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Odd mean labeling of some new families of graphs

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Abstract

A graph $G = (V(G), E(G))$ with p vertices and q edges is said to be an *odd mean graph* if there is an injection $f\colon V(G) \to \{0, 1, 2, \ldots, 2q-1\}$ and the induced function $f^*\colon E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined as $f*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2}; & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2}; & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$ is a bijection. In this paper we investigate some new families of odd mean graphs.

Keywords: Odd mean labeling, step ladder graph, path union graph of cycle C_n.
AMS Subject Classification(2010): 05C78.

1 Introduction

By a graph $G = (V(G), E(G))$ we mean a simple, connected and undirected graph. The terms not defined here are used in the sense of Harary[2]. For a detailed survey on graph labeling readers can refer to Gallian[1].

The concept of mean labeling was introduced by Somasundaram and Ponraj[5]. The notion of odd mean labeling was first discussed by Manikam and Marudai[3].

Definition 1.1. A graph $G = (V(G), E(G))$ with p vertices and q edges is said to be an *odd mean graph* if there is an injection $f : V(G) \to \{0, 1, 2, \ldots, 2q-1\}$ and the induced function $f^*\colon E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined as $f*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2}; & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2}; & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$ is a bijection.

Vasuki and Nagarajan[6] discussed the odd meanness of graphs $P_{a,b}$, P_a^b and $P^b_{<2a>}$.

Definition 1.2. Let P_n be a path on n vertices denoted by $(1, 1), (1, 2), \ldots, (1, n)$ and with $n-1$ edges denoted by $e_1, e_2, \ldots e_{n-1}$ where e_i is the edge joining the vertices $(1, i)$ and $(1, i+1)$. On each edge e_i, $i = 1, 2, \ldots, n-1$ we erect a ladder with $n-(i-1)$ steps including the edge e_i. The graph obtained is called a *step ladder graph* and is denoted by $S(T_n)$, where n denotes the number of vertices in the base.

Definition 1.3. LetG_1,G_2,...,G_n be n copies of the graph $G = (V(G), E(G))$. Then the graph obtained by adding an edge between G_i and G_{i+1}, for $i = 1, 2, ..., n-1$ is called a *path union of graph G*.

Definition 1.4. The *shadow graph* $D_2(G)$ of a connected graph G is obtained by taking two copies of G say G' and G'' and joining each vertex u' in G' to the neighbours of the corresponding vertex u'' in G''.

2 Main Results

Theorem 2.1. The graph $C_n \odot mK_1$ admits an odd mean labeling except when $n \equiv 3(mod4)$ and $m = 1$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of C_n. Let u_{ij} be the newly added vertices in C_n to form $C_n \odot mK_1$, where $1 \leq i \leq n$ and $1 \leq j \leq m$. To define $f : V(C_n \odot mK_1) \to \{0, 1, 2, \ldots, 2q-1\}$, four cases are to be considered.

Case 1: $n \equiv 0(mod4)$.

For $1 \leq i \leq \frac{n}{2}$,

$$f(v_i) = \begin{cases} 2i + 2m(i-1); \ i \ \text{ is odd,} \\ 4m + 2(m+1)(i-2); \ i \text{ is even.} \end{cases}$$

For $\frac{n}{2} + 1 \leq i \leq n-1$,

$$f(v_i) = \begin{cases} 2i + 2m(i-1); \ i \text{ is odd,} \\ 4(m+1) + 2(m+1)(i-2); \ i \text{ is even.} \end{cases}$$

$$f(v_n) = 4m + 2(m+1)(n-2) + 3.$$

For $1 \leq i \leq \frac{n}{2}$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + 4(j-1); \ i \text{ is odd,} \\ 2(m+1)(i-2) + (4j+2); \ i \text{ is even.} \end{cases}$$

For $\frac{n}{2} + 1 \leq i \leq n-1$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + (4j); \ i \text{ is odd,} \\ 2(m+1)(i-2) + (4j+2); \ i \text{ is even.} \end{cases}$$

$$f(u_{nj}) = 2(m+1)(n-2) + (4j+2); \quad \text{for } 1 \leq j \leq m.$$

Case 2: $n \equiv 1(mod4)$.

