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Cordialness of arbitrary supersubdivision of graphs

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Abstract

In this paper we prove that arbitrary supersubdivision of ladder, cyclic ladder, triangular snake and certain double triangular snake are cordial.

Keywords: Ladder, cyclic ladder, triangular snake, double triangular snake, subdivision of a graph, supersubdivision of a graph, cordial labeling.
AMS Subject Classification(2010): 05C78.

1 Introduction

By a graph we mean simple, finite and undirected graph $G = (V, E)$. The concept of cordial labeling was introduced by Cahit [1]. Sethuraman and Selvaraju [5] proved gracefulness of supersubdivision of graphs. Kathiresan [3] has proved subdivision of ladders are graceful. Ramchandran and Sekar [4] have discussed graceful labeling of supersubdivision of ladder. Vaidya [8] proved cordial labeling of snakes.

A ladder is defined by $P_n \times P_2$, where P_n is a path of length $n-1$ and is denoted by L_n. The ladder L_n has $2n$ vertices and $3n-2$ edges. Cyclic ladder is obtained by taking cartesian product of cycle C_n and P_2. The triangular snake T_n is a graph containing a path of length n with vertices $u_1, u_2, ...u_n, u_{n+1}$ and each pair of consecutive vertices u_i, u_{i+1} is joined to a common vertex v_i, $i = 1, 2, \ldots, n$. Thus it consists of $2n+1$ vertices and $3n$ edges. The double triangular snake $D(T_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to two new verices v_i and w_i respectively, $1 \leq i \leq n-1$.

Definition 1.1. Let G be a graph with p vertices and q edges. A graph H is said to be a subdivision of G if H is obtained by subdividing every edge of G exactly once. H is denoted by $S(G)$. Thus, $|V| = p + q$ and $|E| = 2q$.

Definition 1.2. Let G be a graph with p vertices and q edges. A graph H is said to be a supersubdivision of G if it is obtained from G by replacing every edge e of G by a complete bipartite graph $K_{2,m}$. H is denoted by $SS(G)$. Thus, $|V| = p + mq$ and $|E| = 2mq$.

Definition 1.3. Let G be a graph with p vertices and q edges. A graph H is said to be a arbitrary supersubdivision of G if it is obtained from G by replacing every edge e_i of G by a complete bipartite graph $K_{2,m_i}, i = 1, 2, \ldots, q$. H is denoted by $ASS(G)$. Thus, $|V| = p + \sum_{i=1}^{q} m_i$ and $|E| = \sum_{i=1}^{q} 2m_i$.

Definition 1.4. Let f be a function from the vertices of G to $\{0, 1\}$ and for an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$. f is said to be cordial labeling of G if $|V_0 - V_1| \leqslant 1$ and $|E_0 - E_1| \leqslant 1$ where V_0 and V_1 are the number of vertices of G having labels 0 and 1 respectively under f and E_0 and E_1 be the number of edges having labels 0 and 1 respectively under induced labeling f^*

In the complete bipartite graph $K_{2,m}$, we call the part consisting of two vertices as 2–vertices part of $K_{2,m}$ and the part consisting of m vertices as m–vertices part of $K_{2,m}$.

2 Main Results

Theorem 2.1. Arbitrary supersubdivision of L_n, $ASS\,(L_n)$ is cordial.

Proof: Let $u_i, \quad i = 1, 2, \ldots, n$ and $v_i, \quad i = 1, 2, \ldots, n$ be the vertices of two paths of Ladder L_n. Let $x_i^k, k = 1, 2, \ldots, m_i^1$ be the vertices of the m_i^1-vertices part of K_{2,m_i^1} replacing the edge $u_i u_{i+1}$, for $i = 1, 2, \ldots, n-1$. Let $y_i^k, k = 1, 2, \ldots, m_i^2$ be the vertices of the m_i^2-vertices part of K_{2,m_i^2} replacing the edge $v_i v_{i+1}$, for $i = 1, 2, \ldots, n-1$. Let $w_i^k, k = 1, 2, \ldots, m_i^3$ be the vertices of the m_i^3-vertices part of K_{2,m_i^3} replacing the edge $u_i v_i$, for $i = 1, 2, \ldots, n$.

