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Cordial labeling of m-splitting graph of a graph

S. J. Gajjar[1], **A. K. Desai**[2]

[1] General Department, Government Polytechnic
Himmatnagar - 383001, India
gjr.sachin@gmail.com

[2] Department of Mathematics, Gujarat University,
Ahmedabad - 380009, India,
desai_ak@yahoo.com

Abstract

In this paper, we investigate a condition for a splitting graph of a graph, to be cordial. We have extended the concept of splitting graph to m-splitting graph of a graph and proved that the m-splitting graph of every cordial graph is cordial. We also prove that the splitting graph of W_n is cordial for all $n \geqslant 3$, splitting graph of K_n is cordial for $n = t^2$ and $t^2 \pm 2$ and splitting graph of shadow graph of any graph is cordial.

Keywords: Graph labeling, cordial Labeling.
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1 Introduction

We consider only simple, finite, undirected and non-trivial graph $G = (V, E)$ with vertex set V and edge set E. The number of elements of V, denoted as $|V|$ is called the order of the graph G while the number of elements of E, denoted as $|E|$ is called the size of the graph G. For various graph theoretic notations and terminology we follow Gross and Yellen [1] whereas for number theoretic notions we follow D. M. Burton [2]. We give brief summary of definitions and other information which are useful for the present investigations.

Definition 1.1. If the vertices of the graph are assigned values subject to certain conditions then it is known as *graph labeling*.

Vast amount of literature is available on different types of graph labeling and more than 2000 research papers have been published. For latest survey on graph labeling one can refer to Gallian [3].

The aim of the present work is to discuss one such labeling known as cordial labeling.

Definition 1.2. Suppose $G = (V, E)$ is a graph with vertex set V and edge set E. A binary vertex labeling $f : V(G) \to \{0, 1\}$ induces an edge labeling $f^* : E(G) \to \{0, 1\}$ defined by $f^*(e) = |f(u) - f(v)|$. For $i = 0$ and 1, let $v_f(i)$ be the number of vertices of G having label

i under f and $e_f(i)$ be the number of edges of G having label i under f. Then f is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leqslant 1$ and $|e_f(0) - e_f(1)| \leqslant 1$. A graph G is *cordial* if it admits cordial labeling.

The concept of cordial labeling was introduced by Cahit[4]. In the same paper he proved that tree is cordial and K_n is cordial if and only if $n \leqslant 3$. Ho et al.[5] proved that unicyclic graph is cordial unless it is C_{4k+2}. Andar et al.[6] have discussed cordiality of multiple shells. Vaidya et al.[7, 8] have also discussed the cordiality of various graphs.

If we interchange the 0's and 1's in the binary vertex labeling f of G, the resulting labeling $\overline{f}$, called the dual labeling of f, has the property:

$v_{\overline{f}}(0) = v_f(1), v_{\overline{f}}(1) = v_f(0)$ and $e_{\overline{f}}(0) = e_f(0), e_{\overline{f}}(1) = e_f(1)$

Definition 1.3. For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to v. The graph $S(G)$ thus obtained is called *splitting graph* of G.

If G has p vertices and q edges, then $S(G)$ has $2p$ vertices and $3q$ edges. Lawrence Rozario Raj and Koilraj [9] have proved that the splitting graph of path P_n, cycle C_n for $n \not\equiv 2 (\text{mod } 4), n \geqslant 3$, complete bipartite graph $K_{m,n}$, matching M_n, wheel W_n for $n \not\equiv 2 (\text{mod } 4), n \geqslant 3$, fan F_n and $< K_{1,n}^{(1)} : K_{1,n}^{(2)} : \ldots : K_{1,n}^{(k)} >$ are cordial.

In the following definition, we generalize the concept of splitting graph.

Definition 1.4. For each vertex v_i of a graph G, take m new vertices $v_{i,1}, v_{i,2}, v_{i,3}, \ldots, v_{i,m}$. Join each $v_{i,j}$ to all the vertices of G adjacent to v_i for $1 \leqslant j \leqslant m$. The graph $mS(G)$ thus obtained is called *m-splitting graph* of G.

If G has p vertices and q edges, then $mS(G)$ has $(m+1)p$ vertices and $(2m+1)q$ edges. For $m = 1$, 1-splitting graph $1S(G) =$ splitting graph $S(G)$.

