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On Edge Irregular Fuzzy Graphs

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Abstract

In this paper, edge irregular fuzzy graphs, edge totally irregular fuzzy graphs, highly edge irregular fuzzy graphs, highly edge totally irregular fuzzy graphs are introduced. Some properties of an edge irregular fuzzy graph and highly edge irregular fuzzy graphs are discussed in this paper. Edge irregularity and highly edge irregularity on some fuzzy graphs whose underlying crisp graphs are a cycle, a path, a star, a barbell graph are also studied.

Keywords: Edge degree in fuzzy graph, total edge degree in fuzzy graph, edge regular fuzzy graph, strongly edge irregular fuzzy graph, neighbourly edge irregular fuzzy graph.
AMS Subject Classification(2010): 05C12, 03E72, 05C72.

1 Introduction

Euler first introduced the concept of graph theory in 1736. In 1965, Lofti A. Zadeh[11] introduced the concept of a fuzzy subset of a set as a method for representing the phenomena of uncertainty in real life situation. Azriel Rosenfeld introduced fuzzy graphs in 1975[8]. It has been growing fast and has numerous applications in various fields. Nagoor Gani and Radha [4] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Nagoorgani and Latha [3]introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008. The concept of strongly irregular fuzzy graphs was introduced in [6].

Radha and Kumaravel[7] introduced the concept of edge degree, total edge degree and edge regular fuzzy graph and discussed about the degree of an edge in some fuzzy graphs. Santhi Maheswari and Sekar [9] introduced the concept of strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs and discussed about their properties. Also, Santhi Maheswari and Sekar introduced neighbourly edge irregular fuzzy graphs and neighbourly edge totally irregular fuzzy graphs and discussed about their properties in [10].

Some properties of an edge irregular fuzzy graph and highly edge irregular fuzzy graphs are discussed in this paper. Edge irregularity and highly edge irregularity on some fuzzy graphs whose underlying crisp graphs are a cycle, a path, a star, a barbell graph are also studied.

2 Preliminaries

We present some known definitions and results in fuzzy graph theory for ready reference to go through the work presented in this paper.

Definition 2.1. [8] A Fuzzy graph denoted by $G : (\sigma, \mu)$ on the graph $G^* : (V, E)$ is a pair of functions (σ, μ) where $\sigma : V \to [0, 1]$ is a fuzzy subset of a non empty set V and $\mu : V \times V \to [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V, the relation $\mu(u, v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 2.2. [5] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The degree of a vertex u is $d_G(u) = \sum\limits_{u \neq v} \mu(uv)$, for $uv \in E$ and $\mu(uv) = 0$, for uv not in E; this is equivalent to $d_G(u) = \sum\limits_{uv \in E} \mu(uv)$.

Definition 2.3. [4] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d(v) = k$ for all $v \in V$, then G is said to be regular fuzzy graph of degree k.

Definition 2.4. [4] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total degree of a vertex u is defined as $td(u) = \sum \mu(u, v) + \sigma(u) = d(u) + \sigma(u)$, $uv \in E$. If each vertex of G has the same total degree k, then G is said to be a totally regular fuzzy graph of degree k or k-totally regular fuzzy graph..

Definition 2.5. [3] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct degrees.

Definition 2.6. [3] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a totally irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct total degrees.

Definition 2.7. [6] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a strongly irregular fuzzy graph if every pair of vertices having distinct degrees.

Definition 2.8. [6] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a strongly totally irregular fuzzy graph if every pair of vertices having distinct total degrees.

Definition 2.9. [3] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a highly irregular fuzzy graph if every vertex is adjacent to the vertices having distinct degrees.

Definition 2.10. [3] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a highly totally irregular fuzzy graph if every vertex is adjacent to the vertices having distinct total degrees.

Definition 2.11. [7]Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The degree of an edge uv is defined as $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$. The minimum degree of an edge is $\delta_E(G) = \wedge\{d_G(uv) : uv \in E\}$.
The maximum degree of an edge is $\Delta_E(G) = \vee\{d_G(uv) : uv \in E\}$.

Definition 2.12. [7] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total degree of an edge uv is defined as $td_G(uv) = d_G(u) + d_G(v) - \mu(uv)$. It can also be defined as $td_G(uv) = d_G(uv) + \mu(uv)$. The minimum total degree of an edge is $\delta_{tE}(G) = \wedge\{td_G(uv) : uv \in E\}$.
The maximum total degree of an edge is $\Delta_{tE}(G) = \vee\{td_G(uv) : uv \in E\}$.

