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Cordial labeling in the context of Duplication of some graph elements

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Abstract

Let $G = (V(G), E(G))$ be a graph and let $f : V(G) \to \{0, 1\}$ be a mapping from the set of vertices to {0,1} and for each edge $uv \in E$ assign the label $|f(u) - f(v)|$. If the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labled with 0 and the number of edges labeled with 1 differ by at most 1, then f is called a cordial labeling. In this paper we discuss cordial labeling for duplication of certain graph element of cycle graph and path graph.

Keywords: Graph labeling, cordial labeling, cordial graph.
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1 Introduction

We begin with simple, finite, undirected graph $G = (V(G), E(G))$ where $V(G)$ and $E(G)$ denotes the vertex set and the edge set respectively. For all other terminology we follow Gross [3]. We present the brief summary of definitions which are useful for the present work.

Definition 1.1. The *graph labeling* is an assignment of numbers to the vertices or edges or both subject to certain condition(s).
A detailed survey of various graph labeling is given in Gallian [2].

Definition 1.2. For a graph $G = (V(G), E(G))$, a mapping $f : V(G) \to \{0, 1\}$ is called a *binary vertex labeling* of G and $f(v)$ is called the *label* of the vertex v of G under f. For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ is defined as $f^*(uv) = |f(u) - f(v)|$.
Let $v_f(0), v_f(1)$ be the number of vertices of G with labels 0 and 1 respectively under f and let $e_f(0), e_f(1)$ be the number of edges with labels 0 and 1 respectively under f^*.

Definition 1.3. [7] Duplication of a vertex v of a graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'.

Definition 1.4. [5] Duplication of a vertex v_k by a new edge $e = v'_k v''_k$ in a graph G produces a new graph G' such that $N(v'_k) = \{v_k, v''_k\}$ and $N(v''_k) = \{v_k, v'_k\}$.

Definition 1.5. [5] Duplication of an edge $e = uv$ by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$.

Definition 1.6. [6] Duplication of an edge $e = uv$ of a graph G produces a new graph G' by adding an edge $e' = u'v'$ such that $N(u') = N(u) \cup \{v'\} - \{v\}$and $N(v') = N(v) \cup \{u'\} - \{u\}$.

Definition 1.7. A binary vertex labeling f of a graph G is called *cordial labeling* if $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$. A graph G is said to be cordial if it admits cordial labeling.

The concept of cordial labeling was introduced by Cahit [1] in which he proved that the wheel W_n is cordial if and only if $n \not\equiv 3 \pmod 4$. Vaidya and Dani [6] proved that the graphs obtained by duplication of an arbitrary edge of a cycle and a wheel admit a cordial labeling. Prajapati and Gajjar [4] proved that complement of wheel graph W_n and complement of cycle graph C_n are cordial if $n \not\equiv 4,7 \pmod 8$.

2 Duplication of graph elements in cycle

Theorem 2.1. The graph obtained by duplication of every vertex by an edge in C_n is cordial.

Proof: Let $n \geq 3$. Let $u_0, u_1, \ldots, u_{n-1}$ be the consecutive vertices of C_n and G be the graph obtained by duplication of each of the vertices u_i in C_n by a new edge $v_i w_i$ for $i = 0, 1, \ldots, n-1$. Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = u_i, i = \{0, 1, \ldots, n-1\}; \\ 0 & \text{if } x = v_i, i = \{0, 1, \ldots, n-1\}; \\ 0 & \text{if } x = w_i, 0 \leq i \leq \left\lfloor \dfrac{n-1}{2} \right\rfloor; \\ 1 & \text{if } x = w_i, \left\lfloor \dfrac{n-1}{2} \right\rfloor < i \leq n-1. \end{cases}$$

In view of the function f we have $|v_f(1) - v_f(0)| \leq 1$.
We have the following cases:

(A) If $e = u_i v_i$ for all $i = 0, 1, \ldots, n-1$ then there are n edges with label 1 and no edge with label 0.

(B) If $e = v_i w_i$ for all $i = 0, 1, \ldots, n-1$ then there are $\left\lfloor \frac{n}{2} \right\rfloor$ edges with label 1 and $\left\lceil \frac{n}{2} \right\rceil$ edges with label 0.

(C) If $e = u_i w_i$ for all $i = 0, 1, \ldots, n-1$ then there are $\left\lceil \frac{n}{2} \right\rceil$ edges with label 1 and $\lfloor \frac{n}{2} \rfloor$ edges with label 0.