We use the notation $mS^n(G)$ for the n^{th}- order m-splitting graph of a graph G, where $mS^n(G) = mS(mS^{n-1}(G))$ for all $n > 1$ and $mS^1(G) = mS(G)$.

Definition 1.5. The *shadow graph* of a connected graph G is constructed by taking two copies of G say G' and G'' . Join each vertex u' in G' to the neighbours of the corresponding vertex u'' in G'' . The graph obtained is denoted as $D_2(G)$.

If G has p vertices and q edges, then $D_2(G)$ has $2p$ vertices and $4q$ edges.

2 Cordial labeling of splitting graph

The following theorem shows the condition for splitting graph of a graph, to be cordial.

Theorem 2.1. If there is a binary vertex labeling $f : V(G) \to \{0, 1\}$ for a graph G such that $|e_f(1) - e_f(0)| \leqslant 1$ then the splitting graph of G is cordial.

Proof: Let $G(V,E)$ be a graph such that there is a binary vertex labeling $f : V(G) \to \{0,1\}$ and $|e_f(1) - e_f(0)| \leqslant 1$. Let $S(G)$ be the splitting graph of G. Let $v_i, i = 1,2,3,\ldots,n$ be the vertices of G and v_i' are the new vertices corresponding to each vertex v_i of G in $S(G)$. Now define a binary vertex labeling $h : V(S(G)) \to \{0,1\}$ of $S(G)$ as follows:

$$h(v) = \begin{cases} f(v) & \text{if} \quad v \in V(G); \\ \overline{f}(v_i) & \text{if} \quad v = v_i'. \end{cases}$$

From the above labeling, we can check that for the graph $S(G)$, $v_h(0) = v_f(0) + v_f(1) = v_h(1)$.

Now for each edge of the graph G with label 0, we have two new edges of the graph $S(G)$ with label 1 as shown in Figure $1(a)$ and $1(b)$ and for each edge of the graph G with label 1, we have two new edges of the graph $S(G)$ with label 0 as shown in Figure $1(c)$.

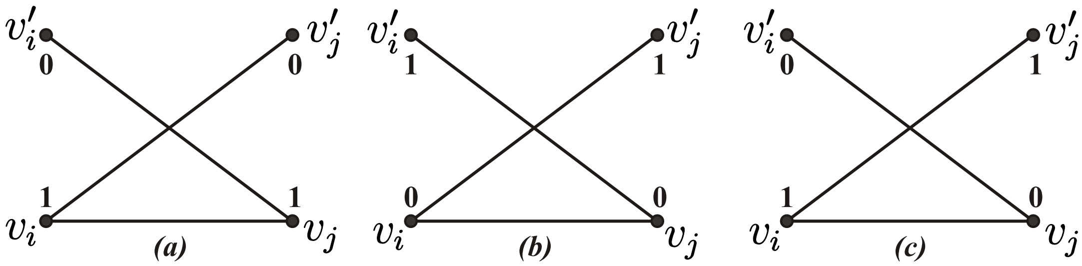

Figure 1

Thus for the graph $S(G)$, we have $e_h(1) = e_f(1) + 2e_f(0)$ and $e_h(0) = e_f(0) + 2e_f(1)$. Therefore , $|e_h(1) - e_h(0)| = |e_f(1) + 2e_f(0) - e_f(0) + 2e_f(1)| = |e_f(0) - e_f(1)| \leqslant 1$, as it is given. So the graph $S(G)$ is cordial. ■

The following theorem shows the condition for m-splitting graph of a graph, to be cordial.

Theorem 2.2. If there is a binary vertex labeling $f : V(G) \to \{0,1\}$ for a graph G such that $|e_f(1) - e_f(0)| \leqslant 1$ then the m-splitting graph of G is cordial, for odd m.

Proof: Let $G(V,E)$ be a graph such that there is a binary vertex labeling $f : V(G) \to \{0,1\}$ and $|e_f(1) - e_f(0)| \leqslant 1$. Let $mS(G)$ be the m-splitting graph of G. Let $v_i, i = 1,2,3,\ldots,n$ be the vertices of G and $v_{i,j}, j = 1,2,3,\ldots,m$ are the new vertices corresponding to each vertex v_i of G in $mS(G)$. Now define a binary vertex labeling $h : V(mS(G)) \to \{0,1\}$ of $mS(G)$ as follows:

$$h(v) = \begin{cases} f(v_i) & \text{if} \quad v_i \in V(G); \\ f(v_i) & \text{if} \quad v = v_{i,j} \text{ for } 1 \leqslant j \leqslant \dfrac{m-1}{2} \text{ and } 1 \leqslant i \leqslant n; \\ \overline{f}(v_i) & \text{if} \quad v = v_{i,j} \text{ for } \dfrac{m+1}{2} \leqslant j \leqslant m \text{ and } 1 \leqslant i \leqslant n. \end{cases}$$