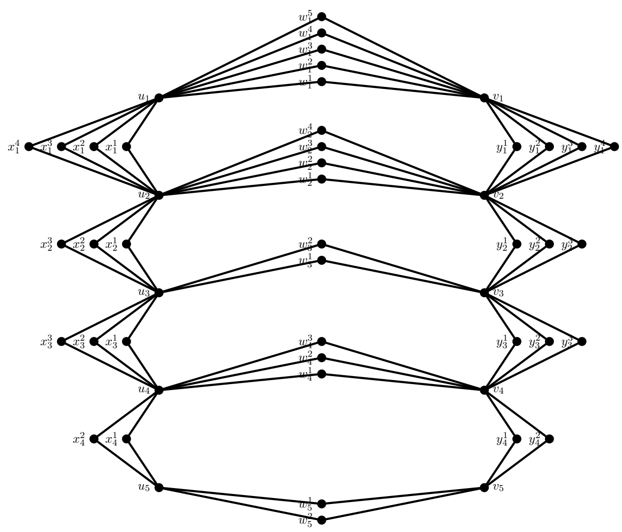

Figure 1: $ASS\,(L_5)$ with vertex labels.

$$|V| = 2n + \left(\sum m_i^1 + \sum m_i^2 + \sum m_i^3\right) \qquad |E| = 2\left(\sum m_i^1 + \sum m_i^2 + \sum m_i^3\right)$$

Define a labeling $f : V \to \{0, 1\}$ as follows.

$$\begin{aligned} f(u_i) &= 0 \quad if \;\; i \equiv 1\,(mod \;\; 2) \\ &= 1 \quad if \;\; i \equiv 0\,(mod \;\; 2), \quad 1 \le i \le n. \\ f(v_i) &= 1 \quad if \;\; i \equiv 1\,(mod \;\; 2) \\ &= 0 \quad if \;\; i \equiv 0\,(mod \;\; 2), \quad 1 \le i \le n. \end{aligned}$$

$$\begin{aligned} f(x_1^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2), \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2), \quad 1 \le k \le m_1^1. \end{aligned}$$

For $i = 1$ to $n - 2$,

If $f\left(x_i^{m_i^1}\right) = 0$ then,

$$\begin{aligned} f(x_{i+1}^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2), \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2), \quad 1 \le k \le m_{i+1}^1. \end{aligned}$$

If $f\left(x_i^{m_i^1}\right) = 1$ then,

$$\begin{aligned} f(x_{i+1}^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2), \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2), \quad 1 \le k \le m_{i+1}^1. \end{aligned}$$

If $f\left(x_{n-1}^{m_{n-1}^1}\right) = 1$ then,

$$\begin{aligned} f(y_1^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2), \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2), \quad 1 \le k \le m_1^2. \end{aligned}$$

If $f\left(x_{n-1}^{m_{n-1}^1}\right) = 0$ then,

$$\begin{aligned} f(y_1^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2), \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2), \quad 1 \le k \le m_1^2. \end{aligned}$$

For $i = 1$ to $n - 1$,

If $f\left(y_i^{m_i^2}\right) = 0$ then,

$$\begin{aligned} f(y_{i+1}^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2), \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2), \quad 1 \le k \le m_{i+1}^2. \end{aligned}$$

If $f\left(y_i^{m_i^2}\right) = 1$ then,

$$\begin{aligned} f(y_{i+1}^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2), \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2), \quad 1 \le k \le m_{i+1}^2. \end{aligned}$$

If $f\left(y_{n-1}^{m_{n-1}^2}\right) = 0$ then,

$$\begin{aligned} f(w_1^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_1^3. \end{aligned}$$

If $f\left(y_{n-1}^{m_{n-1}^2}\right) = 1$ then,

$$\begin{aligned} f(w_1^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_1^3. \end{aligned}$$

For $i = 1$ to n,

If $f\left(w_i^{m_i^3}\right) = 0$ then,

$$\begin{aligned} f(w_{i+1}^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^3. \end{aligned}$$

If $f\left(w_i^{m_i^3}\right) = 1$ then,

$$\begin{aligned} f(w_{i+1}^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^3. \end{aligned}$$

Let V_0 and V_1 be the number of vertices of G having labels 0 and 1 respectively under f and E_0 and E_1 be the number of edges having labels 0 and 1 respectively under induced labeling f^*.