Subcase 1: m is even.

$$f(v_1) = 1$$

For $2 \leq i \leq \frac{n+1}{2}$,

$$f(v_i) = \begin{cases} 4m + 2(m+1)(i-2); \ i \text{ is even,} \\ 2i + 2m(i-1); \ i \text{ is odd.} \end{cases}$$

For $\frac{n+3}{2} \leq i \leq n-1$,

$$f(v_i) = \begin{cases} 4(m+1) + 2(m+1)(i-2); \ i \text{ is even,} \\ 2(i) + 2m(i-1); \ i \text{ is odd.} \end{cases}$$

$$f(v_n) = 4m + 2(m+1)(n-2) + 3$$

For $1 \leq i \leq \frac{n-1}{2}$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + 4(j-1); \ i \text{ is odd,} \\ 2(m+1)(i-2) + (4j+2); \ i \text{ is even.} \end{cases}$$

For $i = \frac{n+1}{2}$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + 4(j-1); \ \text{ for } 1 \leq j \leq \frac{m}{2}. \\ 2(m+1)(i-1) + (4j); \ \text{ for } \frac{m}{2} + 1 \leq j \leq m, \end{cases}$$

For $\frac{n+3}{2} \leq i \leq n-1$ and for $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-2) + (4j+2); \ i \text{ is even,} \\ 2(m+1)(i-1) + (4j); \ i \text{ is odd.} \end{cases}$$

$$f(u_{nj}) = \begin{cases} 2(m+1)(n-2) + (4j-1); \ \text{ for } 1 \leq j \leq \frac{m}{2}. \\ 2(m+1)(n-2) + (4j+2); \ \text{ for } \frac{m}{2} + 1 \leq j \leq m, \end{cases}$$

Subcase 2: m is odd.

For $1 \leq i \leq \frac{n+1}{2}$,

$$f(v_i) = \begin{cases} 2i + 2m(i-1); \ i \text{ is odd.} \\ 4m + 2(m+1)(i-2); \ i \text{ is even,} \end{cases}$$

For $\frac{n+3}{2} \leq i \leq n-2$,

$$f(v_i) = \begin{cases} 4(m+1) + 2(m+1)(i-2); \ i \text{ is even,} \\ 2i + 2m(i-1); \ i \text{ is odd.} \end{cases}$$

For $n-1 \leq i \leq n$,

$$f(v_i) = 4m + 2(m+1)(i-2) + 3$$

For $1 \leq i \leq \frac{n-1}{2}$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + 4(j-1); \ i \text{ is odd,} \\ 2(m+1)(i-2) + (4j+2); \ i \text{ is even.} \end{cases}$$

For $i = \frac{n+1}{2}$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + 4(j-1); \ \text{ for } 1 \leq j \leq \frac{m+1}{2}, \\ 2(m+1)(i-1) + (4j); \ \text{ for } \frac{m+3}{2} \leq j \leq m. \end{cases}$$

For $\frac{n+3}{2} \leq i \leq n-1$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-2) + (4j+2); \ i \text{ is even,} \\ 2(m+1)(i-1) + (4j); \ i \text{ is odd.} \end{cases}$$

$$f(u_{nj}) = \begin{cases} 2(m+1)(n-2)+(4j-1); & \text{for } 1 \leq j \leq \frac{m-1}{2}, \\ 2(m+1)(n-2)+(4j+2); & \text{for } \frac{m+1}{2} \leq j \leq m. \end{cases}$$

Case 3: $n \equiv 2(mod4)$.

For $1 \leq i \leq \frac{n}{2}$,

$$f(v_i) = \begin{cases} 2i+2m(i-1); \ i \text{ is odd}, \\ 4m+2(m+1)(i-2); \ i \text{ is even}. \end{cases}$$

For $\frac{n}{2}+1 \leq i \leq n-1$,

$$f(v_i) = \begin{cases} 4m+2(m+1)(i-2); \ i \text{ is even}, \\ 2(i+2)+2m(i-1); \ i \text{ is odd}. \end{cases}$$

$$f(v_n) = 4m+2(m+1)(n-2)+3.$$

For $1 \leq i \leq \frac{n}{2}$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1)+4(j-1); \ i \text{ is odd}, \\ 2(m+1)(i-2)+(4j+2); \ i \text{ is even}. \end{cases}$$