From the above labeling, we can check that for the graph $mS(G)$,

$$\begin{aligned} v_h(0) &= v_f(0) + \left(\frac{m-1}{2}\right) v_f(0) + \left(\frac{m+1}{2}\right) v_f(1) \\ &= \left(\frac{m+1}{2}\right) v_f(0) + \left(\frac{m+1}{2}\right) v_f(1) \\ &= \left(\frac{m+1}{2}\right) (v_f(0) + v_f(1)) \end{aligned}$$

and

$$\begin{aligned} v_h(1) &= v_f(1) + \left(\frac{m-1}{2}\right) v_f(1) + \left(\frac{m+1}{2}\right) v_f(0) \\ &= \left(\frac{m+1}{2}\right) v_f(1) + \left(\frac{m+1}{2}\right) v_f(0) \\ &= \left(\frac{m+1}{2}\right) (v_f(1) + v_f(0)) \end{aligned}$$

So for the graph $mS(G)$, $|v_h(0) - v_h(1)| = 0$.

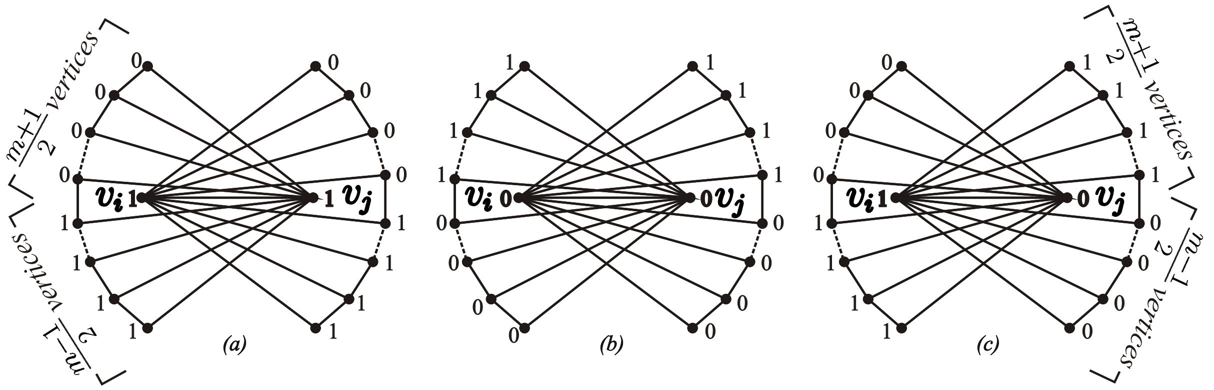

Figure 2

Now every edge of G gives $2m$ more edges of the graph $mS(G)$. For each edge of the graph G with label 0, we have $m-1$ new edges with label 0 and $m+1$ new edges with label 1 in the graph $mS(G)$ as shown in Figures $2(a)$ and $2(b)$ and for each edge of the graph G with label 1, we have $m-1$ new edges with label 1 and $m+1$ new edges with label 0 in the graph $mS(G)$ as shown in Figure $2(c)$.

Thus for the graph $mS(G)$, we have

$e_h(0) = e_f(0) + (m-1)e_f(0) + (m+1)e_f(1) = me_f(0) + (m+1)e_f(1)$ and

$e_h(1) = e_f(1) + (m-1)e_f(1) + (m+1)e_f(0) = me_f(1) + (m+1)e_f(0)$.

Therefore , $|e_h(0)-e_h(1)| = |me_f(0)+(m+1)e_f(1)-me_f(1)-(m+1)e_f(0)| = |e_f(1)-e_f(0)| \leqslant 1$.
So the graph $mS(G)$ is cordial. ∎

Corollary 2.3. m-splitting graph of every cordial graph is cordial for all m.

Proof: Let G be a cordial graph, then there is a binary vertex labeling $f : V(G) \to \{0,1\}$ such that $|v_f(1) - v_f(0)| \leqslant 1$ and $|e_f(1) - e_f(0)| \leqslant 1$. Let $mS(G)$ be the m-splitting graph of G. Let $v_i, i = 1,2,3,\ldots,n$ be the vertices of G and $v_{i,j}, j = 1,2,3,\ldots,m$ be the new vertices corresponding to each vertex v_i of G in $mS(G)$. Now we have the following cases:

Case 1: m is odd.