Definition 2.13. [1]The degree of an edge uv in the underlying graph is defined as $d_G(uv) = d_G(u) + d_G(v) - 2$.

Definition 2.14. [9] Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a strongly edge irregular fuzzy graph if every pair of adjacent edges have the same degree.

Definition 2.15. [9] Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a strongly edge totally irregular fuzzy graph if every pair of edges have the same total degree.

Definition 2.16. [10]Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a neighbourly edge irregular fuzzy graph if every pair of adjacent edges have the same degree.

Definition 2.17. [10] Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a neighbourly edge totally irregular fuzzy graph if every pair of adjacent edges have the same total degree.

Theorem 2.18. [9] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on $2m$ $(m > 1)$ vertices. If the membership value of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ are respectively $c_1, c_2, c_3, \ldots,$ c_{2m-1} such that $c_1 < c_2 < c_3 < \ldots, c_{2m-1}$, then G is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph.

Theorem 2.19. [9] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a cycle on n $(n \geq 4)$ vertices. If the membership value of the edges $e_1, e_2, e_3, \ldots, e_n$ are respectively $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 < c_2 < c_3 < \ldots, c_n$, then G is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph.

Theorem 2.20. [9] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a star $K_{1,n}$. If the membership values of no two edges are same, then G is strongly edge irregular fuzzy graph and G is totally edge regular fuzzy graph.

Theorem 2.21. [9] Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a Barbell graph $B_{n,m}$. If the membership values of no two edges are same, then G is strongly edge irregular fuzzy graph.

3 Edge irregular fuzzy graph and Highly edge irregular fuzzy graph

In this section, we define an edge irregular fuzzy graph and an edge totally irregular fuzzy graph, highly edge irregular fuzzy graph and highly edge totally irregular fuzzy graph and discussed some of their properties.

Definition 3.1. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be an edge irregular fuzzy graph if there exists atleast one edge which is adjacent to the edges having distinct degrees.

Definition 3.2. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be an edge totally irregular fuzzy graph if there exists atleast one edge which is adjacent to the edges having distinct total degrees.

Definition 3.3. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a highly edge irregular fuzzy graph if every edge is adjacent to the edges having distinct degrees.

Definition 3.4. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a highly edge totally irregular fuzzy graph if every edge is adjacent to the edges having distinct total degrees.

Theorem 3.5. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. If G is highly edge irregular fuzzy graph, then G is an edge irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. Let us assume that G is highly edge irregular fuzzy graph$\Rightarrow$every edge in G is adjacent to the edges having distinct degrees$\Rightarrow$there exists atleast one edge which is adjacent to the edges having distinct degrees. Hence G is an edge irregular fuzzy graph. ■

Theorem 3.6. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. If G is highly edge totally irregular fuzzy graph, then G is an edge totally irregular fuzzy graph.

Proof: Proof is similar to Theorem 3.5. ■

Remark 3.7. Converse of Theorems 3.5 and 3.6 need not be true.

Example 3.8. The following graph is an example for edge irregular fuzzy graph, edge totally irregular fuzy graph, highly edge irregular fuzzy graph and highly edge totally irregular fuzzy graph.
Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a cycle of length five.

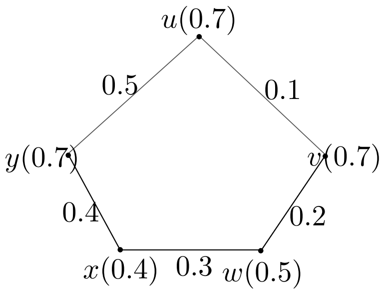

Figure 1

From Figure 1, $d_G(u) = 0.6, d_G(v) = 0.3, d_G(w) = 0.5, d_G(x) = 0.7, d_G(y) = 0.9.$ Degrees of the edges are calculated below.

$$d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv) = 0.6 + 0.3 - 2(0.1) = 0.7.$$
$$d_G(vw) = d_G(v) + d_G(w) - 2\mu(vw) = 0.3 + 0.5 - 2(0.2) = 0.4.$$
$$d_G(wx) = d_G(w) + d_G(x) - 2\mu(wx) = 0.5 + 0.7 - 2(0.3) = 0.6.$$
$$d_G(xy) = d_G(x) + d_G(y) - 2\mu(xy) = 0.7 + 0.9 - 2(.4) = 0.8.$$
$$d_G(yu) = d_G(y) + d_G(u) - 2\mu(yu) = 0.9 + 0.6 - 2(.5) = 0.5.$$