Along both paths vertex labels are 0 and 1 alternately. As labeling of m_i-part of each K_{2,m_i^j} is done alternately edge weights get balanced in every K_{2,m_i^1} and K_{2,m_i^2}. Also in K_{2,m_i^3}, labels of $2-$vertices part are 0 and 1, thus balance edge weights. Thus, $|E_0| = |E_1|$. Vertex labels are given alternately starting with 1, so if it ends with 0 we get $|V_0| = |V_1|$ or else we get $|V_0| - |V_1| = 1$. Thus,$|\,|V_0| - |V_1|\,| \le 1$. ■

Theorem 2.2. Arbitrary supersubdivision of cyclic ladder, $ASS\,(C_n \times P_2)$ is cordial if $m_n^2 \equiv 0\,(mod \;\; 2)$ and $m_n^3 \equiv 0\,(mod \;\; 2)$.

Proof: Let $c_1^1, c_2^1, \ldots, c_n^1$ be the vertices of the inner cycle and $c_1^2, c_2^2, \ldots, c_n^2$ be the vertices of the outer cycle. Let $K_{2,{m_i}^1}$ be the graph replacing edges $c_i^1 c_i^2, i = 1, 2, \ldots, n$ and x_i^k be the vertices of ${m_i}^1-$part of $K_{2,{m_i}^1}$. Let $K_{2,{m_i}^2}$ be the graph replacing edges $c_i^1 c_{i+1}^1, i = 1, 2, \ldots, n-1$ and y_i^k be the vertices of ${m_i}^2-$part of $K_{2,{m_i}^2}$. Let $K_{2,{m_n}^2}$ be the graph replacing edges $c_n^1 c_1^1$ and y_n^k be the vertices of ${m_n}^2-$part of $K_{2,{m_n}^2}$. Let $K_{2,{m_i}^3}$ be the graph replacing edges $c_i^2 c_{i+1}^2, i = 1, 2, \ldots, n-1$ and z_i^k be the vertices of ${m_i}^3-$part of $K_{2,{m_i}^3}$. Let $K_{2,{m_n}^3}$ be the graph replacing edges $c_n^2 c_1^2$ and z_n^k be the vertices of ${m_n}^3-$part of $K_{2,{m_n}^3}$.

Define a labeling $f : V \to \{0, 1\}$ as follows.

$$\begin{aligned} f(c_i^1) &= 1 \quad if \;\; i \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; i \equiv 0\,(mod \;\; 2)\,, \quad 1 \le i \le n. \\ f(c_i^2) &= 0 \quad if \;\; i \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; i \equiv 0\,(mod \;\; 2)\,, \quad 1 \le i \le n. \end{aligned}$$

$$\begin{aligned} f(x_1^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_i^1. \end{aligned}$$

For $i = 1 \;\; to \;\; n-1$,

If $f\left(x_i^{m_i^1}\right) = 0$ then,

$$\begin{aligned} f(x_{i+1}^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^1. \end{aligned}$$

If $f\left(x_i^{m_i^1}\right) = 1$ then,

$$\begin{aligned} f(x_{i+1}^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^1. \end{aligned}$$

If $f\left(x_n^{m_n^1}\right) = 0$ then,

$$\begin{aligned} f(y_1^k) &= 1 if \quad k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_1^2. \end{aligned}$$

If $f\left(x_n^{m_n^1}\right) = 1$ then,

$$\begin{aligned} f(y_1^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_1^2. \end{aligned}$$

For $i = 1 \;\; to \;\; n-1$,

If $f\left(y_i^{m_i^2}\right) = 0$ then,

$$\begin{aligned} f(y_{i+1}^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^2. \end{aligned}$$

If $f\left(y_i^{m_i^2}\right) = 1$ then,

$$\begin{aligned} f(y_{i+1}^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^2. \end{aligned}$$

If $f\left(y_n^{m_n^2}\right) = 0$ then,

$$\begin{aligned} f(z_1^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_1^3. \end{aligned}$$

If $f\left(y_n^{m_n^2}\right) = 1$ then,

$$\begin{aligned} f(z_1^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_1^3. \end{aligned}$$

For $i = 1 \;\; to \;\; n-1$,

If $f\left(z_i^{m_i^3}\right) = 0$ then,

$$\begin{aligned} f(z_{i+1}^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^3. \end{aligned}$$

If $f\left(z_i^{m_i^3}\right) = 1$ then,

$$\begin{aligned} f(z_{i+1}^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^3. \end{aligned}$$

Let V_0 and V_1 be the number of vertices of G having labels 0 and 1 respectively under f and E_0 and E_1 be the number of edges having labels 0 and 1 respectively under induced labeling f^*.