(D) If $e = u_i u_{i+1}$ for all $i = 0, 1, \ldots, n-2$ or $e = u_{n-1}u_0$ then there are no edge with label 1 and n edges with label 0.

From all the above cases, we have
$e_f(1) = n + \left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil + 0 = n + \left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil$ and $e_f(0) = 0 + \left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil + n = \left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil + n$.
So, $|e_f(1) - e_f(0)| = \left| \left(n + \left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil \right) - \left(\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil + n \right) \right| = 0 \leq 1$.
So, G admits a cordial labeling f and hence G is cordial. ■

Theorem 2.2. The graph obtained by duplication of every edge by a vertex in C_n is cordial if $n \not\equiv 2 \pmod 4$.

Proof: Let $n \geq 3$. Let $u_0, u_1, \ldots, u_{n-1}$ be the consecutive vertices of C_n and G be the graph obtained by duplication of each of the edges $u_0u_1, u_1u_2, \ldots, u_{n-2}u_{n-1}, u_{n-1}u_0$ in C_n by the corresponding new vertices $v_0, v_1, \ldots, v_{n-1}$ respectively.
We have the following cases:
Case 1: $n \equiv 0 \pmod 4$. Thus $n = 4k$ for some $k \in N$.
Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = v_i, i = \{0, 1, \ldots, k-1\}; \\ 0 & \text{if } x = v_i, i = \{k, k+1, \ldots, n-1\}; \\ 1 & \text{if } x = u_i, i = \{0, 1, \ldots, 2k-1\}; \\ 1 & \text{if } x = u_i, i = \{2k, 2k+2, \ldots, n-2\}; \\ 0 & \text{if } x = u_i, i = \{2k+1, 2k+3, \ldots, n-1\}. \end{cases}$$

It is easy to check that $v_f(1) = 4k$ and $v_f(0) = 4k$.Thus $|v_f(1) - v_f(0)| \leq 1$. The induced edge labeling $f^* : E(G) \to \{0, 1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$.
We have the following sub-cases:

(A) If $e = u_i v_i$ for all $i = 0, 1, \ldots, n-1$ then there are $2k$ edges with label 1 and $2k$ edges with label 0.

(B) If $e = u_i v_{i-1}$ for all $i = 1, 2, \ldots, n-1$ or $e = u_0 v_{n-1}$ then there are $2k$ edges with label 1 and $2k$ edges with label 0.

(C) If $e = u_iu_{i+1}$ for all $i = 0, 1, \dots, n-2$ or $e = u_{n-1}u_0$ then there are $2k$ edges with label 1 and $2k$ edges with label 0.

From all the sub cases, we have $e_f(1) = 2k + 2k + 2k = 6k$ and $e_f(0) = 2k + 2k + 2k = 6k$.
So, $|e_f(1) - e_f(0)| = |6k - 6k| \leq 1$.
Case 2: $n \equiv 1 \pmod 4$. Thus $n = 4k + 1$ for some $k \in N$.
Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = v_i, i = \{0, 1, \dots, k-1\}; \\ 0 & \text{if } x = v_i, i = \{k, k+1, \dots, n-1\}; \\ 1 & \text{if } x = u_i, i = \{0, 1, \dots, 2k\}; \\ 1 & \text{if } x = u_i, i = \{2k+1, 2k+3, \dots, n-2\}; \\ 0 & \text{if } x = u_i, i = \{2k+2, 2k+4, \dots, n-1\}. \end{cases}$$

It is easy to check that $v_f(1) = 4k + 1$ and $v_f(0) = 4k + 1$. Thus $|v_f(1) - v_f(0)| \leq 1$.
We have the following sub-cases:

(A) If $e = u_iv_i$ for all $i = 0, 1, \dots, n-1$ then there are $2k + 1$ edges with label 1 and $2k$ edges with label 0.

(B) If $e = u_iv_{i-1}$ for all $i = 1, 2, \dots, n-1$ or $e = u_0v_{n-1}$ then there are $2k + 1$ edges with label 1 and $2k$ edges with label 0.

(C) If $e = u_iu_{i+1}$ for all $i = 0, 1, \dots, n-2$ or $e = u_{n-1}u_0$ then there are $2k$ edges with label 1 and $2k + 1$ edges with label 0.

