aerodynamic wingtips. In this regard, the distribution of the lifting force over the entire wing span with the AT winglets wingtips is more even. And this leads to a decrease in the amplitude of wing oscillations and noise during flight. Note that the tip itself is also a profiled winglet and from its end there is a flow of air towards the fuselage at the upper wingtips and from the fuselage at the lower wingtips. These currents, forming with the main flow of air against motion, create vortices of relatively low intensity, flowing from the ends of the tips. The inductive drag of the wing is affected only by the longitudinal component of the total aerodynamic force of the tips, defined by formula (3).

Force can be represented through the high-speed head,

$$R_{zx} = \overline{C}_{zx} q_\infty S_z = C_{zx} q_\infty S$$

where $q_\infty = \dfrac{\rho V_\infty^2}{2}$ — dynamic head, S – wing area without wingtips, and C_{zx} — the coefficient of longitudinal force of the wingtip. The coefficient of this force is:

$$\overline{C}_{zx} = C_{zx} \frac{S}{S_z}$$

where S_z — is the sum of the areas of the wingtips. Then the force ΔQ_i can be written in the form:

$$\Delta Q_i = C_{zx} q_\infty S$$

The coefficient of inductive drag of the wing with the tips is written in the form of the difference in the inductive drag of the wing without the tip and the force factor ΔQ_i

$$C_{zxi} = \frac{C_y^2}{\pi\lambda}(1+\delta) - C_{zx}$$

We express this expression in the following form:

$$C_{zxi} = \frac{C_y^2}{\pi\lambda}(1+\delta)\left[1 - \frac{\pi\lambda}{C_y^2(1+\delta)} C_{zx}\right],$$

Or

$$C_{zxi} = \frac{C_y^2}{\pi\tilde{\lambda}}(1+\delta)$$

where

$$\tilde{\lambda} = \lambda\left[1 - \frac{\pi\lambda}{C_y^2(1+\delta)}C_{zx}\right]^{-1}$$

is the effective lengthening of the wing. As can be seen, the value of the effective elongation makes it possible to record the inductive drag of the wing with the tips in the usual form. It is easy to see the inequality $\lambda < \tilde{\lambda}$, which shows a decrease in the inductive drag of the wing under the influence of the wingtips.

Whereas $\frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)} < 1$, then a square bracket with negative degree is the sum of a geometric progression:

$$1 + \frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)} + \left(\frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)}\right)^2 + \left(\frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)}\right)^3 + \cdots$$

Substituting this into the expressions for the effective elongation and leaving only the first two terms, we have:

$$\tilde{\lambda} = \lambda\left[1 + \frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)}\right]$$

or, taking into account the coefficient of inductive drag of the wing without wingtips:

$$\tilde{\lambda} = \lambda\left(1 + \frac{C_{zx}}{C_{xi}}\right)$$

Taking into account this formula:

$$C_{zx} = 2C_{zx}^{в}\cos\varphi\sin\alpha_z + 2C_{zx}^{н}\cos\phi\sin\beta$$

we get

$$\tilde{\lambda} = \lambda\left(1 + \frac{2C_{zx}^{в}\cos\varphi\sin\alpha_z + 2C_{zx}^{н}\cos\phi\sin\beta}{C_{xi}}\right)$$

This is an approximate expression of the effective elongation of a wing with double tips. The relative elongation of the wing is in the form:

$$\frac{\Delta\lambda}{\lambda}=\frac{C_{zx}}{C_{xi}}. \tag{6}$$

This result can be obtained in another way. We transform the expression for the effective elongation:

$$\tilde{\lambda}=\lambda\left[1-\frac{\pi\lambda}{C_y^2\left(1+\delta\right)}C_{zx}\right]^{-1}=\lambda\frac{1}{1-\dfrac{C_{zx}}{C_{xi}}}=\lambda\frac{C_{xi}}{C_{xi}-C_{zx}}=\lambda\frac{C_{xi}}{C_{zxi}}$$

Thus:

$$\tilde{\lambda}=\lambda\frac{C_{xi}}{C_{zxi}}$$

or

$$\tilde{\lambda}C_{zxi}=\lambda C_{xi}$$

Taking into account formula $C_{zx}=C_{zx}^{в}+C_{zx}^{н}$, the last equation can be extended:

$$\tilde{\lambda}C_{zxi}=\tilde{\lambda}^{в}C_{zxi}^{в}=\lambda C_{xi}. \tag{7}$$

Here both $\tilde{\lambda}^{в}$ and $C_{zxi}^{в}$ are the effective elongation, and the inductive drag of the wing with some upper wingtips. As can be seen, the product of effective lengthening of the wings with double and only upper tips on their coefficients of inductive resistance is equal to the product of the lengthening of the wing without wingtip by its coefficient of inductive resistance. Thus, with increasing effective lengthening of the wing, its inductive resistance decreases and vice versa. This fundamental result can also be said as follows: under the condition that the weight of the aircraft is constant, the product of the effective aspect ratio of the wing with its ends by its coefficient of inductive resistance is constant. Alternatively, provided the weight of the aircraft is constant, the ratio of effective wing extensions is equal to the inverse ratio of their inductive resistance coefficients.

We also note the following proof of this result. From the expression for the coefficient of inductive resistance for all wings:

$$C_{zxi}=\frac{C_y^2}{\pi\tilde{\lambda}}\left(1+\delta\right)$$

we get

$$\tilde{\lambda}C_{zxi}=\frac{C_y^2}{\pi}\left(1+\delta\right)$$

which is constant with a constant weight of the aircraft.

Assuming the effective lengthening of the wing as known, from formulas (6–7) one can determine the loss of inductive resistance:

$$\Delta C_{xi} = C_{xi} - C_{zxi} = C_{xi} - \frac{\lambda}{\tilde{\lambda}} C_{xi} = \frac{\tilde{\lambda} - \lambda}{\tilde{\lambda}} C_{xi} = \frac{\Delta\lambda}{\lambda} C_{xi}$$

Conclusions

1. The aerodynamic forces created by the AT winglet wingtips during the flight are determined.
2. A system of algebraic equations containing all the forces acting on an airplane with a steady horizontal flight, which constitutes the mathematical model of the problem under investigation, is recorded.
3. It is shown that the wing tips reduce the inductive resistance.
4. The effective elongation of the wing is determined.
5. It is shown that in case of constant weight of the aircraft, the product of the effective elongation of the wing with its wingtips by its coefficient of inductive resistance is constant.

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