For $\frac{n}{2}+1 \leq i \leq n-1$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-2)+(4j+6); \ i \text{ is even}, \\ 2(m+1)(i-1)+4(j-1); \ i \text{ is odd}. \end{cases}$$

$$f(u_{nj}) = \begin{cases} 2(m+1)(n-2)+(4j-1); & \text{for } j = 1, \\ 2(m+1)(n-2)+(4j+2); & \text{for } 2 \leq j \leq m. \end{cases}$$

Case 4: $n \equiv 3(mod4)$, $n \neq 3$ and $m > 1$.

Subcase 1: m is even

$$f(v_1) = 1.$$

For $2 \leq i \leq \frac{n-1}{2}$,

$$f(v_i) = \begin{cases} 4m+2(m+1)(i-2); \ i \text{ is even}, \\ 2i+2m(i-1); \ i \text{ is odd}. \end{cases}$$

For $\frac{n+1}{2} \leq i \leq n-1$

$$f(v_i) = \begin{cases} 4m+2(m+1)(i-2); \ i \text{ is even}, \\ 2(i+2)+2m(i-1); \ i \text{ is odd}. \end{cases}$$

$$f(v_n) = 4m+2(m+1)(n-2)+3$$

For $1 \leq i \leq \frac{n-1}{2}$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + 4(j-1); \, i \text{ is odd,} \\ 2(m+1)(i-2) + (4j+2); \, i \text{ is even.} \end{cases}$$

For $i = \frac{n+1}{2}$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-2) + (4j+2); & \text{for } 1 \leq j \leq \frac{m}{2} \\ 2(m+1)(i-2) + (4j+6); & \text{for } \frac{m}{2} + 1 \leq j \leq m \end{cases}$$

For $\frac{n+3}{2} \leq i \leq n-1$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + 4(j-1); \, i \text{ is odd,} \\ 2(m+1)(i-2) + (4j+6); \, i \text{ is even.} \end{cases}$$

$$f(u_{nj}) = \begin{cases} 2(m+1)(n-2) + (4j-1); & \text{for } 1 \leq j \leq \frac{m}{2} - 1, \\ 2(m+1)(n-2) + (4j+2); & \text{for } \frac{m}{2} \leq j \leq m. \end{cases}$$

Subcase 2: m is odd.

For $1 \leq i \leq \frac{n-1}{2}$,

$$f(v_i) = \begin{cases} 4m + 2(m+1)(i-2); \, i \text{ is even,} \\ 2i + 2m(i-1); \, i \text{ is odd.} \end{cases}$$

For $\frac{n+1}{2} \leq i \leq n-2$,

$$f(v_i) = \begin{cases} 4m + 2(m+1)(i-2); \, i \text{ is even,} \\ 2(i+2) + 2m(i-1); \, i \text{ is odd.} \end{cases}$$

For $n-1 \leq i \leq n$,

$$f(v_i) = \begin{cases} 4m + 2(m+1)(i-2) - 1; \, i \text{ is even,} \\ 4m + 2(m+1)(i-2) + 3; \, i \text{ is odd.} \end{cases}$$

For $1 \leq i \leq \frac{n-1}{2}$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + 4(j-1); \, i \text{ is odd,} \\ 2(m+1)(i-2) + (4j+2); \, i \text{ is even.} \end{cases}$$

For $i = \frac{n+1}{2}$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-2) + (4j+2); & \text{for } 1 \leq j \leq \frac{m+1}{2} \\ 2(m+1)(i-2) + (4j+6); & \text{for } \frac{m+3}{2} \leq j \leq m \end{cases}$$

For $\frac{n+3}{2} \leq i \leq n-1$ and $1 \leq j \leq m$,

$$f(u_{ij}) = \begin{cases} 2(m+1)(i-1) + 4(j-1); \, i \text{ is odd,} \\ 2(m+1)(i-2) + (4j+6); \, i \text{ is even.} \end{cases}$$

For $m = 3$,

$$f(u_{nj}) = \begin{cases} 2(m+1)(n-2) + (4j+3); & \text{for } 1 \leq j \leq 2, \\ 2(m+1)(n-2) + 14; & \text{for } j = 3. \end{cases}$$

For $m \geq 5$,

$$f(u_{nj}) = \begin{cases} 2(m+1)(n-2) + (4j-1); & \text{for } 1 \leq j \leq \frac{m-3}{2}, \\ 2(m+1)(n-2) + (4j+3); & \text{for } \frac{m-1}{2} \leq j \leq m-1, \\ 2(m+1)(n-2) + (4j+2); & \text{for } j = m. \end{cases}$$

Case 5: $n = 3$; $m > 1$.

