

On Intuitionistic Fuzzy Semihyperring

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Abstract

In this paper, the concepts of an intuitionistic fuzzy semihyperrings and an intuitionistic fuzzy subsemihypermodule of R , where R is a semihyperring, are introduced and some of their characteristics are proved. In particular, fully idempotent semihyperrings are investigated in the context of intuitionistic fuzzy concept.

Keywords: Intuitionistic fuzzy semihyperrings, Intuitionistic fuzzy subsemihypermodule, intuitionistic fuzzy prime hyperideals, intuitionistic fuzzy irreducible.

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1 Introduction

The concept of hyperstructure theory was initiated in 1934, at the eighth congress of Scandinavian Mathematicians, when F. Marty [5] first defined a hypergroup as a set equipped with an associative and reproductive hyperoperations and analysed their properties. Algebraic hyperstructures are suitable generalization of classical algebraic structures.

The theory of fuzzy sets was first proposed by Zadeh [7] in 1965, has provided a useful mathematical tool for describing too complex or ill defined mathematical analysis by classical methods. In this aspect, the concept of fuzzy groups was defined by Rosenfeld [6] and its structures were investigated. Since, then many papers have been published in the field of fuzzy algebra. Generalizing the concepts of fuzzy algebras, fuzzy algebraic hyperstructures are studied.

On the other hand, Atanassov [1] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets and later there has been much progress in the study of intuitionistic fuzzy sets by many authors. Some basic results on intuitionistic fuzzy sets were published in [2]. The book [3] provides a comprehensive coverage of virtually all results in the area of the theory as well as the applications of intuitionistic fuzzy sets. Many scientists and economists are used the concepts of intuitionistic fuzzy sets for their research.

In this paper, our aim is to introduce and study the concept of intuitionistic fuzzy hyperideal and intuitionistic fuzzy subsemihypermodule as a generalization of the usual fuzzy hyperideal and the usual fuzzy subsemihypermodule. Moreover, examples and some of the characterization theorems of intuitionistic fuzzy hyperideal are constructed and proved.

2 Preliminaries

In this section, basic definitions on hyperstructures, fuzzy hyperstructures are summarized.

Definition 2.1. [5] A *hyperstructure* is a non-empty set H together with a mapping $\circ : H \times H \rightarrow P^*(H)$, where $P^*(H)$ is the set of all non-empty subsets of H . If $x \in H$ and $A, B \in P^*(H)$, then $A \circ B$, $A \circ x$ and $x \circ B$ are defined as $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$, $A \circ x = A \circ \{x\}$ and $x \circ B = \{x\} \circ B$ respectively.

Definition 2.2. [4] A *semihyperring* is an algebraic hyperstructure (R, \oplus, \cdot) consisting of a non-empty set R together with one hyperoperation " \oplus " and one binary operation \cdot on R , such that the following conditions hold.

- (i) (R, \oplus) is a commutative semihypergroup,
- (ii) (R, \cdot) is a semigroup,
- (iii) An element $0 \in R$, is an absorbing element, such that $0 \oplus x = x = x \oplus 0$ and $0 \cdot x = 0 = x \cdot 0$ for all $x \in R$,
- (iv) For all $x, y, z \in R$, the binary operation of multiplication is distributive over hyperoperation from bothsides. That is, $x \cdot (y \oplus z) = x \cdot y \oplus x \cdot z$ and $(x \oplus y) \cdot z = x \cdot z \oplus y \cdot z$.

Definition 2.3. [4] A non-empty set M which is commutative semihypergroup with respect to addition, with an absorbing element 0 is called a *right R -semihypermodule* M_R , if R is a semihyperring and there is a function $\alpha : M \times R \rightarrow P^*(M)$, such that if $\alpha(m, x)$ is denoted by mx and $mx \subseteq M$, for all $x \in R, m \in M$. Then, the following conditions hold, for all $x, y \in R, m, m_1, m_2 \in M$,

- (i) $(m_1 \oplus m_2)x = m_1x \oplus m_2x$,
- (ii) $m(x \oplus y) = mx \oplus my$,
- (iii) $m(xy) = (mx)y$,
- (iv) $0 \cdot x = m \cdot 0 = 0$.