We noted that every edge is adjacent to the edges having distinct degrees. Hence G is highly edge irregular fuzzy graph as well as an edge irregular fuzzy graph. Total degrees of the edges are calculated below.

$$td_G(uv) = d_G(uv) + \mu(uv) = 0.7 + 0.1 = 0.8.$$
$$td_G(vw) = d_G(vw) + \mu(vw) = 0.4 + 0.2 = 0.6.$$
$$td_G(wx) = d_G(wx) + \mu(wx) = 0.6 + 0.3 = 0.9.$$
$$td_G(xy) = d_G(xy) + \mu(xy) = 0.8 + 0.4 = 1.2.$$
$$td_G(yu) = d_G(yu) + \mu(yu) = 0.5 + 0.5 = 1.$$

It is noted that every edge is adjacent to the edges having distinct total degrees. Hence G is highly edge totally irregular fuzzy graph and also an edge totally irregular fuzzy graph.

Example 3.9. This example illustrates that a highly edge irregular fuzzy graph need not be a highly edge totally irregular fuzzy graph. Consider a fuzzy graph $G : (\sigma, \mu)$ such that $G^* : (V, E)$, a star on four vertices.

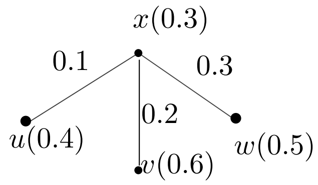

Figure 2

From Figure 2, $d_G(u) = 0.1, d_G(v) = 0.2, d_G(w) = 0.3, d_G(x) = 0.6$.

$d_G(ux) = 0.5$, $d_G(vx) = 0.4$, $d_G(wx) = 0.3$.

$td_G(ux) = 0.6$, $td_G(vx) = 0.6$, $td_G(wx) = 0.6$.

Here, each edge is adjacent to the edges having distinct degrees. So, G is highly edge irregular fuzzy graph. But G is not highly edge totally irregular fuzzy graph, since all edges are having same total degree.

Example 3.10. Highly edge totally irregular fuzzy graph need not be a highly edge irregular fuzzy graph. Consider $G^* : (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$.

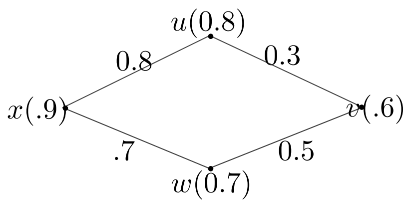

Figure 3

From Figure 3, $d_G(u) = 1.1, d_G(v) = 0.8, d_G(w) = 1.2, d_G(x) = 1.5$. $d_G(uv) = 1.3, d_G(vw) = 1, d_G(wx) = 1.3, d_G(xu) = 1.1$. Note that the edge vw is adjacent to the edges uv and vw having same degree. Hence G is not highly edge irregular fuzzy graph. Also, $td_G(uv) = 1.6, td_G(vw) = 1.5, td_G(wx) = 2, td_G(uv) = 1.9$. It is observed that every edge is adjacent to the edges having distinct total degrees. So, G is highly edge totally irregular fuzzy graph. Hence highly edge totally irregular fuzzy graph need not be highly edge irregular fuzzy graph.

Remark 3.11. Edge irregular fuzzy graph need not be an edge totally irregular fuzzy graph.

Remark 3.12. Edge totally irregular fuzzy graph need not be an edge irregular fuzzy graph.

Theorem 3.13. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is constant function. If G is highly edge irregular fuzzy graph, then G is highly edge totally irregular fuzzy graph.

Proof: Assume that μ is a constant function, let $\mu(uv) = c$ for all uv in E, where c is a constant. Suppose that G is highly edge irregular fuzzy graph. Then every edge is adjacent to the edges having distinct degrees. Let uv is any edge which is adjacent to the edges uw, ux which are incident at the vertex u and vy is the edge incident with the vertex v. Then $d_G(uw) \neq d_G(ux) \neq d_G(vy)$, where uw, ux and vy are adjacent to the edge uv in E.