Subcase 1: m is even.

$$f(v_1) = 1,\ f(v_2) = 4m,\ f(v_3) = 4m + 2(m+1) + 3$$
$$f(u_{1j}) = 4(j-1); \quad \text{for } \ 1 \leq j \leq m$$
$$f(u_{2j}) = \begin{cases} 4j+2; & \text{for } 1 \leq j \leq \frac{m}{2}, \\ 4j+6; & \text{for } \frac{m}{2}+1 \leq j \leq m. \end{cases}$$

For $m = 2$,

$$f(u_{3j}) = \begin{cases} 13; & \text{for } j = 1, \\ 16; & \text{for } j = 2. \end{cases}$$

For $m \geq 4$,

$$f(u_{3j}) = \begin{cases} 2(m+1) + (4j-1); & \text{for } 1 \leq j \leq \frac{m}{2} - 1, \\ 2(m+1) + (4j+2); & \text{for } \frac{m}{2} \leq j \leq m. \end{cases}$$

Subcase 2: m is odd.

$$f(v_1) = 2,\ f(v_2) = 4m-1,\ f(v_3) = 4m + 2(m+1) + 3$$
$$f(u_{1j}) = 4(j-1); \quad \text{for } 1 \leq j \leq m$$
$$f(u_{2j}) = \begin{cases} 4j+2; & \text{for } 1 \leq j \leq \frac{m+1}{2}, \\ 4j+6; & \text{for } \frac{m+3}{2}+1 \leq j \leq m. \end{cases}$$

For $m = 3$,

$$f(u_{3j}) = \begin{cases} 2(m+1) + (4j+3); & \text{for } 1 \leq j \leq 2, \\ 22; & \text{for } j = 3. \end{cases}$$

For $m \geq 5$,

$$f(u_{3j}) = \begin{cases} 2(m+1) + (4j-1); & \text{for } 1 \leq j \leq \frac{m-3}{2}, \\ 2(m+1) + (4j+3); & \text{for } \frac{m-1}{2} \leq j \leq m-1, \\ 2(m+1) + (4j+2); & \text{for } j = m. \end{cases}$$

It can be verified that f is an odd mean labeling in all the cases. Hence $C_n \odot mK_1$ is an odd mean graph except for $n \equiv 3(mod 4)$ and $m = 1$. ■

Illustration 2.2. An odd mean labeling of $C_{10} \odot 3K_1$ is shown in Figure 1.

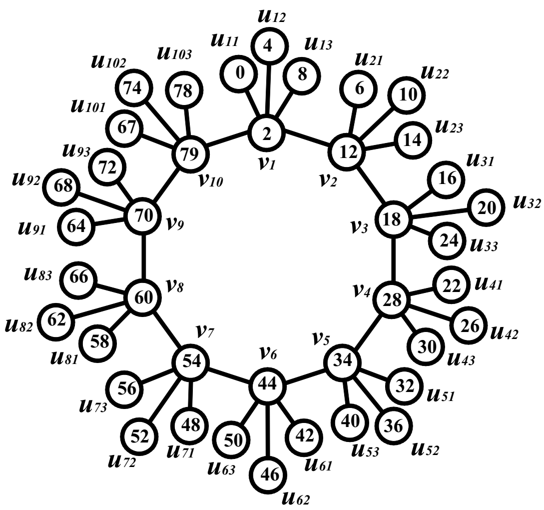

Figure 1: An odd mean labeling of $C_{10} \odot 3K_1$.

Theorem 2.3. The shadow graph $D_2(B_{n,n})$ is an odd mean graph.

Proof: Consider two copies of the bistar $B_{n,n}$.