Similarly, we can define a left R -semihypermodule ${}_R M$.

Note 2.4. The following results are proved in [4].

1. A semihyperring R is a right semihypermodule over itself which is denoted by R_R .
2. A non-empty subset N of a right R -semihypermodule M is called a subsemihypermodule of M , if (N, \oplus) is a subsemihypergroup of (M, \oplus) and $RN \subseteq P^*(N)$.
3. The right (left) subsemihypermodules $R_R ({}_R R)$ are right (left) hyperideals of R .

4. Every hyperideal of a semihyperring R is a semihypermodule of R .

Definition 2.5. [4] Let R be a semihyperring and μ be a fuzzy set in R . Then, μ is said to be a *fuzzy hyperideal* of R if for all $r, x, y \in R$, the following axioms hold.

- (i) $\inf_{z \in x \oplus y} \mu(z) \geq \mu(x) \wedge \mu(y)$ for all $x, y \in R$,
- (ii) $\mu(xy) \geq \mu(x)$ and $\mu(yx) \geq \mu(x)$ for all $x, y \in R$.

Definition 2.6. [4] Let M be a right (left) R -semihypermodule. A function $\mu : M \rightarrow [0, 1]$ is called a *fuzzy subsemihypermodule* of M_R (${}_R M$), if the following conditions hold for all $m, m_1, m_2 \in M, r \in R$.

- (i) $\mu(0_M) = 1$,
- (ii) $\inf_{m' \in m_1 \oplus m_2} \mu(m') \geq \mu(m_1) \wedge \mu(m_2)$,
- (iii) $\mu(mr) \geq \mu(m), (\mu(rm) \geq \mu(m))$.

Definition 2.7. [4] Let λ and μ be fuzzy hyperideals of R . The fuzzy subset $\lambda \oplus \mu$ of R is defined by, $(\lambda \oplus \mu)(x) = \vee_{x \in y \oplus z} [\lambda(y) \wedge \mu(z)]$, for $x \in R$.

Definition 2.8. [4] A semihyperring R is called *fully idempotent* if each hyperideal of R is idempotent (a hyperideal I is idempotent if $I^2 = I$).

A semihyperring R is said to be *regular* if for each $x \in R$, there exist $a \in R$ such that $x = xax$.

Definition 2.9. [4] A hyperideal I of a semihyperring R is called a *prime hyperideal* of R , if for all hyperideals A, B of R , $AB \subseteq I$ implies that either $A \subseteq I$ or $B \subseteq I$.

Definition 2.10. [4] A hyperideal I of a semihyperring R is called *irreducible* if for all hyperideals A, B of R , $A \cap B = I$, implies $A = I$ or $B = I$.

Lemma 2.11. [4] A semihyperring R is regular if and only if for any right hyperideal I and for any left hyperideal L of R , we have $IL = I \cap L$.

Definition 2.12. [4] A fuzzy hyperideal η of a semihyperring R is called a *fuzzy prime hyperideal* of R if for fuzzy hyperideals λ and μ of R , $\lambda \mu \leq \eta \Rightarrow \lambda \leq \eta$ or $\mu \leq \eta$. Also, η is called *fuzzy irreducible* if for fuzzy hyperideals λ, μ of R , $\lambda \cap \mu = \eta \Rightarrow \lambda = \eta$ or $\mu = \eta$.

Definition 2.13. [1] An Intuitionistic Fuzzy Set (IFS) A in X is an object of the form $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle | x \in X\}$, where the functions $\mu_A : X \rightarrow [0, 1]$ and $\gamma_A : X \rightarrow [0, 1]$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of non-membership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.14. [1] Let A and B be two Intuitionistic Fuzzy Sets of the forms $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle | x \in X\}$. Then,

- (i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,
- (ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
- (iii) The complement of A is denoted by \bar{A} and is defined by $\bar{A} = \{\langle x, \gamma_A(x), \mu_A(x) \rangle | x \in X\}$,
- (iv) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle | x \in X\}$,
- (v) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle | x \in X\}$.

