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Two stage approach to solve extended transportation problem

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Abstract

The main objective of transportation problem is to minimise the cost of shipping homogeneous commodity from various origins to various destinations. The classical transportation problem is extended by considering multiple incommensurate inputs and outputs for each shipment link. The relative efficiency concept is defined for each possible shipment link. Two linear programming models are proposed to determine the optimal transportation plan with maximum efficiency. A numerical example is discussed to show the applicability of said approach.

Keywords: Data Envelopment Analysis (DEA), Decision Making Unit (DMU), Relative Efficiency, Transportation Problem (TP).

AMS Subject Classification (2010): 90C08.

1 Introduction

The transportation problem is an important and commonly applicable in the field of operations research. It is a subclass of linear programming problem and had a great attention in the literature. The main objective of transportation problem is to minimise the cost of shipping homogeneous commodity from various origins to various destinations with respective rim requirements. During the formulation of classical transportation problem, only cost or profit for each possible shipment link is considered. In many real applications, several kinds of variables such as cost, distance, shipment value, manpower, profit etc (i.e. multiple inputs and multiple outputs) may be involved for each possible shipment link, which are to be considered in the shipment plan. The decision makers may have different aims to achieve for each possible shipment link, which may conflict to each other. In such situation, we are interested to decide an optimal transportation plan with maximum relative efficiency.

Relative efficiency is calculated by using Data Envelopment Analysis (DEA). DEA is a mathematical approach which assesses the comparative efficiency of a set of decision making units (DMUs) such as airlines, railways, banks, automobile manufacturers, hospitals, universities, etc. Charnes et al. [3] introduced DEA in the literature. DEA has become popular in the practise and research of efficiency analysis. A number of DEA applications and research have led to many new developments in concepts and methodologies related to the DEA efficiency analysis. Some methods were suggested for estimating technical and scale inefficiencies (e.g., Banker et al.,[2]; Cooper et al. [3,6] in data envelopment analysis. Charnes, Cooper, and Rhodes [3] suggested a model to compute relative efficiency of various DMUs, named as CCR model. The computation of relative efficiency by the CCR model is based on constant returns to scale. Banker, Charnes, and Cooper [2] suggested the other model for DEA, named as BCC model. The BCC model is more flexible and allows variable returns to scale. The treatment of returns to scale is the primary difference between the BCC and the CCR model.

The literature available on transportation problem with multiple inputs and outputs is limited. Chen and Lu [5] extended the assignment problem by considering multiple inputs and outputs. Alireza Amirteimoori [1] has extended transportation problem by DEA based approach. As far as we aware, there is no work regarding our proposed approach in the literature. We extend the transportation problem by considering multiple inputs and outputs by using BCC model for each possible shipment link. The relative efficiency for each possible shipment link is defined. The shipment plan with maximum efficiency is considered as an optimal plan to the transportation problem.

2 BCC Model

The evaluation of a DMU has long been recognized to be a problem of considerable complexity. This evaluation becomes more difficult when it involves multiple inputs and multiple outputs, in that a set of weights has to be determined to aggregate the outputs and inputs separately to form a ratio as efficiency. To do so, DEA approach is proposed, which allows every DMU to select their most favourable weights while requiring the resulted ratio of the aggregated outputs to the aggregated inputs of all DMUs to be less than or equal to 1.

Consider *n* DMUs, each consumes varying amounts of *m*-different inputs to produce s- different outputs. In model formulation, y_{rj} (r=1,2,...s) and x_{ij} (i=1,2,.....m) denotes the non-negative output and input values respectively for j^{th} DMU denoted as DMU_j ; j =1,2,....n. One of the DMUs is considered for evaluation, accorded the designation DMU_o, and placed in the functional form to maximise output, while also leaving it in the constraints. The seminal programming statement for the (input oriented) BCC model is:

$$\text{Maximise} \quad \sum_{r=1}^{s} u_r y_{ro}$$

$$\text{Subject to:} \quad v_0 + \sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{ro} - v_0 - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad \text{for all } j,$$

$$u_r,\ v_i \ge \varepsilon, \qquad \text{for all } r \text{ and } i,$$

$$v_0 \text{ is unrestricted and } \varepsilon > 0. \qquad \text{(M1)}$$

where, $u_r(r=1,2,...,s)$ and $v_i(i=1,2,...,m)$ are the weights associated with output r and input i, respectively and ε is a non-Archimedean infinitesimal. Apart from the restriction that no weight may be zero, weights on inputs and outputs are only restricted by the requirements that they must not make the efficiency of any DMU more than 1.The benefit of allowing such freeness on the weights is that, a best efficiency rating is associated to each DMU. DMU_o is rated as relatively efficient if the optimal value of the objective function is equal to one; otherwise it is rated as relatively inefficient.