Let $\{v_1, v_2, v_{ij}, 1 \leq i \leq 2, 1 \leq j \leq n\}$ and $\{u_1, u_2, u_{ij},\ 1 \leq i \leq 2, 1 \leq j \leq n\}$ be the vertex sets of the two copies of $B_{n,n}$.

Define $f : V(D_2(B_{n,n})) \to \{0, 1, 2, 3, \ldots, 2q - 1\}$ as follows:

$f(v_1) = 0,\ f(v_2) = 16n + 2,$
$f(u_1) = 8n,\ f(u_2) = 16n + 6,$
$f(v_{1j}) = 4(j - 1) + 2;$ for $1 \leq j \leq n,$
$f(v_{2j}) = 8j;$ for $1 \leq j \leq n - 1,$
$f(v_{2j}) = 16n;$ for $j = n,$
$f(u_{1j}) = 4j + 4n - 2;$ for $1 \leq j \leq n,$
$f(u_{2j}) = 8n + 8j;$ for $1 \leq j \leq n - 1,$
$f(u_{2j}) = 16n + 7;$ for $j = n.$

In view of the above defined labeling, $D_2(B_{n,n})$ is an odd mean graph. ■

Illustration 2.4. Figure 2 shows an odd mean labeling of of $D_2(B_{3,3})$.

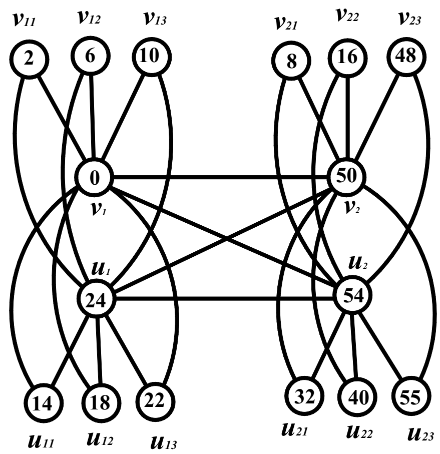

Figure 2: An odd mean labeling of $D_2(B_{3,3})$.

Theorem 2.5. The step ladder graph $S(T_n)$ admits odd mean labeling.

Proof: Let P_n be a path on n vertices denoted by $(1,1),(1,2),\ldots,(1,n)$ and with $n-1$ edges denoted by $e_1,e_2,\ldots,e_{n-1}$ where e_i is the edge joining the vertices $(1,i)$ and $(1,i+1)$. The vertices of the step ladder graph $S(T_n)$ are denoted by $(1,1),(1,2),\ldots,(1,n)$, $(2,1),(2,2),\ldots,(2,n)$, $(3,1),(3,2),\ldots,(3,n-1)$, $(4,1),(4,2),\ldots,(4,n-2),\ldots,(n,1),(n,2)$. In the ordered pair (i,j), i denotes the row (counted from bottom to top) and j denotes the column (from left to right) in which the vertex occurs. Define $f: V(S(T_n)) \to \{0,1,2,\ldots,2q-1\}$ as follows:

$f(1,1) = 2(n^2-1)$,

$f(i,j)=2(n^2-2+i) - 2\sum_{k=1}^{j-1}(n-k-1) - 2\sum_{k=2}^{j}[(n+k)-(j-1)],$

for $1 \leq i \leq n-1, 1 \leq j \leq n$,

$f(i,j)=(2n^2+2i-5)$; for $j=1, i=n$.

Hence the step ladder graph $S(T_n)$ admits odd mean labeling for every n. ■

Illustration 2.6. An odd mean labeling of of $S(T_6)$ is shown in Figure 3.

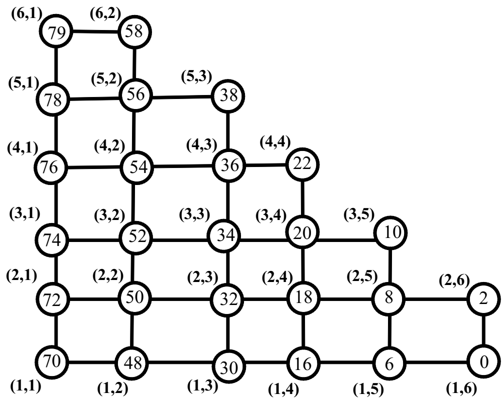

Figure 3: An odd mean labeling of $S(T_6)$.