We can see that vertex labels 0 and 1 on both cycles get balanced. m_n^2 and m_n^3 are even, hence vertex label in K_{2,m_n^2} and K_{2,m_n^3} get balanced. If $\left(\sum_{i=1}^{n} m_i^1 + \sum_{i=1}^{n-1} m_i^2 + \sum_{i=1}^{n-1} m_i^3\right)$ is even then $|V_1| = |V_0|$. If $\left(\sum_{i=1}^{n} m_i^1 + \sum_{i=1}^{n-1} m_i^2 + \sum_{i=1}^{n-1} m_i^3\right)$ is odd then $|V_1| - |V_0| = 1$. In any case, it can be easily seen that $|E_0| = |E_1|$. ■

Notations for Triangular snake

Let $u_i, \;\; i = 1, 2, \ldots, n+1$ be vetices of path of length n and $v_i, \;\; i = 1, 2, \ldots, n$ be the vertices adjecent to u_i and u_{i+1} respectively.

Let $x_i^k, k = 1, 2, \ldots, m_i^1$ be the vertices of the m_i^1-vertices part of K_{2,m_i^1} replacing the edge u_iv_i, for $i = 1, 2, \ldots, n$.

Let $y_i^k, k = 1, 2, \ldots, m_i^2$ be the vertices of the m_i^2-vertices part of K_{2,m_i^2} replacing the edge $u_{i+1}v_i$, for $i = 1, 2, \ldots, n$.

Let even m_i's among K_{2,m_i} replacing the edges $u_iu_{i+1}, \;\; i = 1, 2, \ldots, n$ be renamed as m_i^3, $i = 1, 2, \ldots, l\,(say)$. Let $w_i^k, k = 1, 2, \ldots, m_i^3$ be the vertices of the m_i^3-vertices part of K_{2,m_i^3} replacing the edge u_iu_{i+1}, for $i = 1, 2, \ldots, l$.

Let odd m_i's among K_{2,m_i} replacing the edges $u_iu_{i+1}, \;\; i = 1, 2, \ldots, n$ be renamed as m_i^4, $i = 1, 2, \ldots, n-l$.

Let $z_i^k, k = 1, 2, \ldots, m_i^4$ be the vertices of the m_i^4-vertices part of K_{2,m_i^4} replacing the edge u_iu_{i+1}, for $i = 1, 2, \ldots, n-l$.

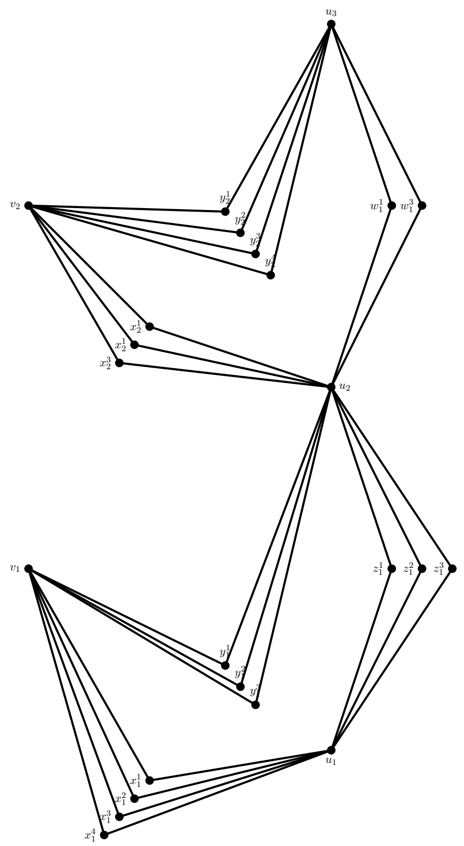

Figure 2: $ASS\,(T_3)$ with vertex labels.

Theorem 2.3. Arbitrary supersubdivision of T_n, $ASS\,(T_n)$ is cordial if there are even number of odd m_i^4's.