3 Classical Transportation Problem

Consider the transportation problem with m-warehouses and n-destinations. The i^{th} warehouse contain a_i homogeneous commodities (i = 1,2,…,m), while j^{th} destination requires b_j homogeneous commodities (j = 1,2,...,n).We assume that the total available commodities equals to the total demand, that is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Let c_{ij} be the cost for shipping a unit commodity from warehouse i to destination j. Let x_{ij} be the number of units shipped from i^{th} warehouse to j^{th} destination. The problem is to determine a feasible shipping plan from warehouses to destinations such that the total transportation cost be minimised. We write transportation problem as the linear programming problem as follows.

$$\text{Minimise } \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{Subject to: } \quad \sum_{j=1}^{n} x_{ij} = a_i \qquad \text{for all } i,$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \text{for all } j,$$

$$x_{ij} \geq 0 \qquad \text{for all } i, j. \qquad \text{(M2)}$$

The simplex algorithm can be used to solve the forgoing transportation problem (Mokhtar S.Bazaraa et al.[8]).

4 The Proposed Approach

This paper extends a classical transportation problem by considering multiple incommensurate inputs and outputs for each possible shipping link. Consider m-warehouses, where i^{th} warehouse has availability of a_i units of a commodity. Consider n-destinations, where j^{th} destination has requirements of b_j units of a commodity. For each possible shipping link (i , j) , the inputs and outputs are denoted as $X_{ij} = \left(x_{ij}^{(1)}, x_{ij}^{(2)}, \cdots, x_{ij}^{(s)}\right)$ and $Y_{ij} = \left(y_{ij}^{(1)}, y_{ij}^{(2)}, \cdots, y_{ij}^{(t)}\right)$ respectively. So, for each possible link(i, j),there are s+t attributes, s- inputs $x_{ij}^{(k)}$, k = 1,2, ...s and t-outputs $y_{ij}^{(l)}$, l = 1,2,....t. We solve such problem in two stages as below.

Stage-I:

In this stage, the DEA technique is used to calculate efficiencies for each possible shipment plan in the problem. For each warehouse i, we consider all destinations j, (j = 1,2,…, n) and suppose that each possible link (i , j) is a DMU$^{(1)}$. With the warehouse i as a target, the efficiency of the unit shipment from i to j (j=1,2,..., n) can be determined by using the DEA technique(BCC model).

According to the BCC model (M1), we have relative efficiency of i^{th} warehouse with link (i ,j) as follows.

$$e_{ij}^{(1)} = \text{Max} \sum_{r=1}^{t} u_r \, y_{ij}^{(r)}$$

Subject to:

$$v_0 + \sum_{k=1}^{s} v_k \, x_{ij}^{(k)} = 1$$

$$\sum_{r=1}^{t} u_r \, y_{ij}^{(r)} - v_0 - \sum_{k=1}^{s} v_k \, x_{ij}^{(k)} \le 0 \quad \text{for all } j,$$

$$u_r \ge \varepsilon \quad \text{for all } r,$$

$$v_k \ge \varepsilon \quad \text{for all } k,$$

$$v_0 \text{ is unrestricted and } \varepsilon > 0. \qquad \text{(M3)}$$

Using model (M3), we can obtain the relative efficiency of i^{th} warehouse as $e_{i1}^{(1)}, e_{i2}^{(1)}, ...e_{in}^{(1)}$ by changing the target warehouse in the model.

Meanwhile, for each destination j we consider all warehouses i (i = 1,2,...m) and each possible link (i ,j) is considered as DMU$^{(2)}$. With destination j as target, the efficiency of the unit shipment from i to j (i =1,2,....m) can be determined by using the DEA technique(BCC model).