If m is odd then by Theorem 2.2, $mS(G)$ is cordial.

Case 2: m is even.

If m is even, then define a binary vertex labeling $h : V(mS(G)) \to \{0,1\}$ of $mS(G)$ as follows:

$$h(v) = \begin{cases} f(v_i) & \text{if} \quad v_i \in V(G); \\ f(v_i) & \text{if} \quad v = v_{i,j} \text{ for } 1 \leqslant j \leqslant \dfrac{m}{2} \text{ and } 1 \leqslant i \leqslant n; \\ \overline{f}(v_i) & \text{if} \quad v = v_{i,j} \text{ for } \dfrac{m}{2} \leqslant j \leqslant m \text{ and } 1 \leqslant i \leqslant n. \end{cases}$$

From the above labeling, we can verify that for the graph $mS(G)$,

$v_h(0) = v_f(0) + \left(\frac{m}{2}\right) v_f(0) + \left(\frac{m}{2}\right) v_f(1)$ and $v_h(1) = v_f(1) + \left(\frac{m}{2}\right) v_f(1) + \left(\frac{m}{2}\right) v_f(0)$.

Therefore , $|v_h(0) - v_h(1)| = |v_f(0) - v_f(1)| \leqslant 1$, as G is cordial.

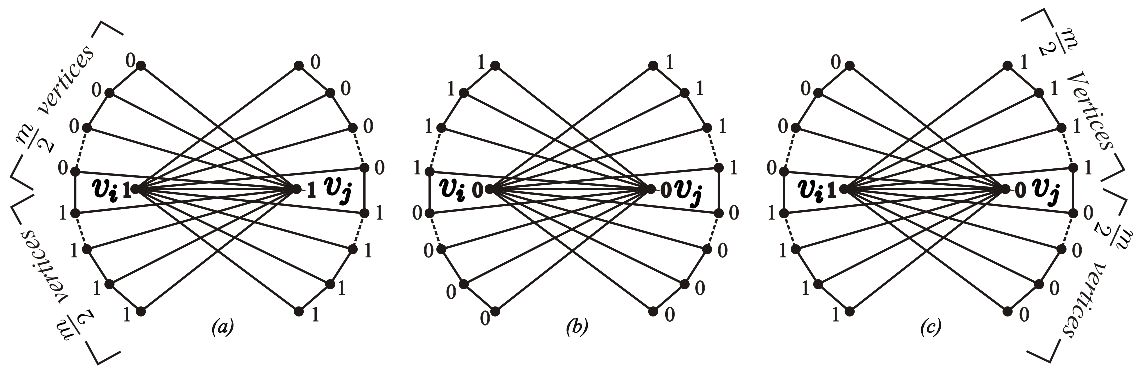

Figure 3

Now every edge of G gives $2m$ more edges of the graph $mS(G)$. For each edge of the graph G with label 0, we have m new edges with label 0 and m new edges with label 1 in the graph $mS(G)$ as shown in Figures 3(a) and 3(b) and for each edge of the graph G with label 1, we have m new edges with label 1 and m new edges with label 0 of the graph $mS(G)$ as shown in Figure 3(c).

Thus for the graph $mS(G)$, we have $e_h(0) = e_f(0) + me_f(0) + me_f(1)$ and $e_h(1) = e_f(1) + me_f(1) + me_f(0)$.

Therefore, $|e_h(0) - e_h(1)| = |e_f(1) - e_f(0)| \leqslant 1$, as G is cordial.

Thus the m-splitting graph $mS(G)$ of G is cordial for all m. ∎

Corollary 2.4. If there is a binary vertex labeling $f : V(G) \to \{0,1\}$ for a graph G such that $|e_f(1) - e_f(0)| \leqslant 1$ then $mS^n(kS(G))$ is cordial for odd k and for all m and n.

Proof: G is a graph with the binary vertex labeling $f : V(G) \to \{0,1\}$ such that $|e_f(1) - e_f(0)| \leqslant 1$. So from Theorem 2.2, k-splitting graph $kS(G)$ of the graph G is cordial for odd k. Now $kS(G)$ is cordial, so by Corollary 2.3, n^{th} order m-splitting graph $mS^n(kS(G))$ of the graph $kS(G)$ is cordial for all m and n. ■

Corollary 2.5. m-splitting graph of the following graphs are cordial.

