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Broadcasting in Bloom Graph

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Abstract

Broadcasting is one of the types of global communication in interconnection networks. Broadcasting from a vertex of a graph is the simplest communication problem. In this paper, we analyse the broadcasting for bloom graph and prove that the broadcast time of bloom graph is at most two time units more than the optimal time.

Keywords: Bloom graph, broadcasting, interconnection network.
AMS Subject Classification(2010): 05C85, 05C90.

1 Introduction

Interconnection networks have been extensively applied in several areas, such as workstation networks and parallel computing. The main characteristic in parallel computing systems is the simulation between interconnection networks. Interconnection networks are used to connect the processors to memory or processors among themselves. An essential implementation of a parallel computing system depends mainly on the interconnection network. The design of an interconnection network is one of the important research problem in parallel computing. Graphs can be used to model interconnection networks in which vertices correspond to processors of the networks and the edges correspond to communication links. A new interconnection network topology called the bloom graph has been recently introduced in [1]. Bloom graph is planar, 4-regular and tripartite. The Hamiltonicity of bloom graph is studied in [1]. Communication-friendly topological structure is one of the most desirable property of an interconnection network. An efficient algorithm for routing messages between any two nodes of the bloom graph was studied in [2].

Broadcasting is the process of disseminating a message originating at one node of a network to all other nodes, with the condition that each node can only transmit the message to at most one of its neighbors at a time. The minimum number of time units needed to broadcast an information on a graph with n vertices and one originator is at least $\lceil log(n) \rceil$. The broadcast time for a vertex u of G, denoted $b(G, u)$, is the minimal length t of a broadcast protocol for u. The broadcast time of the graph G is $b(G) = max(b(G, u)|u \in G)$. Furthest-Distance-First Protocol is a common strategy used to solve the broadcasting problem [3, 8]. In this technique, message is broadcasted from the source node to an uninformed adjacent node which leads to longest path in a tree. Broadcasting has various applications in virus spreading, internet messaging and management of distributed systems etc. Broadcasting in cube connected cycles and shuffle-exchange networks was discussed in [6, 7]. Lower bound on the broadcast time of the butterfly graph was presented in [13]. Broadcasting in undirected graphs has been studied in [4, 5, 12]. In this paper, we study the broadcasting for bloom graph $B_{m,n}$ and a broadcast algorithm of bloom graph $B_{m,n}$ with a fixed source vertex. Also, we prove that when $m > n$, the broadcast time for bloom graph $B_{m,n}$ is optimal and when $m \leq n$, the broadcast time is at most two time units more than the optimal time.

2 Bloom Graph

Definition 2.1. [1] The bloom graph $B_{m,n}; m, n > 2$ is defined as follows:
$V(B_{m,n}) = \{(x, y) : 0 \leq x \leq m-1, 0 \leq y \leq n-1\}$, two distinct vertices (x_1, y_1) and (x_2, y_2) being adjacent if and only if

(i) $x_2 = x_1 + 1$ and $y_1 = y_2$

(ii) $x_1 = x_2 = 0$ and $y_1 + 1 \equiv y_2 (mod n)$

(iii) $x_1 = x_2 = m-1$ and $y_1 + 1 \equiv y_2 (mod n)$

(iv) $x_2 = x_1 + 1$ and $y_1 + 1 \equiv y_2 (mod n)$

The first condition describes the vertical edges, the second and third condition describe the horizontal edges in the top most and the lower most rows respectively. Condition four describes the slant edges (See Figure 1).

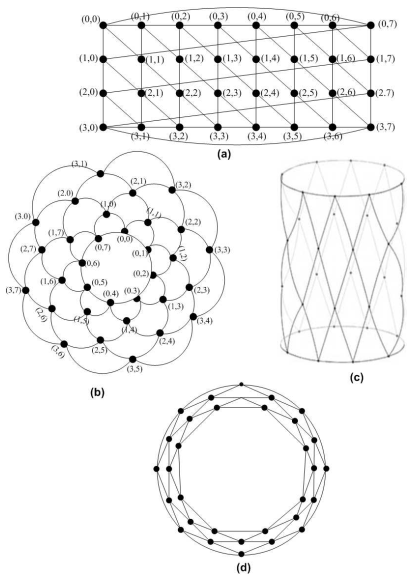

Figure 1: Bloom graph $B_{4,8}$ and its isomorphic figures

Bloom graph has mn vertices and $2mn$ edges. The diameter of bloom graph is

$$diam(B_{m,n}) = \begin{cases} m-1, & m > n \\ \lceil \frac{m+n}{2} \rceil - 1, & m \leq n \end{cases}.$$

The vertex connectivity and the edge connectivity of bloom graph is 4.