According to the BCC model (M1), we have relative efficiency of j^{th} destination with link (i ,j) as follows.

$$e_{ij}^{(2)} = \text{Max} \sum_{r=1}^{t} u_r \, y_{ij}^{(r)}$$

Subject to:

$$v_0 + \sum_{k=1}^{s} v_k \, x_{ij}^{(k)} = 1$$

$$\sum_{r=1}^{t} u_r \, y_{ij}^{(r)} - v_0 - \sum_{k=1}^{s} v_k \, x_{ij}^{(k)} \le 0 \quad \text{for all } i,$$

$$u_r \ge \varepsilon \quad \text{for all } r,$$

$$v_k \ge \varepsilon \quad \text{for all } k,$$

$$v_0 \text{ is unrestricted and } \varepsilon > 0. \qquad \text{(M4)}$$

Using model (M4), we can obtain the relative efficiency of j^{th} destination to each warehouse as $e_{1j}^{(2)}$, $e_{2j}^{(2)}$, ..., $e_{mj}^{(2)}$ by changing the target destination in the model.

The two groups of relative efficiencies are obtained for the comparisons from either the warehouse side or the destinations side. For the transportation problem with multiple inputs and outputs, we need to optimise the total efficiency for entire shipment. Therefore, we construct a composite efficiency index to integrate two kinds of relative efficiencies as follows.

$$e_{ij} = \frac{e_{ij}^{(1)} + e_{ij}^{(2)}}{2} \qquad \text{for all } i, j. \qquad \text{(M5)}$$

We consider the values of the composite efficiency index e_{ij} as performance measure of DMUs for each shipment link (i, j).

Also, we consider a performance measure of DMUs for each shipment link (i, j) by assigning maximum efficiency e_{ij}^{*} for each shipment link (i, j) as follows.

$$e_{ij}^{*} = \text{Max}\{ e_{ij}^{(1)}, e_{ij}^{(2)} \} \qquad \text{for all } i, j. \qquad \text{(M6)}$$

Stage-II:

In this stage, we determine the shipment plan with the maximum efficiency. This is achieved by solving the model as

$$\text{Minimise } \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - e_{ij}\right) x_{ij}$$

$$\text{Subject to: } \sum_{j=1}^{n} x_{ij} = a_i \qquad \text{for all } i,$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \text{for all } j,$$

$$x_{ij} \geq 0 \qquad \text{for all } i, j. \qquad \text{(M7)}$$

Similarly for e_{ij}^{*}, we solve the model as

$$\text{Minimise } \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - e_{ij}^{*}\right) x_{ij}$$

$$\text{Subject to: } \sum_{j=1}^{n} x_{ij} = a_i \qquad \text{for all } i,$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \text{for all } j,$$

$$x_{ij} \geq 0 \qquad \text{for all } i, j. \qquad \text{(M8)}$$

The problems (M7) and (M8) are classical transportation problems and can be solved by usual algorithm.

5 Numerical Examples

Suppose the manufacturer of two wheeler motorbike has four plants located at places A, B, C and D. The production send to seven major cities E, F, G, H, I, J and K. The company manager considers one input (shipping cost) and two outputs (the value of shipment and profit).The appropriate input-output, availabilities (a_i) and requirements (b_j) are listed in [Table 1]. Each ordered triplet (x_1, y_1, y_2) shows shipping cost, value of shipment and profit respectively.

Table 1: Extended Balanced transportation problem.

Places	Major City							a_i
	E	F	G	H	I	J	K	
A	(9,80,425)	(1,75,390)	(5,79,424)	(6,80,495)	(1,75,360)	(1,93,408)	(1,70,345)	35
B	(4,77,260)	(4,73,498)	(9,92,420)	(5,94,322)	(7,73,365)	(3,92,280)	(6,72,230)	25
C	(6,85,345)	(6,79,290)	(2,87,333)	(1,87,445)	(2,74,390)	(2,82,360)	(8,83,456)	18
D	(3,79,250)	(4,91,442)	(3,91,365)	(9,82,288)	(8,92,415)	(3,74,295)	(5,95,409)	22
b_j	15	25	10	18	7	16	9	

We have solved the data given in Table 1 by using models (M3) and (M4). The efficiencies for the set of DMUs$^{(1)}$ and DMUs$^{(2)}$ are obtained and given in Table 2.

Table 2: Efficiencies by using model (M3) and (M4) for the set of DMUs$^{(1)}$ and DMUs$^{(2)}$ for the data given in Table 1.