From all the above sub-cases, we have
$e_f(1) = (2k + 1) + (2k + 1) + 2k = 6k + 2$ and $e_f(0) = 2k + 2k + (2k + 1) = 6k + 1$.
So, $|e_f(1) - e_f(0)| \leq 1$.
Case 3: $n \equiv 3 \pmod 4$. Thus $n = 4k + 3$ for some $k \in N$.
Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = v_i, i = \{0, 1, \dots, k\}; \\ 0 & \text{if } x = v_i, i = \{k+1, k+2, \dots, n-1\}; \\ 1 & \text{if } x = u_i, i = \{0, 1, \dots, 2k\}; \\ 1 & \text{if } x = u_i, i = \{2k+1, 2k+3, \dots, n-2\}; \\ 0 & \text{if } x = u_i, i = \{2k+2, 2k+4, \dots, n-1\}. \end{cases}$$

It is easy to check that $v_f(1) = 4k + 3$ and $v_f(0) = 4k + 3$. Thus $|v_f(1) - v_f(0)| \leq 1$.
We have the following sub-cases:

(A) If $e = u_iv_i$ for all $i = 0, 1, \ldots, n-1$ then there are $2k+1$ edges with label 1 and $2k+2$ edges with label 0.

(B) If $e = u_iv_{i-1}$ for all $i = 1, 2, \ldots, n-1$ or $e = u_0v_{n-1}$ then there are $2k+1$ edges with label 1 and $2k+2$ edges with label 0.

(C) If $e = u_iu_{i+1}$ for all $i = 0, 1, \ldots, n-2$ or $e = u_{n-1}u_0$ then there are $2k+2$ edges with label 1 and $2k+1$ edges with label 0.

From all the above sub-cases, we have $e_f(1) = (2k+1) + (2k+1) + (2k+2) = 6k+4$ and $e_f(0) = (2k+2) + (2k+2) + (2k+1) = 6k+5$. Therefore $|e_f(1) - e_f(0)| \leq 1$.
So from each of the cases, G admits a cordial labeling f if $n \not\equiv 2 \pmod 4$. Hence the graph obtained by duplication of every edge by a vertex in C_n is cordial if $n \not\equiv 2 \pmod 4$. ■

Theorem 2.3. The graph obtained by duplication of every edge by a vertex in C_n is not cordial if $n \equiv 2 \pmod 4$.

Proof: Here $n = 4k+2$ for some $k \in N$. Let G be the graph obtained by duplication of every edge by a vertex of C_n. G has $8k+4$ vertices and $12k+6$ edges. The graph G is an edge disjoint union of $4k+2$ triangles .
Assume that G is cordial.
For each $i \in \{0, 1, 2, 3\}$, let T_i be the number of triangles of G having exactly i vertices with label 1. So, $T_0 + T_1 + T_2 + T_3 = 4k+2$.
For a triangle with each vertex with label 0, there is no edge with label 1 and 3 edges with label 0. For a triangle with one vertex with label 1 and two vertices with label 0, there are 2 edges with label 1 and 1 edge with label 0. For a triangle with two vertices with label 1 and one vertex with label 0, there are 2 edges with label 1 and 1 edge with label 0. For a triangle with each vertex with label 1, there is no edge with label 1 and 3 edges with label 0. As G is a cordial with even number of edges, the number of edges with label 0 and the number of edges with label 1 are same.
Thus, $3T_0 + T_1 + T_2 + 3T_3 = 2T_1 + 2T_2$
$\Rightarrow 3(T_0 + T_3) = T_1 + T_2$
$\Rightarrow 3(T_0 + T_3) = (4k+2) - (T_0 + T_3)$
$\Rightarrow (T_0 + T_3) = \frac{2k+1}{2}$, is not an integer, which is a contradiction. Therefore, G is not cordial. ■

3 Duplication of graph elements in path

Theorem 3.1. The graph obtained by duplication of every vertex by an edge in P_n is cordial.

Proof: Let $u_0, u_1, \ldots, u_{n-1}$ be the consecutive vertices of P_n and G be the graph obtained by duplication of each of the vertices u_i in P_n by a new edge v_iw_i for $i = 0, 1, \ldots, n-1$.

Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = u_i, i = \{0, 1, \dots, n-1\}; \\ 0 & \text{if } x = v_i, i = \{0, 1, \dots, n-1\}; \\ 0 & \text{if } x = w_i, 0 \le i \le \left\lfloor \dfrac{n-2}{2} \right\rfloor; \\ 1 & \text{if } x = w_i, \left\lfloor \dfrac{n-2}{2} \right\rfloor < i \le n-1. \end{cases}$$

In view of the previous defined function f we have $|v_f(1) - v_f(0)| \le 1$.
We have the following cases:

(A) If $e = u_i v_i$ for all $i = 0, 1, \dots, n-1$ then there are n edges with label 1 and no edge with label 0.