Bloom graph is planar, tripartite and 4-regular. In order to understand the planarity of the bloom graph, it is redrawn as in Figure 1 (b),(c). In Figure 1 (a), (b),(c) and (d), the graphs are isomorphic to each other. Figure 1(a), (b) and (c) gives a grid view, cylindrical view and a blooming flower view of bloom graph respectively.

3 Broadcasting Algorithm of Bloom Graph

Let $G = (V, E)$ be a bloom graph $B_{m,n}$. Let $O = (\lceil \frac{m}{2} \rceil - 1, \lceil \frac{n}{2} \rceil - 1)$ be the source vertex. The broadcast procedure I is done by constructing a spanning tree for $B_{m,n}$. The source vertex O sends the message along the spanning tree according to Furthest-Distance-First protocol.

Procedure I
Input: A bloom graph $B_{m,n}$, when the vertex O is the source node.

Algorithm:

Step 1: Construct a spanning tree of $B_{m,n}$, which includes:

(i) Delete the wrapping edges between the first and last column.

(ii) Draw lines XOY vertically. This line creates two zones namely, Z_1 and Z_2 respectively.(see Figure 2(a))

(iii) Delete all the edges in Z_1 except the horizontal edges joining the vertices $(m-1, j)$ and $(m-1, j+1)$ where $0 \leq j \leq \lceil \frac{n}{2} \rceil - 1$ and the slant edges.

(iv) Delete all the edges in Z_2 except the edges joining the vertices $(0, i)$ and $(0, i+1)$ where $\lceil \frac{n}{2} \rceil - 1 \leq i \leq n-1$ and the slant edges.

Step 2: Applying Furthest-Distance-First protocol, disseminate the message along the spanning tree.

Step 3: Stop, when all the vertices are informed.

Output: $b(O, B_{m,n}) \leq \lfloor \frac{m-1}{2} \rfloor + \lceil \frac{n-1}{2} \rceil + 2$.

Proof of Correctness:
The source node O disseminates the message according to Furthest-Distance-First protocol. In other words, O sends the message by giving priority to an uninformed adjacent node which leads to longest path in the spanning tree. Since the message is sent from one vertex to another vertex based on Furthest-Distance-First protocol, either one of the vertices $x(m-2, 0)$ or $x'(1, n-1)$

will be informed in $\lfloor \frac{m-1}{2} \rfloor + \lceil \frac{n-1}{2} \rceil + 2$ time units. By the construction of the spanning tree, all the other vertices will also be informed in at most $\lfloor \frac{m-1}{2} \rfloor + \lceil \frac{n-1}{2} \rceil + 2$ time units. □

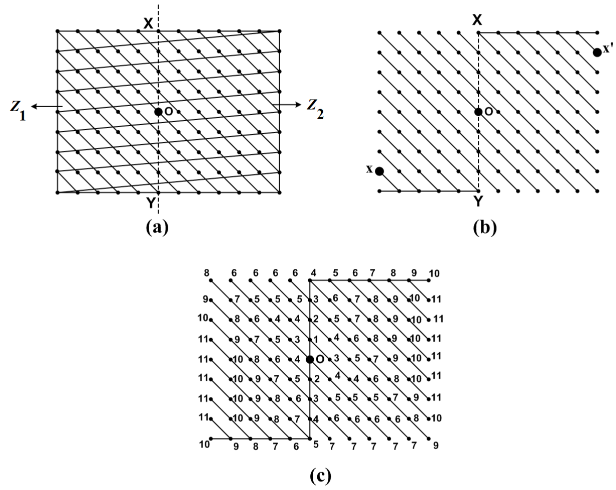

Figure 2: (a) Division of $B_{9,12}$ into two zones Z_1 and Z_2 (b)Spanning tree for $B_{9,12}$ (c) Broadcasting in $B_{9,12}$ when O is the source node

This algorithm leads to the following result.