(i) Path P_n for all n.

(ii) Cycle C_n for $n \not\equiv 2(\text{mod } 4), n \geqslant 3$.

(iii) Complete bipartite graph $K_{m,n}$ for all m and n.

(iv) Fan F_n for all n.

(v) Tree T.

(vi) $S(W_n)$ for $n \geqslant 3$.

Proof: by Corollary 2.3, the m-splitting graphs of (i), (ii), (iii), (iv) and (v) are cordial as the graphs themselves are cordial.

We label all the rim vertices of W_n by 1 and an apex vertex by 0. So for W_n, the number of edges with label 0 and the number of edges with label 1 are equal. Thus it satisfies the condition of Theorem 2.1 and hence $S(W_n)$ is cordial. Therefore, by Corollary 2.3, the m-splitting graph of $S(W_n)$ is cordial. ■

Corollary 2.6. m-splitting graph of K_n, for $n = t^2, t^2 - 2$ and $t^2 + 2$ is cordial for odd m.

Proof: We consider the following three cases:
Case 1: $n = t^2$.

Label the $\dfrac{t(t+1)}{2}$ vertices of K_n by 1 and $\dfrac{t(t-1)}{2}$ vertices of K_n by 0. Total number of edges of graph K_n is $\dfrac{t^2(t^2-1)}{2}$. We can easily check that out of these edges, $\dfrac{t^2(t^2-1)}{4}$ edges have label 0 and the remaining $\dfrac{t^2(t^2-1)}{4}$ edges have label 1. Thus we have a binary vertex labeling of K_n such that $|e_f(0) - e_f(1)| = 0$. So by Theorem 2.2, we can say that $mS(K_n)$ is cordial for $n = t^2$ and for odd m.
Case 2: $n = t^2 - 2$.

Label the $\dfrac{(t+2)(t-1)}{2}$ vertices of K_n by 1 and $\dfrac{(t-2)(t+1)}{2}$ vertices of K_n by 0. Total number of edges of graph K_n is $\dfrac{(t^2-2)(t^2-3)}{2}$. We can easily check that out of these edges,

$\dfrac{(t^4-5t^2+8)}{4}$ edges have label 0 and the remaining $\dfrac{(t^4-5t^2+6)}{4}$ edges have label 1. Thus we have a binary vertex labeling of K_n such that $|e_f(0)-e_f(1)|=1$. So by Theorem 2.2, we can say that $mS(K_n)$ is cordial for $n=t^2-2$ and for odd m.

Case 3: $n=t^2+2$.

Label the $\dfrac{(t^2+t+2)}{2}$ vertices of K_n by 1 and $\dfrac{(t^2-t+2)}{2}$ vertices of K_n by 0. Total number of edges of graph K_n is $\dfrac{(t^2+2)(t^2+1)}{2}$. We can easily check that out of these edges, $\dfrac{(t^4+3t^2)}{4}$ edges have label 0 and the remaining $\dfrac{(t^4+3t^2+4)}{4}$ edges have label 1. Thus we have a binary vertex labeling of K_n such that $|e_f(0)-e_f(1)|=1$. So by Theorem 2.2, we can say that $mS(K_n)$ is cordial for $n=t^2+2$ and for odd m. ■

Theorem 2.7. The shadow graph $D_2(G)$ of every graph G is cordial.

Proof: Let G be a graph with p vertices and q edges. Let $u_1^{'},u_2^{'},u_3^{'},\dots,u_p^{'}$ be the vertices of first copy $G^{'}$ and $u_1^{''},u_2^{''},u_3^{''},\dots,u_p^{''}$ be the vertices of second copy $G^{''}$ of G. Here $|V(D_2(G))|=2p$ and $|E(D_2(G))|=4q$. Define a binary vertex labeling $f:V(G_2(G))\to\{0,1\}$ as follows:

$$f(u)=\begin{cases}1 & \text{if} \quad u=u_i^{'} \text{ for } i=1,2,3,\dots,p;\\ 0 & \text{if} \quad u=u_i^{''} \text{ for } i=1,2,3,\dots,p.\end{cases}$$

From the above labeling pattern it can be verified that $v_f(0)=p=v_f(1)$ and $e_f(0)=2q=e_f(1)$. Thus the shadow graph $D_2(G)$ of every graph G is cordial. ■

Corollary 2.8. The n^{th} order m-splitting graph $mS^n(D_2(G))$ of the shadow graph $D_2(G)$ of any graph G is cordial.