Now, $d_G(uw) \neq d_G(ux) \neq d_G(vy) \Rightarrow d_G(uw) + c \neq d_G(ux) + c \neq d_G(vy) + c \Rightarrow d_G(uw) + \mu(uw) \neq d_G(ux) + \mu(ux) \neq d_G(vy) + \mu(vy) \Rightarrow td_G(uw) \neq td_G(ux) \neq td_G(vy)$, where uw, ux and vy are adjacent with the edge uv in E. Hence G is highly edge totally irregular fuzzy graph. ■

Theorem 3.14. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is highly edge totally irregular fuzzy graph, then G is highly edge irregular fuzzy graph.

Proof: Proof is similar to Theorem 3.13. ■

Theorem 3.15. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is constant function. If G is an edge irregular fuzzy graph, then G is an edge totally irregular fuzzy graph.

Proof: Proof is similar to Theorem 3.13. ■

Theorem 3.16. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is an edge totally irregular fuzzy graph, then G is an edge irregular fuzzy graph.

Proof: Proof is similar to Theorem 3.14. ■

Remark 3.17. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. If G is both highly edge irregular fuzzy graph and highly edge totally irregular fuzzy graph. Then μ need not be constant function.

Example 3.18. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$ which is a cycle of length five.

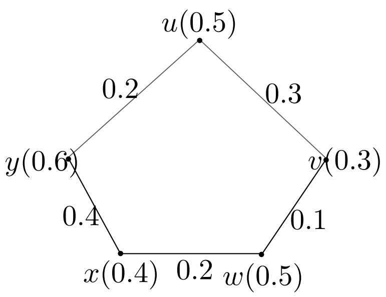

Figure 4

From Figure 4, $d_G(u) = 0.5, d_G(v) = 0.4, d_G(w) = 0.3, d_G(x) = 0.6, d_G(y) = 0.6$.
$d_G(uv) = 0.3$, $d_G(vw) = 0.5$, $d_G(wx) = 0.5$, $d_G(xy) = 0.4$, $d_G(yu) = 0.7$.
$td_G(uv) = 0.6$, $td_G(vw) = 0.6$, $td_G(wx) = 0.7$, $td_G(xy) = .8$, $td_G(yu) = .9$.

It is noted that every edge is adjacent to the edges having distinct degrees. Hence G is highly edge irregular fuzzy graph. Also, every edge is adjacent to the edges having distinct total degrees. So, G is highly edge totally irregular fuzzy graph. Hence G is both highly edge irregular fuzzy graph and highly edge totally irregular fuzzy graph. But μ is not constant.

Theorem 3.19. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. If G is strongly edge irregular fuzzy graph, then G is highly edge irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. Let us assume that G is strongly edge irregular fuzzy graph⇒every pair of edges in G having distinct degrees⇒every edge in G is adjacent to the edges having distinct degrees. Hence G is highly edge irregular fuzzy graph. ■

Theorem 3.20. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. If G is strongly edge totally irregular fuzzy graph, then G is highly edge totally irregular fuzzy graph.

Proof: Proof is similar to Theorem 3.19. ■

Remark 3.21. Example 3.18 shows that the converse of the above theorems 3.19 and 3.20 need not be true. Also it shows that highly edge irregular fuzzy graph need not be neighbourly edge irregular fuzzy graph. Also it shows that highly edge totally irregular fuzzy graph need not be neighbourly edge totally irregular fuzzy graph.

Theorem 3.22. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. If G is strongly edge irregular fuzzy graph, then G is an edge irregular fuzzy graph.

Proof: By Theorems 3.19 and 3.5, G is an edge irregular fuzzy graph. ■

Theorem 3.23. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. If G is strongly edge totally irregular fuzzy graph, then G is an edge totally irregular fuzzy graph.

Proof: By Theorems 3.20 and 3.6, G is an edge totally irregular fuzzy graph. ■

Remark 3.24. Converse of Theorems 3.22 and 3.23 need not be true.

Theorem 3.25. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is highly irregular fuzzy graph, then G is highly edge irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. Assume that μ is a constant function, let $\mu(uv) = c$ for all $uv \in E$, where c is a constant. Let uv is any edge which is adjacent to the edges uw, ux which are incident at the vertex u and vy is an edge incident with the vertex v.