Theorem 3.1. The broadcast time of a bloom graph $B_{m,n}$, with O as source node is, $b(O, B_{m,n}) \leq \lfloor \frac{m+n}{2} \rfloor + 2$.

The above algorithm gives the broadcast time for the fixed source node O. Now we discuss the broadcasting of bloom graph for an arbitrary source node. Consider an arbitrary vertex $S(u, v)$ of $B_{m,n}$, $m, n > 2$ as the source node. By the structure of bloom graph (see Figure 1(c)) it is observed that any vertex with in the same row are transitive to each other. Therefore, instead of the source node $S(u, v)$, let us consider $S'(u, (\lceil \frac{n}{2} \rceil - \lceil \frac{m}{2} \rceil + u) mod\ n)$ to be the source node when $m \leq n$ and when $m > n$.

$$\text{Consider } S' = \begin{cases} (u, (\lceil \frac{n}{2} \rceil - \lceil \frac{m}{2} \rceil + u)\ mod\ n) & if\ \lceil \frac{m}{2} \rceil - \lceil \frac{n}{2} \rceil \leq u \leq \lceil \frac{m}{2} \rceil - \lceil \frac{n}{2} \rceil + (n-1) \\ (u, 0) & if\ u < \lceil \frac{m}{2} \rceil - \lceil \frac{n}{2} \rceil \\ (u, n-1) & if\ u > \lceil \frac{m}{2} \rceil - \lceil \frac{n}{2} \rceil \end{cases}$$

as the source node. The spanning tree is constructed from the source node S' in the similar way as in the algorithm Procedure I (See Figure 3).

When $m \leq n$, we know that either x or x' will receive the maximum broadcast time when O is the source node (See Figure 4(a)). By the choice of S', $d(O, x) = d(S', x)$ and $d(O, x') = d(S', x')$ (See Figure 4(b)). There by the broadcast time from S' remains the same as that of the broadcast time from O. In other words, $b(O, B_{m,n}) = b(S', B_{m,n})$.

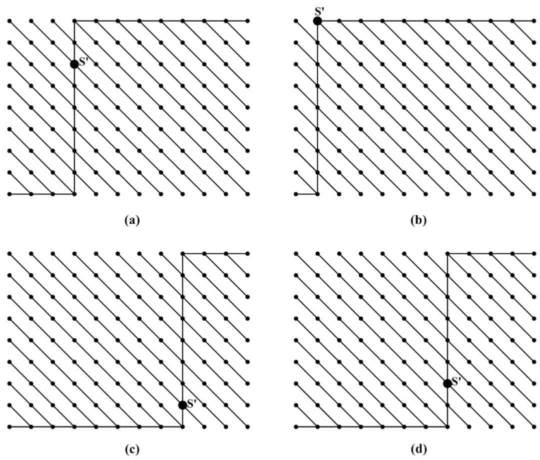

Figure 3: Spanning tree for $B_{9,12}$ when the source is $S^{'}$

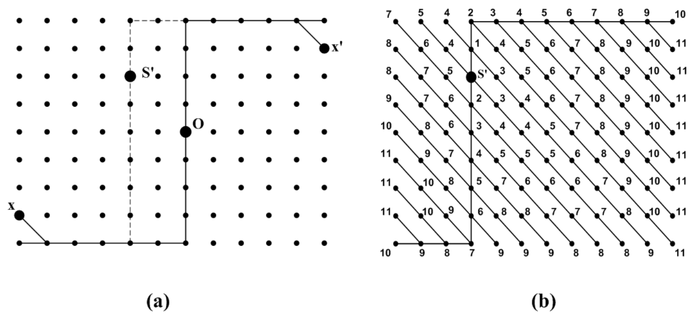

Figure 4: Broadcasting in $B_{9,12}$ when $S^{'}$ is the source node

Now when $m > n$, by the choice of $S^{'}$ the broadcast time from any other vertex will be lesser than that from $(0, 0)$ or $(m - 1, n - 1)$. If $S^{'} = (0, 0)$ the vertex $x^{'}$ will receive the message in m time units. If $S^{'} = (m - 1, n - 1)$ the vertex $y^{'}$ will receive the message in m time units (See Figure 5). All the other remaining vertices will also be informed in at most m time units.