Proof: Define a labeling $f : V \to \{0, 1\}$ as follows.

$$f(u_i) = 1 \qquad if \quad i = 1, 2, \ldots, n+1,$$
$$f(v_i) = 0 \qquad if \quad i = 1, 2 \ldots, n.$$

$$\begin{aligned} f(x_1^k) &= 0 \qquad if \quad k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \qquad if \quad k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_1^1. \end{aligned}$$

For $i = 1$ to $n - 1$,

If $f\left(x_i^{m_i^1}\right) = 0$ then,

$$\begin{aligned} f(x_{i+1}^k) &= 1 \qquad if \quad k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \qquad if \quad k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_{i+1}^1. \end{aligned}$$

If $f\left(x_i^{m_i^1}\right) = 1$ then,

$$\begin{aligned} f(x_{i+1}^k) &= 0 \quad if \;\; k \equiv 1\,(mod\;\;2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod\;\;2)\,, \quad 1 \le k \le m_{i+1}^1. \end{aligned}$$

If $f\left(x_n^{m_n^1}\right) = 1$ then,

$$\begin{aligned} f(y_1^k) &= 0 \quad if \;\; k \equiv 1\,(mod\;\;2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod\;\;2)\,, \quad 1 \le k \le m_1^2. \end{aligned}$$

If $f\left(x_n^{m_n^1}\right) = 0$ then,

$$\begin{aligned} f(y_1^k) &= 1 \quad if \;\; k \equiv 1\,(mod\;\;2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod\;\;2)\,, \quad 1 \le k \le m_1^2. \end{aligned}$$

For $i = 1$ to $n - 1$,

If $f\left(y_i^{m_i^2}\right) = 0$ then,

$$\begin{aligned} f(y_{i+1}^k) &= 1 \quad if \;\; k \equiv 1\,(mod\;\;2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod\;\;2)\,, \quad 1 \le k \le m_{i+1}^2. \end{aligned}$$

If $f\left(y_i^{m_i^2}\right) = 1$ then,

$$\begin{aligned} f(y_{i+1}^k) &= 0 \quad if \;\; k \equiv 1\,(mod\;\;2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod\;\;2)\,, \quad 1 \le k \le m_{i+1}^2. \end{aligned}$$

If $f\left(y_n^{m_n^2}\right) = 0$ then,

$$\begin{aligned} f(w_1^k) &= 1 \quad if \;\; k \equiv 1\,(mod\;\;2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod\;\;2)\,, \quad 1 \le k \le m_1^3. \end{aligned}$$

If $f\left(y_n^{m_n^2}\right) = 1$ then,

$$\begin{aligned} f(w_1^k) &= 0 \quad if \;\; k \equiv 1\,(mod\;\;2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod\;\;2)\,, \quad 1 \le k \le m_1^3. \end{aligned}$$

For $i = 1 \;\; to \;\; l$,

If $f\left(w_i^{m_i^3}\right) = 0$ then,

$$\begin{aligned} f(w_{i+1}^k) &= 1 \quad if \;\; k \equiv 1\,(mod\;\;2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod\;\;2)\,, \quad 1 \le k \le m_{i+1}^3. \end{aligned}$$

If $f\left(w_i^{m_i^3}\right) = 1$ then,

$$\begin{aligned} f(w_{i+1}^k) &= 0 \quad if \quad k \equiv 1 \,(mod \;\; 2)\,, \\ &= 1 \quad if \quad k \equiv 0 \,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^3. \end{aligned}$$

If $f\left(w_l^{m_l^3}\right) = 0$ then,

$$\begin{aligned} f(z_1^k) &= 1 \quad if \quad k \equiv 1 \,(mod \;\; 2)\,, \\ &= 0 \quad if \quad k \equiv 0 \,(mod \;\; 2)\,, \quad 1 \le k \le m_1^4. \end{aligned}$$

If $f\left(w_l^{m_l^3}\right) = 1$ then,

$$\begin{aligned} f(z_1^k) &= 0 \quad if \quad k \equiv 1 \,(mod \;\; 2)\,, \\ &= 1 \quad if \quad k \equiv 0 \,(mod \;\; 2)\,, \quad 1 \le k \le m_1^4. \end{aligned}$$

For $i = 1 \;\; to \;\; n - l$,

If $f\left(z_i^{m_i^4}\right) = 0$ then,

$$\begin{aligned} f(z_{i+1}^k) &= 1 \quad if \quad k \equiv 1 \,(mod \;\; 2)\,, \\ &= 0 \quad if \quad k \equiv 0 \,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^4. \end{aligned}$$

If $f\left(z_i^{m_i^4}\right) = 1$ then,

$$\begin{aligned} f(z_{i+1}^k) &= 0 \quad if \quad k \equiv 1 \,(mod \;\; 2)\,, \\ &= 1 \quad if \quad k \equiv 0 \,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^4. \end{aligned}$$

Let V_0 and V_1 be the number of vertices of G having labels 0 and 1 respectively under f and E_0 and E_1 be the number of edges having labels 0 and 1 respectively under induced labeling f^*.