Let us suppose that G is highly irregular fuzzy graph ⇒ every vertex is adjacent to the vertices have distinct degrees ⇒ $d_G(w) \neq d_G(x) \neq d_G(v)$ and $d_G(u) \neq d_G(y) \Rightarrow d_G(u)+d_G(v) \neq d_G(u) + d_G(w) \neq d_G(u) + d_G(x)$ and $d_G(u) + d_G(v) \neq d_G(v) + d_G(y) \Rightarrow d_G(u) + d_G(v) - 2c \neq d_G(u) + d_G(w) - 2c \neq d_G(u) + d_G(x) - 2c$ and $d_G(u) + d_G(v) - 2c \neq d_G(v) + d_G(y) - 2c \Rightarrow$

$d_G(u) + d_G(v) - 2\mu(uv) \neq d_G(u) + d_G(w) - 2\mu(uw) \neq d_G(u) + d_G(x) - 2\mu(ux)$ and $d_G(u) + d_G(v) - 2\mu(uv) \neq d_G(v) + d_G(y) - 2\mu(vy) \Rightarrow d_G(uv) \neq d_G(uw) \neq d_G(ux)$ and $d_G(uv) \neq d_G(vy)$ $\Rightarrow d_G(uv) \neq d_G(uw) \neq d_G(ux) \neq d_G(vy) \Rightarrow$ any edge uv adjacent with the edges uw, ux and vy having distinct degrees. Hence G is highly edge irregular fuzzy graph. ■

Theorem 3.26. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is highly irregular fuzzy graph, then G is highly edge totally irregular fuzzy graph.

Proof: Proof is similar to Theorem 3.25. ■

Remark 3.27. Converse of Theorems 3.25 and 3.26 need not be true.

Theorem 3.28. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is highly irregular fuzzy graph, then G is an edge irregular fuzzy graph.

Proof: By theorems 3.25 and 3.5, G is highly edge irregular fuzzy graph. ■

Theorem 3.29. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is highly irregular fuzzy graph, then G is an edge totally irregular fuzzy graph.

Proof: By Theorems 3.26 and 3.6, G is an edge totally irregular fuzzy graph. ■

Theorem 3.30. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is an edge irregular fuzzy graph, then G is an irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. Assume that μ is a constant function, let $\mu(uv) = c$ for all $uv \in E$, where c is a constant. Let uv is an edge which is adjacent with edges uw and ux which are incident at the vertex u and vy is the edge incident with the vertex v.

Let us suppose that G is an edge irregular fuzzy graph $\Rightarrow$ there exists an edge adjacent with the edges have distinct degrees $\Rightarrow d_G(uw) \neq d_G(ux) \neq d_G(vy) \Rightarrow d_G(u) + d_G(w) - 2\mu(uw) \neq d_G(u) + d_G(x) - 2\mu(ux) \neq d_G(v) + d_G(y) - 2\mu(vy) \Rightarrow d_G(u) + d_G(w) - 2c \neq d_G(u) + d_G(x) - 2c \neq d_G(v) + d_G(y) - 2c \Rightarrow d_G(u) + d_G(w) \neq d_G(u) + d_G(x) \neq d_G(v) + d_G(y) \Rightarrow d_G(w) \neq d_G(x) \Rightarrow$ there exists a vertex u adjacent to the vertices with distinct degrees. Hence G is an irregular fuzzy graph. ■

Theorem 3.31. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is constant. If G is an edge totally irregular fuzzy graph, then G is an irregular fuzzy graph.

Proof: Proof is similar to Theorem 3.30. ■

Remark 3.32. Converse of Theorems 3.30 and 3.31 need not be true.

Example 3.33. Consider $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a path on four vertices.

$$u(.6) \quad .3 \quad v(.7) \quad .6 \quad w(.4) \quad .3 \quad x(.5)$$

Figure 6

From Figure 6, $d_G(u) = 0.3, d_G(v) = 0.9, d_G(w) = 0.9, d_G(x) = 0.3$. Hence, G is an irregular fuzzy graph. $d_G(uv) = 0.6$, $d_G(vw) = 0.6$, $d_G(wx) = 0.6$. It is noted that $d_G(uv) = d_G(vw) = d_G(wx)$. Hence G is not an edge irregular fuzzy graph.

Theorem 3.34. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is strongly irregular fuzzy graph, then G is an edge irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. Assume that μ is a constant function, let $\mu(uv) = c$ for all $uv \in E$, where c is a constant. Let uv is an edge which is adjacent with edges uw and ux which are incident at the vertex u and vy is the edge incident with the vertex v.