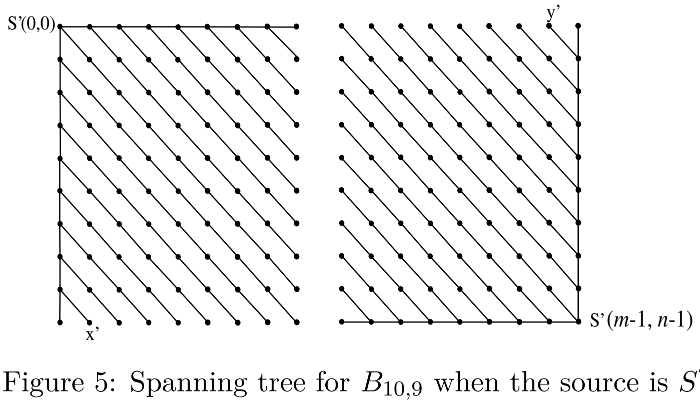

Figure 5: Spanning tree for $B_{10,9}$ when the source is $S^{'}$

The broadcast time for the vertex $S^{'}(0,0)$ is $b(S^{'}(0,0), B_{m,n}) = m$ (see Figure 6). Similarly, $b(S^{'}(m-1, n-1), B_{m,n}) = m$. The broadcast time of a graph is the maximum, taken over all its vertices of their broadcast time. Clearly when $m > n$, $b(S^{'}, B_{m,n}) = max\ \{b(S^{'}(0,0), B_{m,n})$ (or) $b(S^{'}(m-1, n-1), B_{m,n})\} = m$.

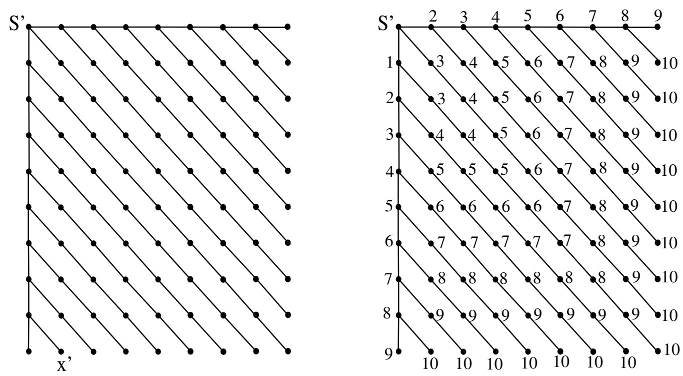

Figure 6: Broadcasting in $B_{10,9}$ when $S^{'}$ is the source node

Since $S^{'}$ is any arbitrary vertex, we give the result of the broadcast time of the bloom graph $B_{m,n}$, in the following theorems.

Theorem 3.2. The broadcast time of the bloom graph $B_{m,n}$, $m \leq n$ with $S^{'}$ as source node, $b(S^{'}, B_{m,n}) \leq \lfloor \frac{m+n}{2} \rfloor + 2$.

Theorem 3.3. The broadcast time of the bloom graph $B_{m,n}$, $m > n$ with $S^{'}$ as source node, $b(S^{'}, B_{m,n}) \leq m$.

Lemma 3.4. [11] In any graph of diameter D, if there exist three different vertices, u, v_1 and v_2 with v_1 and v_2 at a distance D from $u, b(G) \geq D + 1$

Remark: The diameter of $B_{m,n}$ is, $diam(B_{m,n}) = \begin{cases} m-1, & m > n \\ \lceil \frac{m+n}{2} \rceil - 1, & m \leq n. \end{cases}$

Hence by Lemma 3.4, $b(B_{m,n}) \geq \begin{cases} m, & when\ m > n \\ \lceil \frac{m+n}{2} \rceil, & when\ m \leq n. \end{cases}$

Theorem 3.2, Theorem 3.3 and Lemma 3.4 imply the following results.

Theorem 3.5. $\lceil \frac{m+n}{2} \rceil \leq b(B_{m,n}) \leq \lfloor \frac{m+n}{2} \rfloor + 2$, where $m \leq n$.

Theorem 3.6. The broadcast time of the bloom graph $B_{m,n}$, $m > n$ is m.

4 Conclusion

In this paper, we study the broadcasting for bloom graph $B_{m,n}$. When $m \leq n$, we prove that the broadcast time is at most two time units more than the optimal time. Also, we show that when $m > n$, the broadcast time of bloom graph is optimal. Other graph theoretical problems in bloom graph are under investigation.

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