Among $u_i, \;\; i = 1, 2, \ldots, n+1$ and $v_i, \;\; i = 1, 2, \ldots, n$, number of vertices with 0 labels are n and with 1 are $n+1$. As the remaining vertices are labeled alternately we have,

$$\begin{aligned} ||V_1| - |V_0|| &= 0 \quad if \quad f\left(w_n^{m_n^3}\right) = 0, \\ &= 1 \quad if \quad f\left(w_n^{m_n^3}\right) = 1. \end{aligned}$$

Edge weights of K_{2,m_i^1} and K_{2,m_i^2} are balanced as 2−vertices part is labeled as 0 and 1 with alternate labeling to m_i^j−vertices part.For path P_n, if m_i^3 is even then for corresponding K_{2,m_i^3} we will get $|E_o| = |E_1|$. And as number of odd m_i^3 is even, altogether in union of K_{2,m_i^3}, we get, $|E_o| = |E_1|$. Hence in general, $|E_o| = |E_1|$. ∎

Theorem 2.4. Arbitrary supersubdivision of $D(T_n)$, $ASS\,(D(T_n))$ is cordial if $\sum m_i^4 \equiv 0\,(mod \;\; 2)$ and $\sum m_i^5 \equiv 0\,(mod \;\; 2)$.

Proof: Let $u_i, \; i = 1, 2, \ldots, n$ be the vertices of the path. Each u_i and u_{i+1} are joined to v_i and w_i respectively for $i = 1, 2, \ldots, n-1$. Let $K_{2,{m_i}^1}$ be the graph replacing edges $u_iu_{i+1}, i =$

$1, 2, \ldots, n-1$ and x_i^k be the vertices of ${m_i}^1-$part of $K_{2,{m_i}^1}$. Let $K_{2,{m_i}^2}$ be the graph replacing edges $v_i u_{i+1}, i = 1, 2, \ldots, n-1$ and y_i^k be the vertices of ${m_i}^2-$part of $K_{2,{m_i}^2}$. Let $K_{2,{m_i}^3}$ be the graph replacing edges $w_i u_i, i = 1, 2, \ldots, n-1$ and z_i^k be the vertices of ${m_i}^3-$part of $K_{2,{m_i}^3}$.

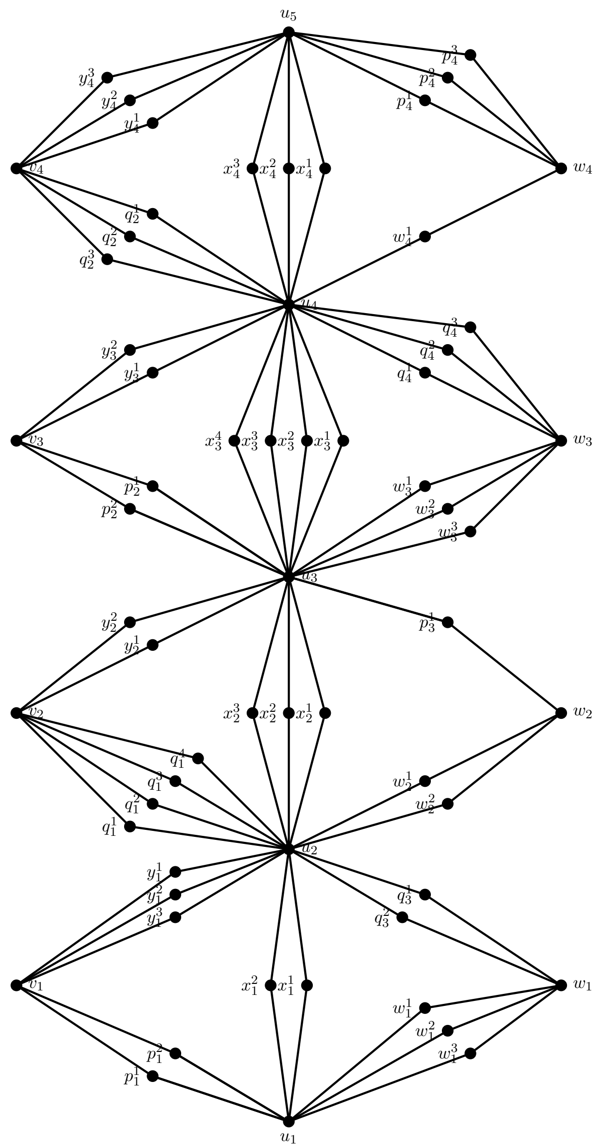

Figure 3: $ASS\,(D(T_5))$ with vertex labels.