The Intuitionistic Fuzzy Sets $0_\sim = \{\langle x, 0, 1 \rangle | x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle | x \in X\}$ are the empty set and the whole set respectively.

3 Intuitionistic Fuzzy Semihyperrings

In this section, we extend the concept of fuzzy semihyperrings to intuitionistic fuzzy semihyperrings and we proved some of the following intuitionistic fuzzy theoretic characterization theorems of intuitionistic fuzzy hyperideals.

Definition 3.1. Let R be a semihyperring and A be an intuitionistic fuzzy subset in R . Then, A is said to be an *intuitionistic fuzzy hyperideal* of R if for all $r, x, y \in R$, the following axioms hold.

- (i) $\inf_{z \in x \oplus y} \mu_A(z) \geq \mu_A(x) \wedge \mu_A(y)$ and $\sup_{z \in x \oplus y} \gamma_A(z) \leq \gamma_A(x) \vee \gamma_A(y)$.
- (ii) $\mu_A(xy) \geq \mu_A(x), \mu_A(yx) \geq \mu_A(x)$ and $\gamma_A(xy) \leq \gamma_A(x), \gamma_A(yx) \leq \gamma(x)$.

Example 3.2. Let $R = \{0, a, b, c\}$ be a semihyperring with a hyperoperation " \oplus " and a binary operation " \cdot " is defined by the following tables.

\oplus	0	a	b	c
0	0	a	b	c
a	a	a	{a, b}	c
b	b	{a, b}	{a, c}	a
c	c	c	a	c

\cdot	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	c	c
c	0	a	b	c

An intuitionistic fuzzy subset A is defined as, $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle | x \in M\}$ where $\mu_A(x) = \frac{0.34}{0} + \frac{0.63}{a} + \frac{0.46}{b} + \frac{0.5}{c}$ and $\gamma_A(x) = \frac{0.52}{0} + \frac{0.25}{a} + \frac{0.3}{b} + \frac{0.1}{c}$ and A is an intuitionistic fuzzy hyperideal of R .

Definition 3.3. Let M be a right (left) R -semihypermodule. The set $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle | x \in M\}$, where $\mu_A : M \rightarrow [0, 1]$ and $\gamma_A : M \rightarrow [0, 1]$ is called an *intuitionistic fuzzy subsemihypermodule* of M_R (${}_R M$), if the following conditions hold for all $m, m_1, m_2 \in M, r \in R$.

- (i) $\mu_A(0_M) = 1, \gamma_A(0_M) = 0,$

- (ii) $\inf_{m' \in m_1 \oplus m_2} \mu_A(m') \geq \mu_A(m_1) \wedge \mu_A(m_2)$ and $\sup_{m' \in m_1 \oplus m_2} \gamma_A(m') \leq \gamma_A(m_1) \vee \gamma_A(m_2)$,
- (iii) $\mu_A(mr) \geq \mu_A(m)$, $[\mu_A(rm) \geq \mu_A(m)]$ and $\gamma_A(mr) \leq \gamma_A(m)$, $[\gamma_A(rm) \leq \gamma_A(m)]$.

Example 3.4. Let $M = \{0, a, b\}$ be a right (left) semihypermodule with a hyperoperation " \oplus " and a binary operation " \cdot " is defined by the following tables.

\oplus	0	a	b
0	0	a	b
a	a	{0, b}	{0, a}
b	b	{0, a}	{0, b}

\cdot	0	a	b
0	0	0	0
a	0	b	a
b	0	b	b

An intuitionistic fuzzy subset A is defined as, $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle | x \in M\}$ where $\mu_A(x) = \frac{1}{0} + \frac{0.56}{a} + \frac{0.73}{b}$ and $\gamma_A(x) = \frac{0}{0} + \frac{0.28}{a} + \frac{0.15}{b}$ and A is an intuitionistic fuzzy subsemihypermodule of R .