Let us suppose that G is strongly irregular fuzzy graph $\Rightarrow$ every pair of vertices have distinct degrees $\Rightarrow d_G(u) \neq d_G(v) \neq d_G(w) \neq d_G(x) \neq d_G(y) \Rightarrow d_G(v) + d_G(y) - 2c \neq d_G(v) + d_G(u) - 2c \neq d_G(u) + d_G(x) - 2c \neq d_G(u) + d_G(w) - 2c \Rightarrow d_G(v) + d_G(y) - 2\mu(vy) \neq d_G(v) + d_G(u) - 2\mu(uv) \neq d_G(u) + d_G(x) - 2\mu(ux) \neq d_G(u) + d_G(w) - 2\mu(uw) \Rightarrow d_G(vy) \neq d_G(uv) \neq d_G(ux) \neq d_G(uw) \Rightarrow$ there exists an edge uv adjacent with the edges uw and ux have distinct degrees. Hence G is an edge irregular fuzzy graph.

Theorem 3.35. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is strongly irregular fuzzy graph, then G is an edge totally irregular fuzzy graph.

Proof: Proof is similar to Theorem 3.34. ∎

Remark 3.36. Converse of Theorems 3.34 and 3.35 need not be true.

4 Highly edge irregularity and edge irregularity on a path, a cycle, a star and a barbell graph with some specific membership functions

Theorem 4.1. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on $2m$, $(m > 1)$ vertices. If all the edges have the same membership value, then G is both an edge irregular fuzzy graph and G is an edge totally irregular fuzzy graph. But we noted that G is not highly edge irregular fuzzy graph and G is not highly edge totally irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on $2m$, $(m > 1)$ vertices. Let $e_1, e_2, e_3, \ldots, e_{2m-1}$ be the edges of the path G^* in that order. If all the edges have the same membership value, then

$d_G(e_1) = c + 2c - 2c = c,$

$d_G(e_i) = 2c + 2c - 2c = 2c$, for $i = 2, 3, 4, 5, 6, \ldots, 2m-3, 2m-2$,
$d_G(e_{2m-1}) = c + 2c - 2c = c$.
It is noted that the adjacent edges of e_2 are e_1 and e_3 which are having distinct degrees and the edges. Hence G is an edge irregular fuzzy graph but not highly edge irregular fuzzy graph.
$td_G(e_1) = c + 2c - c = 2c$,
$td_G(e_i) = 2c + 2c - c = 3c$ for $i = 2, 3, 4, 5, 6, \ldots, 2m-3, 2m-2$,
$td_G(e_{2m-1}) = c + 2c - c = 2c$.
Note that the adjacent edges of e_2 are e_1 and e_3 which are having distinct total degrees. Hence G is an edge totally irregular fuzzy graph but not highly edge totally irregular fuzzy graph. ■

Theorem 4.2. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on $2m$, $(m > 1)$ vertices. If the alternate edges have the same membership value, then G is both an edge irregular fuzzy graph and an edge totally irregular fuzzy graph. Also, G is not highly edge irregular fuzzy graph and G is not highly edge totally irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on $2m$ $(m > 1)$ vertices. If the alternate edges have the same membership values, then

$$\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2, & \text{if } i \text{ is even.} \end{cases} \quad \text{where } c_1 \neq c_2 \text{ and } c_2 \neq 2c_1.$$

$$\begin{aligned} d_G(e_1) &= c_1 + c_1 + c_2 - 2c_1 = c_2, \\ d_G(e_i) &= c_1 + c_2 + c_1 + c_2 - 2c_1 = 2c_2, \text{ for } i = 3, 5, 7, \ldots, 2m-3, \\ d_G(e_i) &= c_1 + c_2 + c_1 + c_2 - 2c_2 = 2c_1, \text{ for } i = 2, 4, 6, \ldots, 2m-2, \\ d_G(e_{2m-1}) &= c_1 + c_2 + c_1 - 2c_1 = c_2. \end{aligned}$$

It is noted that the adjacent edges of e_2 have distinct degrees. Hence G is an edge irregular fuzzy graph. But we note that G is not a highly edge irregular fuzzy graph.

$$\begin{aligned} d_G(e_1) &= c_1 + c_1 + c_2 - c_1 = c_1 + c_2, \\ d_G(e_i) &= c_1 + c_2 + c_1 + c_2 - c_1 = c_1 + 2c_2, \text{ for } i = 3, 5, 7, \ldots, 2m-3, \\ d_G(e_i) &= c_1 + c_2 + c_1 + c_2 - c_2 = c_2 + 2c_1, \text{ for } i = 2, 4, 6, \ldots, 2m-2, \\ d_G(e_{2m-1}) &= c_1 + c_2 + c_1 - c_1 = c_1 + c_2. \end{aligned}$$