For n even,

Let $K_{2,{m_i}^4}$ be the graph replacing edges $u_{2i-1}v_{2i-1}, i = 1, 2, \ldots, \frac{n}{2}$ and edges $w_{2i-n}u_{2i-n+1}, i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n-1$ and p_i^k be the vertices of ${m_i}^4-$part of $K_{2,{m_i}^4}$.

Let $K_{2,{m_i}^5}$ be the graph replacing edges $u_{2i}v_{2i}, i = 1, 2, \ldots, \frac{n}{2}-1$ and edges $w_{2i-n+1}u_{2i-n+2}, i =$

$\frac{n}{2}, \frac{n}{2}+1, \ldots, n-1$ and q_i^k be the vertices of ${m_i}^5-$part of $K_{2,{m_i}^5}$.

For n odd,

Let $K_{2,{m_i}^4}$ be the graph replacing edges $u_{2i-1}v_{2i-1}, i = 1, 2, \ldots, \frac{n-1}{2}$ and edges $w_{2i-n}u_{2i-n+1}, i = \frac{n-1}{2}+1, \frac{n}{2}+2, \ldots, n-1$ and let p_i^k be the vertices of ${m_i}^4-$part of $K_{2,{m_i}^4}$.

Let $K_{2,{m_i}^5}$ be the graph replacing edges $u_{2i}v_{2i}, i = 1, 2, \ldots, \frac{n+1}{2}-1$ and edges $w_{2i-n+1}u_{2i-n+2}, i = \frac{n+1}{2}, \frac{n}{2}+1, \ldots, n-1$ and let q_i^k be the vertices of ${m_i}^5-$part of $K_{2,{m_i}^5}$.

Labeling is as follows.

Define a labeling $f : V \to \{0, 1\}$ as follows.

$$\begin{aligned} f(u_i) &= 0 \quad if \;\; i \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; i \equiv 0\,(mod \;\; 2)\,, \quad 1 \le i \le n. \\ f(v_i) &= 0 \quad if \;\; i \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; i \equiv 0\,(mod \;\; 2)\,, \quad 1 \le i \le n-1. \\ f(w_i) &= 1 \quad if \;\; i \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; i \equiv 0\,(mod \;\; 2)\,, \quad 1 \le i \le n-1. \end{aligned}$$

$$\begin{aligned} f(x_1^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_1^1. \end{aligned}$$

For $i = 1 \;\; to \;\; n-2$,

If $f\left(x_i^{m_i^1}\right) = 0$ then,

$$\begin{aligned} f(x_{i+1}^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^1. \end{aligned}$$

If $f\left(x_i^{m_i^1}\right) = 1$ then,

$$\begin{aligned} f(x_{i+1}^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_{i+1}^1. \end{aligned}$$

If $f\left(x_{n-1}^{m_{n-1}^1}\right) = 0$ then,

$$\begin{aligned} f(y_1^k) &= 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_1^2. \end{aligned}$$

If $f\left(x_{n-1}^{m_{n-1}^1}\right) = 1$ then,

$$\begin{aligned} f(y_1^k) &= 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,, \\ &= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \le k \le m_1^2. \end{aligned}$$

For $i = 1 \;\; to \;\; n-2$,

If $f\left(y_i^{m_i^2}\right) = 0$ then

$$f(y_{i+1}^k) \;\; = 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,,$$
$$= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_{i+1}^2.$$

If $f\left(y_i^{m_i^2}\right) = 1$ then,

$$f(y_{i+1}^k) \;\; = 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,,$$
$$= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_{i+1}^2.$$

If $f\left(y_{n-1}^{m_{n-1}^2}\right) = 0$ then,

$$f(z_1^k) \;\; = 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,,$$
$$= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_1^3.$$

If $f\left(y_{n-1}^{m_{n-1}^2}\right) = 1$ then,

$$f(z_1^k) \;\; = 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,,$$
$$= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_1^3.$$

For $i = 1 \;\; to \;\; n-2$,

If $f\left(z_i^{m_i^3}\right) = 0$ then,

$$f(z_{i+1}^k) \;\; = 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,,$$
$$= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_{i+1}^3.$$