Definition 3.5. Let A be an intuitionistic fuzzy subsemihypermodule of a right semihypermodule M_R and B be an intuitionistic fuzzy hyperideal of R . Then, an intuitionistic fuzzy subset $C [= A_B]$ of M is defined by, $C = \{\langle x, \mu_C(x), \gamma_C(x) \rangle | x \in M\}$, where $y_i \in M, z_i \in R, p \in N$ and

$$\mu_C(x) = \bigvee_{x \in \sum_{i=1}^p y_i z_i} \left\{ \bigwedge_{1 \leq i \leq p} [\mu_A(y_i) \wedge \mu_B(z_i)] \right\}, \quad \gamma_C(x) = \bigwedge_{x \in \sum_{i=1}^p y_i z_i} \left\{ \bigvee_{1 \leq i \leq p} [\gamma_A(y_i) \vee \gamma_B(z_i)] \right\}.$$

Proposition 3.6. If A is an intuitionistic fuzzy subsemihypermodule of M_R and B is an intuitionistic fuzzy hyperideal of R , then an intuitionistic fuzzy subset $C [= A_B]$ is an intuitionistic fuzzy subsemihypermodule of M .

Proof: Let $y_i \in M, z_i \in R, p, q, r, s \in N$. It is enough to prove that,

- (i) $\mu_C(0_M) = 1$ and $\gamma_C(0_M) = 0$,
- (ii) $\inf_{m'' \in m \oplus m'} \mu_C(m'') \geq \mu_C(m) \wedge \mu_C(m')$ and $\sup_{m'' \in m \oplus m'} \gamma_C(m'') \leq \gamma_C(m) \vee \gamma_C(m')$,
- (iii) $\mu_C(mr) \geq \mu_C(m)$ and $\gamma_C(mr) \leq \gamma_C(m)$.

$$\mu_C(0_M) = \bigvee_{0 \in \sum_{i=1}^p y_i z_i} \left\{ \bigwedge_{1 \leq i \leq p} [\mu_A(y_i) \wedge \mu_B(z_i)] \right\} \geq \mu_A(0_M) \wedge \mu_B(0_M) = 1.$$

$$\gamma_C(0_M) = \bigwedge_{0 \in \sum_{i=1}^p y_i z_i} \left\{ \bigvee_{1 \leq i \leq p} [\gamma_A(y_i) \vee \gamma_B(z_i)] \right\} \leq \gamma_A(0_M) \vee \gamma_B(0_M) = 0.$$

This proves condition (i).

By definition, for membership function, $\mu_C(m) = \bigvee_{m \in \sum_{j=1}^q y'_j z'_j} \left\{ \bigwedge_{1 \leq j \leq q} [\mu_A(y'_j) \wedge \mu_B(z'_j)] \right\}$ and

$$\mu_C(m') = \bigvee_{m' \in \sum_{k=1}^r y_k'' z_k''} \left\{ \bigwedge_{1 \leq k \leq r} [\mu_A(y_k'') \wedge \mu_B(z_k'')] \right\} \text{ where } m, m' \in M.$$

$$\begin{aligned} \mu_C(m) \wedge \mu_C(m') &= \left\{ \bigvee_{m \in \sum_{j=1}^q y_j' z_j'} \left\{ \bigwedge_{1 \leq j \leq q} [\mu_A(y_j') \wedge \mu_B(z_j')] \right\} \right\} \wedge \left\{ \bigvee_{m' \in \sum_{k=1}^r y_k'' z_k''} \left\{ \bigwedge_{1 \leq k \leq r} [\mu_A(y_k'') \wedge \mu_B(z_k'')] \right\} \right\} \\ &= \bigvee_{m \in \sum_{j=1}^q y_j' z_j'} \bigvee_{m' \in \sum_{k=1}^r y_k'' z_k''} \left\{ \left[\bigwedge_{1 \leq j \leq q} [\mu_A(y_j') \wedge \mu_B(z_j')] \right] \wedge \left[\bigwedge_{1 \leq k \leq r} [\mu_A(y_k'') \wedge \mu_B(z_k'')] \right] \right\} \\ &\leq \bigvee_{m \oplus m' \subseteq \sum_{l=1}^s y_l''' z_l'''} \left\{ \bigwedge_{1 \leq l \leq s} [\mu_A(y_l''') \wedge \mu_B(z_l''')] \right\} = \inf_{m'' \in m \oplus m'} \mu_C(m''). \end{aligned}$$

