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Switching of a vertex and independent domination number in graphs

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Abstract

The switching of a vertex v of a graph G means removing all the edges incident to v and adding the edges joining v to every vertex which is not adjacent to v in G.The resultant graph is denoted by $\widetilde{G}$. In this paper, we explore the concept of independent domination in the context of switching of a vertex in a graph.

Keywords: Dominating set, Independent dominating set, Independent domination number, Switching of a vertex.
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1 Introduction

The domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it because of its many and varied applications in fields like linear algebra and optimization, design analysis of communication networks, social sciences and military surveillance. Many variants of domination models are available in the existing literature. For a comprehensive bibliography of papers on the concept of domination, the readers are referred to Hedetniemi and Laskar [9]. This paper is focused on independent domination in graphs.

By a graph G we mean a simple, finite and undirected graph G of order n. We denote the vertex set and edge set of a graph G by $V(G)$ and $E(G)$, respectively. The open neighborhood $N(v)$ of $v \in V(G)$ is the set of vertices adjacent to v, and the set $N[v] = N(v) \cup \{v\}$ is the closed neighborhood of v. We denote the degree of a vertex v in a graph G by $deg(v)$. The maximum degree among the vertices of G is denoted by $\triangle(G)$. A vertex of degree one is called a pendant vertex and a vertex which is not the end of any edge is called the isolated vertex.

The set $S \subseteq V(G)$ of vertices in a graph G is called a dominating set if every vertex $v \in V(G)$ is either an element of S or is adjacent to at least one vertex of S. The minimum cardinality of a dominating set in G is called the domination number of G which is denoted by $\gamma(G)$.

An independent set in a graph G is a set of pairwise non-adjacent vertices of G. A set S of vertices in a graph G is called an independent dominating set if S is both an independent and a dominating set of G. The independent domination number of G, denoted by $i(G)$, is the minimum cardinality of an independent dominating set in G.

The theory of independent domination was formalized by Berge [2] and Ore [12] in 1962. The independent domination number, $i(G)$ was introduced by Cockayne and Hedetniemi in [3, 4]. Ching-Hau liu *et al.* [11] discussed the NP-completeness of both the independent dominating set problem and the dominating set problem on at most cubic grid graphs. Duckworth and Wormald [5] found the upper bounds on the independent domination number of random regular graphs while Henning *et al.* [10] established the upper bound on the independent domination number of a bipartite cubic graph of order n and of girth at least 6. Goddard *et al.* [7] studied independent domination in regular graph and proved that if G is a connected graph then $i(G) \leq (\frac{3}{2})\gamma(G)$, with equality if and only if $G = K_{3,3}$ while the independent domination number of some wheel related graphs is discussed by Vaidya and Pandit [15]. Allan and Laskar [1] proved that if G is a claw-free graph then $\gamma(G) = i(G)$ while Vaidya and Pandit [13] found the graphs G containing claw as an induced subgraph with $\gamma(G) = i(G)$.

The wheel W_n is defined to be the join $C_{n-1} + K_1$ where $n \geq 4$. The vertex corresponding to K_1 is known as the apex vertex and the vertices corresponding to cycle are known as the rim vertices.

For any real number n, $\lceil n \rceil$ denotes the smallest integer not less than n and $\lfloor n \rfloor$ denotes the greatest integer not greater than n.

Throughout the paper, P_n, C_n, W_n and K_n denote the path, the cycle, the wheel and the complete graph with n vertices, respectively.

For notation and graph theoretic terminology not defined herein, the reader may refer to West [16] while the terms related to the concept of domination are used in the sense of Haynes *et al.* [8].

Many domination parameters are formed by combining domination with several other graph theoretic properties. We investigate the independent domination number of the graphs obtained by switching of a vertex in the wheel W_n, the complete graph K_n, the shell S_n, the helm H_n and the generalized web graph $W(t, n-1)$.

2 Main Results

Definition 2.1. The switching of a vertex v of G means removing all the edges incident to v and adding edges joining v to every vertex which is not adjacent to v in G. The resultant graph is denoted by $\widetilde{G}$.