Theorem 2.7. The graph obtained by the path union of finite number of copies of cycle C_n admits odd mean labeling except for $n = 3$, 6 and 7.

Proof: Let G be the path union graph of k copies of cycle C_n. Let the successive vertices of the cycle C_i be $u_{i1}, u_{i2}, \ldots, u_{in}$ where $1 \leq i \leq k$. Let $e_i = u_{i1}u_{(i+1)1}$ be the edge joining C_i and C_{i+1} for $i = 1, 2, \ldots, k-1$. To define an odd mean labeling $f : V(G) \rightarrow \{0, 1, 2, \ldots, 2q-1\}$, the following cases are considered.

Case 1: $n \equiv 0(mod 4)$.

Subcase 1: i is odd.

For $1 \leq j \leq \frac{n}{2}$,

$$f(u_{ij}) = \begin{cases} 2(n+1)(i-1) + (4j-4); \ j \text{ is odd,} \\ 2(n+1)(i-1) + (4j-6); \ j \text{ is even.} \end{cases}$$

For $\frac{n}{2} + 1 \leq j \leq n$,

$$f(u_{ij}) = \begin{cases} 2(n+1)(i-1) + (4n-4j+3); \ j \text{ is odd,} \\ 2(n+1)(i-1) + (4n-4j+6); \ j \text{ is even.} \end{cases}$$

Subcase 2: i is even.

For $1 \leq j \leq \frac{n}{2} + 1$,

$$f(u_{ij}) = 2(n+1)(i-2) + (4n-4j+5)$$

For $\frac{n}{2} + 2 \leq j \leq n$

$$f(u_{ij}) = \begin{cases} 2(n+1)(i-2) + (4j); \ j \text{ is even,} \\ 2(n+1)(i-2) + (4j-2); \ j \text{ is odd.} \end{cases}$$

Case 2: $n \equiv 1(mod 4)$.

Subcase 1: i is odd.

For $1 \leq j \leq \frac{n+1}{2}$,

$$f(u_{ij}) = \begin{cases} 2(n+1)(i-1) + (4j-4); \ j \text{ is odd,} \\ 2(n+1)(i-1) + (4j-6); \ j \text{ is even.} \end{cases}$$

For $\frac{n+3}{2} \leq j \leq n$,

$$f(u_{ij}) = 2(n+1)(i-1) + (4n-4j+5)$$

Subcase 2: i is even.

For $1 \leq j \leq \frac{n-1}{2}$,

$$f(u_{ij}) = 2(n+1)(i-2) + (4n-4j+5)$$

For $\frac{n+1}{2} \leq j \leq n$,

$$f(u_{ij}) = \begin{cases} 2(n+1)(i-2) + (4j); \ j \text{ is odd,} \\ 2(n+1)(i-2) + (4j-2); \ j \text{ is even.} \end{cases}$$

Case 3: $n \equiv 2(mod4)$; where $n \geq 10$.
Subcase 1: i is odd.

$f(u_{i1}) = 2(n+1)(i-1) + (4j-4)$,
$f(u_{i2}) = 2(n+1)(i-1) + (4j-6)$.

For $3 \leq j \leq \frac{n+2}{2}$,

$f(u_{ij}) = 2(n+1)(i-1) + (4j-5)$.

For $\frac{n+4}{2} \leq j \leq n-2$,

$$f(u_{ij}) = \begin{cases} 2(n+1)(i-1) + (4n-4j+6); \; j \text{ is odd}, \\ 2(n+1)(i-1) + (4n-4j+4); \; j \text{ is even}. \end{cases}$$

For $n-1 \leq j \leq n$,

$f(u_{ij}) = 2(n+1)(i-1) + (4n-4j+5)$.

Subcase 2: i is even

For $1 \leq j \leq \frac{n-2}{2}$,
$f(u_{ij}) = 2(n+1)(i-2) + (4n-4j+5)$,
$f(u_{ij}) = 2(n+1)(i-2) + (4j+4)$, for $j = \frac{n}{2}$,
$f(u_{ij}) = 2(n+1)(i-2) + (4j-2)$, for $j = \frac{n+2}{2}$.