Note that the adjacent edges of e_2 have distinct total degrees. Hence G is a edge totally irregular fuzzy graph but not highly edge totally irregular fuzzy graph. ■

Theorem 4.3. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on $2m$ $(m > 1)$ vertices. If the membership value of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ are respectively $c_1, c_2, c_3, \ldots, c_{2m-1}$ such that $c_1 < c_2 < c_3 < \ldots, c_{2m-1}$, then G is both highly edge irregular fuzzy graph and highly edge totally irregular fuzzy graph and G is both an edge irregular fuzzy graph and an edge totally irregular fuzzy graph.

Proof: By Theorems 2.18, 3.19 and 3.20, G is both highly edge irregular fuzzy graph and highly edge totally irregular fuzzy graph. Also by Theorems 2.18, 3.22 and 3.23 , G is oth an edge irregular fuzzy graph and edge totlly irregular fuzzy graph. ■

Theorem 4.4. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is an even cycle of length $2m+2$. If the alternate edges have the same membership value, then G is not an edge irregular fuzzy graph and not an edge totally irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, an even cycle of length $2m + 2$. If the alternate edges have the same membership values, then

$$\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2, & \text{if } i \text{ is even.} \end{cases} \quad \text{where } c_1 \neq c_2.$$

$$d_G(e_i) = c_1 + c_2 + c_1 + c_2 - 2c_2 = 2c_1, \text{ for } i = 2, 4, 6, \ldots, 2m-2,$$

$$d_G(e_i) = c_1 + c_2 + c_1 + c_2 - 2c_1 = 2c_2, \text{ for } i = 1, 3, 5, 7, \ldots, 2m-3, 2m-1.$$

It is noted that the alternate edges have same degrees. Hence G is not an edge irregular fuzzy graph and G is not highly edge irregular fuzzy graph, but it is neighbourly edge irregular fuzzy graph.

$$td_G(e_i) = c_1 + c_2 + c_1 + c_2 - c_2 = 2c_1 + c_2, \text{ for } i = 2, 4, 6, \ldots, 2m-2,$$

$$td_G(e_i) = c_1 + c_2 + c_1 + c_2 - c_1 = 2c_2 + c_1, \text{ for } i = 1, 3, 5, 7, \ldots, 2m-3, 2m-1.$$

It is noted that the alternate edges have same degrees. Hence G is not an edge totally irregular fuzzy graph and not highly edge totally irregular fuzzy graph, but it is neighbourly edge totally irregular fuzzy graph. ∎

Remark 4.5. From the above proof we observed that neighbourly edge irregular fuzzy graphs need not be highly edge irregular fuzzy graphs and neighbourly edge totally irregular fuzzy graphs need not be highly edge totally irregular fuzzy graphs.

Theorem 4.6. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a cycle on n $(n \geq 4)$ vertices. If the membership value of the edges $e_1, e_2, e_3, \ldots, e_n$ are respectively $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 < c_2 < c_3 < \ldots, c_n$, then G is both an edge irregular fuzzy graph and an edge totally irregular fuzzy graph.

Proof: By Theorems 2.19, 3.19 and 3.20, G is both higly edge irregular fuzzy graph and highly edge totally irregular fuzzy graph. Also by Theorems 2.19,3.22 and 3.23, G is both an edge irregular fuzzy graph and an edge totally irregular fuzzy graph. ∎

Theorem 4.7. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a star $K_{1,n}$. If the membership values of no two edges are same, then G is highly edge irregular fuzzy graph and an edge irregular fuzzy graph.

Proof: By aheorems 2.20, 3.19 and 3.20, G is highly edge irregular fuzzy graph and highly edge totally irregular fuzzy graph. Also by Theorems 2.20, 3.22 and 3.23, G is an edge irregular fuzzy graph and an edge totally irregular fuzzy graph. ∎

Theorem 4.8. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a Barbell graph $B_{n,m}$. If the membership values of no two edges are same, then G is highly edge irregular fuzzy graph and an edge irregular fuzzy graph.

Proof: By Theorems 2.21, 3.19, G is highly edge irregular fuzzy graph. Also by Theorems 2.21, 3.22, G is an edge irregular fuzzy graph. ■

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