Proof: The result is obvious by Corollary 2.3. ■

Theorem 2.9. Let H be a path union of the graphs $G_1,G_2,\dots,G_n$. If there is a binary vertex labeling $f_i:V(G_i)\to\{0,1\}$ for each G_i, such that $|e_{f_i}(0)-e_{f_i}(1)|\leqslant 1$ for $1\leqslant i\leqslant n$, then the splitting graph of H is cordial.

Proof: H is the graph obtained by any path union of the graphs $G_1,G_2,\dots,G_n$ and for $1\leqslant i\leqslant n$, $f_i:V(G_i)\to\{0,1\}$ is a binary vertex labeling of G_i such that $|e_{f_i}(0)-e_{f_i}(1)|\leqslant 1$. Let $u_{i,1}$ and $u_{i,2}$ be the vertices of G_i for $2\leqslant i\leqslant n-1$ and $u_{1,1}$ and $u_{n,2}$ be the vertices of G_1 and G_n respectively such that $u_{i,1}$ and $u_{i+1,2}$ are adjacent in H for $1\leqslant i\leqslant n-1$. Without loss of generality, we can assume that $f_i(u_{i,1})=f_{i+1}(u_{i+1,2})$ for $i=1,2,\dots,n-1$. Suppose $\sum\limits_{i=1}^{n}v_{f_i}(0)-\sum\limits_{i=1}^{n}v_{f_i}(1)=k$, where $-n\leqslant k\leqslant n$. We consider the following cases.

Case 1: $\left\lceil\dfrac{n+k-1}{2}\right\rceil\geq 1$.

Define a binary vertex labeling $h:V(H)\to\{0,1\}$ as follows:

$$h(v)=\begin{cases}\overline{f}_i(v) & \text{if} \quad i \text{ is even, where } 1\leqslant i\leqslant \left\lceil\frac{n+k-1}{2}\right\rceil;\\ \overline{f}_n(v) & \text{if} \quad \left\lceil\frac{n+k-1}{2}\right\rceil \text{ is odd };\\ f_i(v) & \text{for} \quad \text{remaining} \quad i\leqslant n.\end{cases}$$

Case 2: $\left\lceil\frac{n+k-1}{2}\right\rceil = 1.$

Define a binary vertex labeling $h: V(H)\to\{0,1\}$ as follows:

$$h(v)=\begin{cases} f_i(v) & \text{for} \quad 1\leqslant i\leqslant n-1;\\ \overline{f}_i(v) & \text{for} \quad i=n.\end{cases}$$

Case 3: $\left\lceil\frac{n+k-1}{2}\right\rceil \leq 1.$

Define a binary vertex labeling $h: V(H)\to\{0,1\}$ as follows:

$$h(v)=f_i(v), \forall i.$$

It can be verified that that, the graph H with the above labeling, satisfies the condition $|e_h(0)-e_h(1)|\leqslant 1$. So by Theorem 2.1, the splitting graph of H is cordial. ■

Theorem 2.10. Every graph G can be embedded as an induced subgraph of a graph H, whose splitting graph is cordial.

Proof: Let $G(p,q)$ be any graph and H be a graph obtained by embedding the graph G as attaching new q pendent edges to any vertex of G. Thus H has $p+q$ vertices and $2q$ edges. Define a binary function $f: V(H)\to\{0,1\}$ as follows:

$$f(v)=\begin{cases} 0 & \text{if} \quad v\in G;\\ 1 & \text{if} \quad v\notin G.\end{cases}$$

From the above labeling, we can easily check that $e_f(0)=q=e_f(1)$. So by Theorem 2.1, the splitting graph of H is cordial. ■

Theorem 2.11. If there is a binary vertex labeling $f: V(G)\to\{0,1\}$ for a graph G such that $|e_f(1)-e_f(0)|\leqslant 1$ then the m-splitting graph of $G+\overline{K}_{2n}$ is cordial for odd m.