If $f\left(z_i^{m_i^3}\right) = 1$ then,

$$f(z_{i+1}^k) \;\; = 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)$$
$$= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_{i+1}^3.$$

If $f\left(z_{n-1}^{m_{n-1}^3}\right) = 0$ then,

$$f(p_1^k) \;\; = 1 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,,$$
$$= 0 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_1^4.$$

If $f\left(z_{n-1}^{m_{n-1}^3}\right) = 1$ then,

$$f(p_1^k) \;\; = 0 \quad if \;\; k \equiv 1\,(mod \;\; 2)\,,$$
$$= 1 \quad if \;\; k \equiv 0\,(mod \;\; 2)\,, \quad 1 \leq k \leq m_1^4.$$

For $i = 1 \;\; to \;\; n-2$,

If $f\left(p_i^{m_i^4}\right)=0$ then,

$$\begin{aligned} f(p_{i+1}^k) &= 1 \quad if \;\; k\equiv 1\,(mod\;\;2)\,, \\ &= 0 \quad if \;\; k\equiv 0\,(mod\;\;2)\,, \quad 1\leq k\leq m_{i+1}^4. \end{aligned}$$

If $f\left(p_i^{m_i^3}\right)=1$ then,

$$\begin{aligned} f(p_{i+1}^k) &= 0 \quad if \;\; k\equiv 1\,(mod\;\;2)\,, \\ &= 1 \quad if \;\; k\equiv 0\,(mod\;\;2)\,, \quad 1\leq k\leq m_{i+1}^4. \end{aligned}$$

If $f\left(p_{n-1}^{m_{n-1}^5}\right)=0$ then,

$$\begin{aligned} f(q_1^k) &= 1 \quad if \;\; k\equiv 1\,(mod\;\;2)\,, \\ &= 0 \quad if \;\; k\equiv 0\,(mod\;\;2)\,, \quad 1\leq k\leq m_1^5. \end{aligned}$$

If $f\left(p_{n-1}^{m_{n-1}^5}\right)=1$ then,

$$\begin{aligned} f(q_1^k) &= 0 \quad if \;\; k\equiv 1\,(mod\;\;2)\,, \\ &= 1 \quad if \;\; k\equiv 0\,(mod\;\;2)\,, \quad 1\leq k\leq m_1^5. \end{aligned}$$

For $i=1\;\; to\;\; n-2$,

If $f\left(q_i^{m_i^5}\right)=0$ then,

$$\begin{aligned} f(q_{i+1}^k) &= 1 \quad if \;\; k\equiv 1\,(mod\;\;2)\,, \\ &= 0 \quad if \;\; k\equiv 0\,(mod\;\;2)\,, \quad 1\leq k\leq m_{i+1}^5. \end{aligned}$$

If $f\left(q_i^{m_i^5}\right)=1$ then,

$$\begin{aligned} f(q_{i+1}^k) &= 0 \quad if \;\; k\equiv 1\,(mod\;\;2)\,, \\ &= 1 \quad if \;\; k\equiv 0\,(mod\;\;2)\,, \quad 1\leq k\leq m_{i+1}^5. \end{aligned}$$

Let V_0 and V_1 be the number of vertices of G having labels 0 and 1 respectively under f and E_0 and E_1 be the number of edges having labels 0 and 1 respectively under induced labeling f^*.

In above labeling, vertices on path are given labels 0 and 1 alternately. Also, number of $0's$ and $1's$ in v_i and w_i are balanced with each other. Labeling of K_{2,m_i^j} is done alternatively as 0 and 1 with condition that $\sum m_i^4\equiv 0\,(mod\;\;2)$ and $\sum m_i^5\equiv 0\,(mod\;\;2)$, thus balancing number of $0's$ and$1's$.

If n is even, with $\sum\left(m_i^1+mi^2+m_i^3\right)$ also even, we get $|V_0|=|V_1|$ and with $\sum\left(m_i^1+mi^2+m_i^3\right)$ is odd we have $|V_1|-|V_0|=1$.

If n is odd, with $\sum\left(m_i^1+mi^2+m_i^3\right)$ also even, we get $|V_0|-|V_1|=1$ and with $\sum\left(m_i^1+mi^2+m_i^3\right)$

is odd then $|V_1| = |V_0|$. In any case, it can be clearly seen that $|E_0| = |E_1|$. ■

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