For $\frac{n+4}{2} \leq j \leq n-2$,

$f(u_{ij}) = 2(n+1)(i-2) + (4j-1)$,
$f(u_{ij}) = 2(n+1)(i-2) + (4j-2)$, for $j = n-1$,
$f(u_{ij}) = 2(n+1)(i-2) + (4j)$, for $j = n$.

Case 4: $n \equiv 3(mod4)$; where $n \geq 11$.
Subcase 1: i is odd.

$f(u_{i1}) = 2(n+1)(i-1) + (4j-4)$,
$f(u_{i2}) = 2(n+1)(i-1) + (4j-6)$,
$f(u_{ij}) = 2(n+1)(i-1) + (4j-5)$, for $j = 3, 4$.

For $5 \leq j \leq \frac{n+1}{2}$,

$$f(u_{ij}) = \begin{cases} 2(n+1)(i-1) + (4j-6); \; j \text{ is odd}, \\ 2(n+1)(i-1) + (4j-4); \; j \text{ is even}. \end{cases}$$

For $\frac{n+3}{2} \leq j \leq n$,
$f(u_{ij}) = 2(n+1)(i-1) + (4n-4j+5)$.

Subcase 2: i is even.

For $1 \leq j \leq \frac{n-5}{2}$,
$f(u_{ij})$=$2(n+1)(i-2) + (4n-4j+5)$.

For $\frac{n-3}{2} \leq j \leq \frac{n-1}{2}$,

$$f(u_{ij}) = \begin{cases} 2(n+1)(i-2) + (4n-4j+4); \ j \text{ is even}, \\ 2(n+1)(i-2) + (4n-4j+6); \ j \text{ is odd}. \end{cases}$$

For $\frac{n+1}{2} \leq j \leq n-2$,

$f(u_{ij})$=$2(n+1)(i-2) + (4j-1)$,
$f(u_{ij})$=$2(n+1)(i-2) + (4j-2)$, for $j = n-1$
$f(u_{ij})$=$2(n+1)(i-2) + (4j)$, for $j = n$.

In all the four cases f is odd mean and hence G is an odd mean graph. ■

Illustration 2.8. Figure 4 shows an odd mean labeling of the path union graph of 4 copies of cycle C_8.

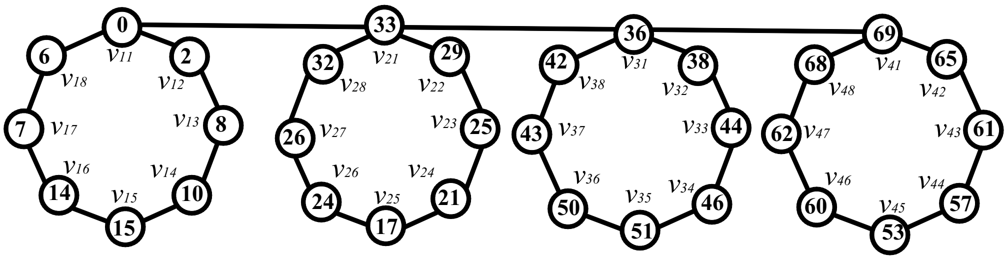

Figure 4: An odd mean labeling of path union graph of 4 copies of cycle C_8.

References

[1] J A Gallian, *A dynamic survey of graph labeling*, The Electronics Journal of Combinatorics, 16(2015) $\sharp DS6$.

[2] F Harary, *Graph Theory*, Addison Wesley, Reading, Massachusetts, 1972.

[3] K Manikam and M Marudai, *Odd mean labelings of graphs*, Bulletin of Pure and Applied Sciences, 25E (1)(2006), 149-153.

[4] S C Shee and Y S Ho, *The cordiality of the path union of n copies of a graph*, Discrete Mathematics, 151(1996), 221-229.

[5] S. Somasundaram and R. Ponraj, *Some results on mean graphs*, Pure and Applied Mathematical Sciences, 58 (2003), 29-35.

[6] R Vasuki and A Nagarajan, *Odd mean labeling of the graphs $P_{a,b}$, P_a^b and $P_{<2a>}^b$*, Kragujevac Journal of Mathematics, 36(1)(2012), 141-150.