Places	Efficiency	Major City						
		E	F	G	H	I	J	K
A	$e_{ij}^{(1)}$	.9321	.9559	.9246	1	.8824	1	.8456
	$e_{ij}^{(2)}$	1	1	1	1	1	1	1
B	$e_{ij}^{(1)}$	.8368	1	1	1	.8445	1	.7659
	$e_{ij}^{(2)}$	.9506	1	1	1	.8884	.9893	.7579
C	$e_{ij}^{(1)}$	.9770	.9080	1	1	.8733	.9425	1
	$e_{ij}^{(2)}$	1	.8681	1	1	1	.8824	1
D	$e_{ij}^{(1)}$	.8681	1	1	.8632	.9843	.8132	1
	$e_{ij}^{(2)}$	1	1	1	.8759	1	.7957	1

We calculate efficiencies for the set of DMUs$^{(1)}$ and DMUs$^{(2)}$ by using Alireza Amirteimoori [1] approach and are given in Table 3.

Table 3: Efficiencies by Alireza Amirteimoori approach:

Places	Efficiency	Major City						
		E	F	G	H	I	J	K
A	$e_{ij}^{(1)}$	.1157	.9559	.2078	.2022	.8824	1	.8456
	$e_{ij}^{(2)}$	.5667	1	.5093	.1854	1	1	1
B	$e_{ij}^{(1)}$	.6654	1	.4247	.6552	.4599	1	.4019
	$e_{ij}^{(2)}$	.7800	.3192	.2803	.2161	.1448	.3297	.1714
C	$e_{ij}^{(1)}$	.1628	.1513	.5	1	.4382	.4713	.1281
	$e_{ij}^{(2)}$	.69	.1756	1	1	.5417	.4412	.1652
D	$e_{ij}^{(1)}$	.8681	.9082	1	.3004	.4264	.8132	.6723
	$e_{ij}^{(2)}$	1	.3033	.7307	.1047	.1533	.2652	.2714

We determine composite efficiency index (e_{ij}) associated with the particular shipment link (i ,j) by using (M5). The efficiency index (e_{ij}^{*}) associated with the particular link (i ,j) is decided by using M(6).

The optimal shipment plan is obtained by solving model (M7) of proposed approach and is as below:
X_{AF}=1; X_{AH}=18; X_{AJ}=16; X_{BF}=18; X_{BG}=7; X_{CE}=15; X_{CG}=3; X_{DF}=6; X_{DI}=7; X_{DK}=9, with minimum value of objective function 0.2495.

Similarly, the optimal shipment plan is obtained by solving model (M8) of proposed approach and is as below:
X_{AE}=2; X_{AF}=17; X_{AJ}=16; X_{BF}=8; X_{BG}=10; X_{BH}=7; X_{CH}=11; X_{CI}=7; X_{DE}=13; X_{DK}=9, with minimum value of objective function zero.

We solved our numerical problem by using an approach suggested by Alireza Amirteimoori (2012) and the solution for model (M7) is obtained as:
X_{AF}=3; X_{AI}=7; X_{AJ}=16; X_{AK}=9; X_{BE}=3; X_{BF}=22; X_{BH}=0; X_{CH}=18; X_{DE}=12; X_{DG}=10,with minimum value of objective function 11.6312.

The problem given in Table 1 is solved by proposed approach and by Alireza Amirteimoori approach. The results are summarised in Table 3.

Table 4: Summary of solutions of models (M7) and (M8).

Problem cited in	Solution of Model (M7)				Solution of Model (M8)	
	Proposed method		Alireza Amirteimoori approach		Proposed method	
	Value of objective function	Efficiency of shipment plan	Value of objective function	Efficiency of shipment plan	Value of objective function	Efficiency of shipment plan
Table 1	0.2495	99.7508	11.6312	88.3689	0	100

5 Conclusion

The relative efficiency of each possible link is considered as a measure of performance to decide a transportation plan with maximum efficiency. The proposed approach is useful when the decision maker has multiple goals to achieve for each possible shipment link. These goals may be in conflict to each other. A numerical example is considered to show the applicability of the said approach to real life situation. In this example, we have seen that our proposed approach provides more efficient solution than the approach suggested by Alireza Amirteimoori [1].

In case of unbalanced problem, the efficiencies of shipment link related to dummy source and dummy destination are found to be zero and remaining efficiencies are unchanged. For unbalanced transportation problem, Alireza Amirteimoori[1] approach is not applicable. So, we suggest proposed approach to solve both balanced and unbalanced transportation problem with multiple flexible inputs and multiple outputs.

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