By definition, for non-Membership function, $\gamma_C(m) = \bigwedge_{m \in \sum_{j=1}^q y_j' z_j'} \left\{ \bigvee_{1 \leq j \leq q} [\gamma_A(y_j') \vee \gamma_B(z_j')] \right\}$ and

$$\gamma_C(m') = \bigwedge_{m' \in \sum_{k=1}^r y_k'' z_k''} \left\{ \bigvee_{1 \leq k \leq r} [\gamma_A(y_k'') \vee \gamma_B(z_k'')] \right\} \text{ where } m, m' \in M.$$

$$\begin{aligned} \gamma_C(m) \vee \gamma_C(m') &= \left\{ \bigwedge_{m \in \sum_{j=1}^q y_j' z_j'} \left\{ \bigvee_{1 \leq j \leq q} [\gamma_A(y_j') \vee \gamma_B(z_j')] \right\} \right\} \wedge \left\{ \bigwedge_{m' \in \sum_{k=1}^r y_k'' z_k''} \left\{ \bigvee_{1 \leq k \leq r} [\gamma_A(y_k'') \vee \gamma_B(z_k'')] \right\} \right\} \\ &= \bigwedge_{m \in \sum_{j=1}^q y_j' z_j'} \bigwedge_{m' \in \sum_{k=1}^r y_k'' z_k''} \left\{ \left[\bigvee_{1 \leq j \leq q} [\gamma_A(y_j') \vee \gamma_B(z_j')] \right] \vee \left[\bigvee_{1 \leq k \leq r} [\gamma_A(y_k'') \vee \gamma_B(z_k'')] \right] \right\} \\ &\leq \bigwedge_{m \oplus m' \subseteq \sum_{l=1}^s y_l''' z_l'''} \left\{ \bigvee_{1 \leq l \leq s} [\gamma_A(y_l''') \vee \gamma_B(z_l''')] \right\} = \sup_{m'' \in m \oplus m'} \gamma_C(m''). \end{aligned}$$

This proves condition (ii).

To prove condition (iii), for membership function,

$$\begin{aligned} \mu_C(m) &= \bigvee_{m \in \sum_{j=1}^q y_j' z_j'} \left\{ \bigwedge_{1 \leq j \leq q} [\mu_A(y_j') \wedge \mu_B(z_j')] \right\} \\ &\leq \bigvee_{m \in \sum_{j=1}^q y_j' z_j'} \left\{ \bigwedge_{1 \leq j \leq q} [\mu_A(y_j') \wedge \mu_B(z_j' r)] \right\}, \text{ where } r \text{ is any element of } R \\ &\leq \bigvee_{mr \subseteq \sum_{n=1}^t y_n'' z_n''} \left\{ \bigwedge_{1 \leq n \leq t} [\mu_A(y_n'') \wedge \mu_B(z_n'')] \right\} = \mu_C(mr). \end{aligned}$$

For non-membership function,

$$\begin{aligned} \gamma_C(m) &= \bigwedge_{m \in \sum_{j=1}^q y_j' z_j'} \left\{ \bigvee_{1 \leq j \leq q} [\gamma_A(y_j') \vee \gamma_B(z_j')] \right\} \\ &\geq \bigwedge_{m \in \sum_{j=1}^q y_j' z_j'} \left\{ \bigvee_{1 \leq j \leq q} [\gamma_A(y_j') \vee \gamma_B(z_j' r)] \right\}, \text{ where } r \text{ is any element of } R \end{aligned}$$

$$\geq \bigwedge_{mr \subseteq \sum_{n=1}^t y''_j z''_j} \left\{ \bigvee_{1 \leq n \leq t} [\gamma_A(y''_n) \vee \gamma_B(z''_n)] \right\} = \gamma_C(mr).$$

This proves condition (iii).

Hence, C is an intuitionistic fuzzy subsemihypermodule of M . ■

Corollary 3.7. If A, B are intuitionistic fuzzy hyperideals of R , then $C [= A_B]$ is an intuitionistic fuzzy hyperideal of R , called the product of A and B .