(B) If $e = v_i w_i$ for all $i = 0, 1, \dots, n-1$ then there are $\left\lceil \dfrac{n}{2} \right\rceil$ edges with label 1 and $\lfloor \frac{n}{2} \rfloor$ edges with label 0.

(C) If $e = u_i w_i$ for all $i = 0, 1, \dots, n-1$ then there are $\left\lfloor \dfrac{n}{2} \right\rfloor$ edges with label 1 and $\left\lceil \dfrac{n}{2} \right\rceil$ edges with label 0.

(D) If $e = u_i u_{i+1}$ for all $i = 0, 1, \dots, n-2$ then there are no edge with label 1 and $n-1$ edges with label 0.

From all the above cases, we have $e_f(1) = n + \left\lceil \dfrac{n}{2} \right\rceil + \left\lfloor \dfrac{n}{2} \right\rfloor + 0 = n + \left\lceil \dfrac{n}{2} \right\rceil + \left\lfloor \dfrac{n}{2} \right\rfloor$ and $e_f(0) = 0 + \left\lceil \dfrac{n}{2} \right\rceil + \left\lfloor \dfrac{n}{2} \right\rfloor + (n-1) = \left\lceil \dfrac{n}{2} \right\rceil + \left\lfloor \dfrac{n}{2} \right\rfloor + (n-1)$.
So, $|e_f(1) - e_f(0)| = \left| \left(n + \left\lceil \dfrac{n}{2} \right\rceil + \left\lfloor \dfrac{n}{2} \right\rfloor \right) - \left(\left\lceil \dfrac{n}{2} \right\rceil + \left\lfloor \dfrac{n}{2} \right\rfloor + (n-1) \right) \right| = 1 \le 1$.
So, G admits a cordial labeling f and hence G is cordial. ∎

Theorem 3.2. The graph obtained by duplication of every edge by a vertex in P_n is cordial if $n \not\equiv 3 \pmod 4$.

Proof: Let $u_0, u_1, \dots, u_{n-1}$ be the consecutive vertices of P_n and G be the graph obtained by duplication of each of the edges $u_0u_1, u_1u_2, \dots, u_{n-2}u_{n-1}$ in P_n by the corresponding new vertices $v_0, v_1, \dots, v_{n-2}$ respectively. We have the following cases:
Case 1: $n \equiv 0 \pmod 4$.
Thus $n = 4k$ for some $k \in N$.

Define a vertex labeling $f : V(G) \to \{0,1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = v_i, i = \{0,1,\ldots,k-1\}; \\ 0 & \text{if } x = v_i, i = \{k,k+1,\ldots,n-2\}; \\ 1 & \text{if } x = u_i, i = \{0,1,\ldots,2k-1\}; \\ 0 & \text{if } x = u_i, i = \{2k,2k+2,\ldots,n-2\}; \\ 1 & \text{if } x = u_i, i = \{2k+1,2k+3,\ldots,n-1\}. \end{cases}$$

It is easy to check $v_f(1) = 4k$ and $v_f(0) = 4k-1$. Thus $|v_f(1) - v_f(0)| \leq 1$.
We have the following sub-cases:

(A) If $e = u_i v_i$ for all $i = 0,1,\ldots,n-2$ then there are $2k-1$ edges with label 1 and $2k$ edges with label 0.

(B) If $e = u_i v_{i-1}$ for all $i = 1,2,\ldots,n-1$ then there are $2k-1$ edges with label 1 and $2k$ edges with label 0.

(C) If $e = u_i u_{i+1}$ for all $i = 0,1,\ldots,n-2$ then there are $2k$ edges with label 1 and $2k-1$ edges with label 0.

From all the above sub-cases, we have $e_f(1) = (2k-1) + (2k-1) + 2k = 6k-2$ and $e_f(0) = 2k + 2k + (2k-1) = 6k-1$. So, $|e_f(1) - e_f(0)| = |(6k-2) - (6k-1)| \leq 1$.
Case 2: $n \equiv 1 \pmod 4$.
Thus $n = 4k+1$ for some $k \in N$.
Define a vertex labeling $f : V(G) \to \{0,1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = v_i, i = \{0,1,\ldots,k-1\}; \\ 0 & \text{if } x = v_i, i = \{k,k+1,\ldots,n-2\}; \\ 1 & \text{if } x = u_i, i = \{0,1,\ldots,2k-1\}; \\ 1 & \text{if } x = u_i, i = \{2k,2k+2,\ldots,n-1\}; \\ 0 & \text{if } x = u_i, i = \{2k+1,2k+3,\ldots,n-2\}. \end{cases}$$

It is easy to check $v_f(1) = 4k+1$ and $v_f(0) = 4k$. Thus $|v_f(1) - v_f(0)| \leq 1$.
We have the following sub-cases:

(A) If $e = u_i v_i$ for all $i = 0,1,\ldots,n-2$ then there are $2k$ edges with label 1 and $2k$ edges with label 0.