Proof: Here, G is the graph with the binary vertex labeling $f: V(G)\to\{0,1\}$ such that $|e_f(1)-e_f(0)|\leqslant 1$. For the graph $H=G+\overline{K}_{2n}$, let $u_1,u_2,u_3,\ldots,u_{2n}$ be the vertices of $\overline{K}_{2n}$ in H. Define a binary vertex labeling function as follows:

$$h(u)=\begin{cases} f(u) & \text{for} \quad \text{all } u\in G;\\ 0 & \text{for} \quad u=u_i \text{ where } 1\leqslant i\leqslant n;\\ 1 & \text{for} \quad u=u_i \text{ where } n+1\leqslant i\leqslant 2n;\end{cases}$$

From the above labeling, we can easily verify that,

$$e_h(0) = e_f(0) + v_f(0)n + v_f(1)n \text{ and } e_h(1) = e_f(1) + v_f(0)n + v_f(1)n.$$

Therefore, $|e_h(0) - e_h(1)| = |e_f(0) - e_f(1)| \leqslant 1$ as given. So by Theorem 2.2, the m-splitting graph of H is cordial for odd m. ∎

Theorem 2.12. If there is a binary vertex labeling $f : V(G) \to \{0,1\}$ for a graph $G(p,q)$ with $|p-q| \leqslant 1$ such that $|e_f(1) - e_f(0)| \leqslant 1$ then the m-splitting graph of $G + \overline{K}_{2n-1}$ is cordial for odd m.

Proof: Here $G(p,q)$ where, $|p-q| \leqslant 1$ is the graph with the binary vertex labeling $f : V(G) \to \{0,1\}$ such that $|e_f(1) - e_f(0)| \leqslant 1$. Let for the graph $H = G + \overline{K}_{2n-1}$, $u_1, u_2, u_3, \ldots, u_{2n-1}$ be the vertices of $\overline{K}_{2n-1}$ in H. Define a binary vertex labeling function as follows:

$$h(u) = \begin{cases} 0 & \text{for} \quad \text{all } u \in G; \\ 1 & \text{for} \quad u = u_i \text{ where } 1 \leqslant i \leqslant n; \\ 0 & \text{for} \quad u = u_i \text{ where } n+1 \leqslant i \leqslant 2n-1; \end{cases}$$

From the above labeling, we can easily verify that, $e_h(0) = q + (n-1)p$ and $e_h(1) = np$.

Therefore, $|e_h(0) - e_h(1)| = |p - q| \leqslant 1$ as given. So by Theorem 2.2, the m-splitting graph of H is cordial for odd m. ∎

Corollary 2.13. (1) For every cycle C_n, m-splitting graph of $C_n + \overline{K}_{2t-1}$ is cordial for odd m.
(2) For every tree T, the m-splitting graph of $T + \overline{K}_{2t-1}$ is cordial for odd m.

Concluding Remarks: We derived a condition for m-splitting graph of a graph to be a cordial and also proved that the m-splitting graph of every cordial graph is cordial. If a graph G is cordial, then its m-splitting graph $mS(G)$ is also cordial and hence $mS(mS(G))$ is also cordial. Thus from a given cordial graph, we can construct a sequence of larger cordial graphs $mS^n(G)$ inductively.

References

[1] J. Gross and J. Yellen, *Handbook of Graph Theory,* CRC Press, 2004.

[2] D. M. Burton, *Elementary Number Theory*, 6^{th} Ed., McGraw-Hill, New York, 2007.

[3] J. A. Gallian, *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinatorics, 17 (2015), # DS6 .

[4] I. Cahit, *Cordial Graphs: A weaker version of graceful and harmonious Graphs*, Ars Combinatoria, 23(1987), 201-207.

[5] Y S Ho, S M Lee and S C Shee, *Cordial labeling of unicyclic graphs and generalized Petersen graph*, Congress. Numer., 68(1989), 109-122.

[6] M Andar, S Boxwala and N B Limaye, **A Note on cordial labeling of multiple shells**, Trends Math., (2002), 77-80.

[7] S K Vaidya, G V Ghodasara, Sweta Srivastav and V J Kaneria, *Some new cordial graphs*, Int. J. of Sci. Comp., 2(1)(2008), 81-92.

[8] S K Vaidya, Sweta Srivastav, G V Ghodasara and V J Kaneria, *Cordial labeling for cycle with one chord and its related graphs*, Indian J. of Math. and Math. Sciences, 4(2)(2008), 145-156.

[9] P. Lawrence Rozario Raj and S. Koilraj, *Cordial labeling for the splitting graph of some standard graphs*, International Journal of Mathematics and Soft Computing, Vol.1, No.1 (2011), 105 - 114.