Definition 3.8. If A, B are intuitionistic fuzzy subsets of R , then the sum of A and B is defined by, $A \oplus B = \{ \langle x, \mu_{A \oplus B}(x), \gamma_{A \oplus B}(x) \rangle \mid x \in M \}$, where $\mu_{A \oplus B}(x) = \bigvee_{x \in y \oplus z} [\mu_A(y) \wedge \mu_B(z)]$ and $\gamma_{A \oplus B}(x) = \bigwedge_{x \in y \oplus z} [\gamma_A(y) \vee \gamma_B(z)]$. $A \oplus B$ is also an intuitionistic fuzzy subset of R .

Proposition 3.9. For an intuitionistic fuzzy hyperideals A, B of R , $A \oplus B$ is an intuitionistic fuzzy hyperideal of R .

Proof: Let $x, x' \in R$. We prove that,

- (i) $\inf_{x'' \in x \oplus x'} \mu_{A \oplus B}(x'') \geq \mu_{A \oplus B}(x) \wedge \mu_{A \oplus B}(x')$ and $\sup_{x'' \in x \oplus x'} \gamma_{A \oplus B}(x'') \leq \gamma_{A \oplus B}(x) \vee \gamma_{A \oplus B}(x')$,
 (ii) $\mu_{A \oplus B}(xa) \geq \mu_{A \oplus B}(x)$ and $\gamma_{A \oplus B}(xa) \leq \gamma_{A \oplus B}(x)$.

$$\begin{aligned} \mu_{A \oplus B}(x) \wedge \mu_{A \oplus B}(x') &= \left\{ \bigvee_{x \in y \oplus z} [\mu_A(y) \wedge \mu_B(z)] \right\} \wedge \left\{ \bigvee_{x' \in y' \oplus z'} [\mu_A(y') \wedge \mu_B(z')] \right\} \\ &= \bigvee_{x \in y \oplus z} \bigvee_{x' \in y' \oplus z'} \{ [\mu_A(y) \wedge \mu_B(z)] \wedge [\mu_A(y') \wedge \mu_B(z')] \} \\ &= \bigvee_{x \in y \oplus z} \bigvee_{x' \in y' \oplus z'} \{ [\mu_A(y) \wedge \mu_A(y')] \wedge [\mu_B(z) \wedge \mu_B(z')] \} \\ &\leq \bigvee_{x \in y \oplus z} \bigvee_{x' \in y' \oplus z'} \left\{ \inf_{y'' \in y \oplus y'} \mu_A(y'') \wedge \inf_{z'' \in z \oplus z'} \mu_B(z'') \right\} \\ &\leq \inf_{x'' \in x \oplus x'} \mu_{A \oplus B}(x''). \end{aligned}$$

$$\begin{aligned} \gamma_{A \oplus B}(x) \vee \gamma_{A \oplus B}(x') &= \left\{ \bigwedge_{x \in y \oplus z} [\gamma_A(y) \vee \gamma_B(z)] \right\} \vee \left\{ \bigwedge_{x' \in y' \oplus z'} [\gamma_A(y') \vee \gamma_B(z')] \right\} \\ &= \bigwedge_{x \in y \oplus z} \bigwedge_{x' \in y' \oplus z'} \{ [\gamma_A(y) \vee \gamma_B(z)] \vee [\gamma_A(y') \vee \gamma_B(z')] \} \\ &= \bigwedge_{x \in y \oplus z} \bigwedge_{x' \in y' \oplus z'} \{ [\gamma_A(y) \vee \gamma_A(y')] \vee [\gamma_B(z) \vee \gamma_B(z')] \} \\ &\geq \bigwedge_{x \in y \oplus z} \bigwedge_{x' \in y' \oplus z'} \left\{ \sup_{y'' \in y \oplus y'} \gamma_A(y'') \vee \sup_{z'' \in z \oplus z'} \gamma_B(z'') \right\} \\ &\geq \sup_{x'' \in x \oplus x'} \gamma_{A \oplus B}(x''). \end{aligned}$$