(B) If $e = u_i v_{i-1}$ for all $i = 1,2,\ldots,n-1$ then there are $2k$ edges with label 1 and $2k$ edges with label 0.

(C) If $e = u_i u_{i+1}$ for all $i = 0, 1, \ldots, n-2$ then there are $2k$ edges with label 1 and $2k$ edges with label 0.

From all the above sub cases, we have $e_f(1) = 2k+2k+2k = 6k$ and $e_f(0) = 2k+2k+2k = 6k$.
So, $|e_f(1) - e_f(0)| \leq 1$.
Case 3: $n \equiv 2 \pmod 4$.
Thus $n = 4k + 2$ for some $k \in N$.
Define a vertex labeling $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = v_i, i = \{0, 1, \ldots, k-1\}; \\ 0 & \text{if } x = v_i, i = \{k, k+1, \ldots, n-2\}; \\ 1 & \text{if } x = u_i, i = \{0, 1, \ldots, 2k-1\}; \\ 0 & \text{if } x = u_i, i = \{2k, 2k+2, \ldots, n-2\}; \\ 1 & \text{if } x = u_i, i = \{2k+1, 2k+3, \ldots, n-1\}. \end{cases}$$

It is easy to check $v_f(1) = 4k + 1$ and $v_f(0) = 4k + 2$. Thus $|v_f(1) - v_f(0)| \leq 1$.
We have the following sub-cases:

(A) If $e = u_i v_i$ for all $i = 0, 1, \ldots, n-2$ then there are $2k$ edges with label 1 and $2k+1$ edges with label 0.

(B) If $e = u_i v_{i-1}$ for all $i = 1, 2, \ldots, n-1$ then there are $2k$ edges with label 1 and $2k+1$ edges with label 0.

(C) If $e = u_i u_{i+1}$ for all $i = 0, 1, \ldots, n-2$ then there are $2k+2$ edges with label 1 and $2k-1$ edges with label 0.

From all the above sub-cases, we have $e_f(1) = 2k + 2k + (2k + 2) = 6k + 2$ and $e_f(0) = (2k+1) + (2k+1) + (2k-1) = 6k+1$. Therefore $|e_f(1) - e_f(0)| \leq 1$.
Thus from each of the cases G admits a cordial labeling f if $n \not\equiv 3 \pmod 4$.
Hence the graph obtained by duplication of every edge by a vertex in P_n is cordial if $n \not\equiv 3 \pmod 4$. ■

Theorem 3.3. The graph obtained by duplication of every edge by a vertex in P_n is not cordial if $n \equiv 3 \pmod 4$.

Proof: Here $n = 4k + 3$ for some $k \in N$. Let G be the graph obtained by duplication of every edge by a vertex of P_n. G has $8k+5$ vertices and $12k+6$ edges. The graph G is an edge disjoint union of $4k + 2$ triangles.
Assume that G is cordial.

For each $i \in \{0, 1, 2, 3\}$, let T_i be the number of triangles of G having exactly i many vertices with label 1. So, $T_0 + T_1 + T_2 + T_3 = 4k + 2$.
For a triangle with each vertex with label 0, there is no edge with label 1 and 3 edges with label 0. For a triangle with one vertex with label 1 and two vertices with label 0, there are two edges with label 1 and one edge with label 0. For a triangle with two vertices with label 1 and one vertex with label 0, there are two edges with label 1 and 1 edge with label 0. For a triangle with each vertex with label 1, there is no edge with label 1 and 3 edges with label 0.
As G is a cordial with even number of edges, the number of edges with label 0 and label 1 are same.
Thus, $3T_0 + T_1 + T_2 + 3T_3 = 2T_1 + 2T_2$
$\Rightarrow 3(T_0 + T_3) = T_1 + T_2$
$\Rightarrow 3(T_0 + T_3) = (4k + 2) - (T_0 + T_3)$
$\Rightarrow (T_0+T_3) = \frac{2k+1}{2}$, is not an integer, which is a contradiction. Therefore, G is not cordial. ■

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