$$\begin{aligned} \mu_{A \oplus B}(x) &= \bigvee_{x \in y \oplus z} [\mu_A(y) \wedge \mu_B(z)] \\ &\leq \bigvee_{xa \subseteq ya \oplus za} [\mu_A(ya) \wedge \mu_B(za)] \end{aligned}$$

$$\begin{aligned}
&= \mu_{A \oplus B}(xa), \text{ where } a \in R. \\
\gamma_{A \oplus B}(x) &= \bigwedge_{x \in y \oplus z} [\gamma_A(y) \vee \gamma_B(z)] \\
&\geq \bigwedge_{xa \subseteq ya \oplus za} [\gamma_A(ya) \vee \gamma_B(za)] \\
&= \gamma_{A \oplus B}(xa), \text{ where } a \in R.
\end{aligned}$$

Hence, $A \oplus B$ is an intuitionistic fuzzy hyperideal of R . ■

4 Fully Idempotent Semihyperrings

In this section, we investigate [4] fully idempotent semihyperrings, that is, semihyperrings all of whose hyperideals are idempotent in the context of intuitionistic fuzzy concept.

Definition 4.1. An intuitionistic fuzzy hyperideal D of a semihyperring R is called an *intuitionistic fuzzy prime hyperideal* of R if for an intuitionistic fuzzy hyperideals A, B of $R, C [= A_B] \subseteq D \Rightarrow \mu_A(x) \leq \mu_D(x)$ and $\gamma_A(x) \geq \gamma_D(x)$ or $\mu_B(x) \leq \mu_D(x)$ and $\gamma_B(x) \geq \gamma_D(x)$.

Definition 4.2. An intuitionistic fuzzy hyperideal D of a semihyperring R is called an *intuitionistic fuzzy irreducible hyperideal* if for an intuitionistic fuzzy hyperideals A, B of $R, A \cap B = D \Rightarrow A = D$ or $B = D \Rightarrow A \subseteq D$ and $D \subseteq A$ or $B \subseteq D$ and $D \subseteq B$

Theorem 4.3. Let R be a semihyperring. Then the following conditions are equivalent:

- (i) R is fully idempotent.
- (ii) Any intuitionistic fuzzy hyperideal of R is idempotent.
- (iii) For each pair of intuitionistic fuzzy hyperideals A, B of $R, A \cap B = C$, where $C [= A_B]$ is an intuitionistic fuzzy hyperideal of R .

Proof: (i) \Rightarrow (ii): Let R be fully idempotent and A be an intuitionistic fuzzy hyperideal of R . We prove that, $A^2 = A$.

Let $x \in R, A^2 = AA = \{ \langle x, \mu_{AA}(x), \gamma_{AA}(x) \rangle \}$.

$$\begin{aligned}
\mu_{AA}(x) &= \bigvee_{x \in \sum_{i=1}^p y_i z_i} \left\{ \bigwedge_{1 \leq i \leq p} [\mu_A(y_i) \wedge \mu_A(z_i)] \right\} \\
&\leq \bigvee_{x \in \sum_{i=1}^p y_i z_i} \left\{ \bigwedge_{1 \leq i \leq p} [\mu_A(y_i z_i) \wedge \mu_A(y_i z_i)] \right\} \\
&= \bigvee_{x \in \sum_{i=1}^p y_i z_i} \left\{ \bigwedge_{1 \leq i \leq p} \mu_A(y_i z_i) \right\} \wedge \left\{ \bigwedge_{1 \leq i \leq p} \mu_A(y_i z_i) \right\} \\
&\leq \bigvee_{x \in \sum_{i=1}^p y_i z_i} [\mu_A(x) \wedge \mu_A(x)] = \mu_A(x)
\end{aligned}$$

Thus, $\mu_{AA}(x) \leq \mu_A(x)$.

Similarly, we can prove $\gamma_{AA}(x) \geq \gamma_A(x)$.

Since, each hyperideal of R is idempotent, $(x) = (x)^2$ for each $x \in R$ and $x \in (x)^2 = RxRRxR$, it follows that $x \in \sum_{j=1}^q a_j x a'_j b_j x b'_j$ where $a_j, a'_j, b_j, b'_j \in R, q \in N$. Now

$$\begin{aligned} \mu_A(x) &= \mu_A(x) \wedge \mu_A(x) \\ &\leq \mu_A(a_j x a'_j) \wedge \mu_A(b_j x b'_j) \\ \Rightarrow \mu_A(x) &\leq \bigwedge_{1 \leq j \leq q} [\mu_A(a_j x a'_j) \wedge \mu_A(b_j x b'_j)] \\ &\leq \bigvee_{x \in \sum_{j=1}^q a_j x a'_j b_j x b'_j} \left\{ \bigwedge_{1 \leq j \leq q} [\mu_A(a_j x a'_j) \wedge \mu_A(b_j x b'_j)] \right\} \\ &\leq \bigvee_{x \in \sum_{j=1}^q y_j z_j} \left\{ \bigwedge_{1 \leq j \leq q} [\mu_A(y_j) \wedge \mu_A(z_j)] \right\} = \mu_{AA}(x) \end{aligned}$$

Thus, $\mu_A(x) \leq \mu_{AA}(x)$.

Similarly, we can prove $\gamma_A(x) \geq \gamma_{AA}(x)$.

Thus, $\mu_A(x) = \mu_{AA}(x)$ and $\gamma_A(x) = \gamma_{AA}(x)$. Hence, A is idempotent.

(ii) \Rightarrow (i): Let I be a hyperideal of R . Then the characteristic function of I , χ_I is an intuitionistic fuzzy hyperideal of R . That is,

$$\chi_I^2 = \chi_I \Rightarrow \chi_I \chi_I = \chi_{I^2} \Rightarrow I^2 = I.$$

(i) \Rightarrow (iii): Let A, B be any pair of intuitionistic fuzzy hyperideals of R and let $x \in R$. Then, $C = \{ \langle x, \mu_C(x), \gamma_C(x) \rangle \}$.

$$\begin{aligned} \mu_C(x) &= \bigvee_{x \in \sum_{i=1}^p y_i z_i} \left\{ \bigwedge_{1 \leq i \leq p} [\mu_A(y_i) \wedge \mu_B(z_i)] \right\} \\ &\leq \bigvee_{x \in \sum_{i=1}^p y_i z_i} \left\{ \bigwedge_{1 \leq i \leq p} [\mu_A(y_i z_i) \wedge \mu_B(y_i z_i)] \right\} \\ &= \bigvee_{x \in \sum_{i=1}^p y_i z_i} \left\{ \bigwedge_{1 \leq i \leq p} \mu_A(y_i z_i) \right\} \wedge \left\{ \bigwedge_{1 \leq i \leq p} \mu_B(y_i z_i) \right\} \\ &\leq \bigvee_{x \in \sum_{i=1}^p y_i z_i} [\mu_A(x) \wedge \mu_B(x)] \\ &= \mu_A(x) \wedge \mu_B(x) \end{aligned}$$

Thus, $\mu_C(x) \leq \mu_A(x) \wedge \mu_B(x)$.

By assumption, R is fully idempotent: $(x) = (x)^2$ for any $x \in R$ and hence $\mu_A(x) \wedge \mu_B(x) \leq \bigvee_{x \in \sum_{i=1}^p y_i z_i} \left\{ \bigwedge_{1 \leq i \leq p} [\mu_A(y_i) \wedge \mu_B(z_i)] \right\} = \mu_C(x)$.

That is, $\mu_A(x) \wedge \mu_B(x) \leq \mu_C(x)$. Thus, $\mu_C(x) = \mu_A(x) \wedge \mu_B(x)$.

Similarly, we can prove $\gamma_C(x) = \gamma_A(x) \vee \gamma_B(x)$.

Hence, $C = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle \} = A \cap B$.

(iii)⇒(i): Let A, B be any pair of intuitionistic fuzzy hyperideals of R . Then, $C = A \cap B$ is any intuitionistic fuzzy hyperideal of R . Taking $B = A$, we have $C = A \cap A = A$ is any intuitionistic fuzzy hyperideal of R . Thus, R is fully idempotent. ■

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