Proposition 2.2. [6] For the path and cycle, $i(P_n) = i(C_n) = \lceil \frac{n}{3} \rceil$.

Theorem 2.3. If $\widetilde{W_n}$ is the graph obtained by switching of an arbitrary vertex v of wheel W_n

then

$$i(\widetilde{W_n}) = \begin{cases} 2 & \text{if } v \text{ is a rim vertex of } W_n\ (n \neq 5), \\ \left\lceil \frac{n-1}{3} \right\rceil + 1 & \text{if } v \text{ is the apex vertex .} \end{cases}$$

Proof: Let $W_n = C_{n-1} + K_1$ and let $\widetilde{W_n}$ denote the graph obtained by switching of a vertex v of W_n.

Case (i): Let the switched vertex v be a rim vertex of W_n $(n \neq 5)$.

In this case, the apex vertex of W_n dominates all the vertices of $\widetilde{W_n}$ except the switched vertex and there is no vertex in $\widetilde{W_n}$ which can dominate all the vertices of $\widetilde{W_n}$. Moreover, two non-adjacent vertices of $\widetilde{W_n}$, namely, the apex vertex and the switched vertex, dominate all the vertices of $\widetilde{W_n}$. Therefore, for any independent dominating set S of $\widetilde{W_n}$, $\mid S \mid \geq 2$ implying that $i(\widetilde{W_n}) = 2$.

Case (ii): Let the switched vertex be the apex vertex c of W_n.

Since the apex vertex c of W_n is an isolated vertex of $\widetilde{W_n}$, it follows that every independent dominating set of $\widetilde{W_n}$ must contain c. Moreover, $V(\widetilde{W_n}) = V(C_{n-1}) \cup \{c\}$ and by Proposition 2.2, $i(C_{n-1}) = \left\lceil \frac{n-1}{3} \right\rceil$. Hence, at least $\left\lceil \frac{n-1}{3} \right\rceil + 1$ pairwise non-adjacent vertices are required to dominate all the vertices of $\widetilde{W_n}$. Consequently, every independent dominating set of $\widetilde{W_n}$ must contain at least $\left\lceil \frac{n-1}{3} \right\rceil + 1$ vertices implies that $i(\widetilde{W_n}) = \left\lceil \frac{n-1}{3} \right\rceil + 1$.
Hence, we prove that

$$i(\widetilde{W_n}) = \begin{cases} 2 & \text{if } v \text{ is a rim vertex of } W_n\ (n \neq 5), \\ \\ \left\lceil \frac{n-1}{3} \right\rceil + 1 & \text{if } v \text{ is the apex vertex .} \end{cases} \qquad \blacksquare$$

Remark 2.4. Let $\widetilde{W_5}$ denote the graph obtained by switching of a rim vertex of W_5. Since there exists a vertex $u \in V(\widetilde{W_5})$ such that $N[u] = V(\widetilde{W_5})$, it follows that $i(\widetilde{W_5}) = 1$.

Illustration 2.5. In Figure 1, the graph $\widetilde{W_9}$ obtained by switching of a rim vertex v_1 is shown in which the set of solid vertices is its independent dominating set with minimum cardinality.

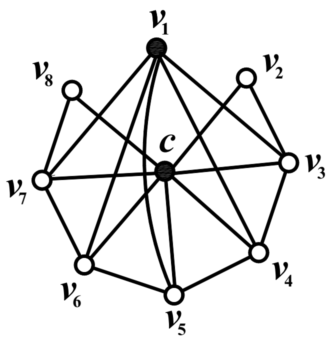

Figure 1

Theorem 2.6. If $\widetilde{K_n}$ is the graph obtained by switching of an arbitrary vertex of complete graph K_n then $i(\widetilde{K_n}) = 2$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the successive vertices of K_n and $\widetilde{K_n}$ denote the graph obtained by switching of a vertex of K_n. Then, $\mid V(\widetilde{K_n}) \mid = n$.

Without loss of generality, let the switched vertex be v_1. The vertex v_1 of K_n becomes the isolated vertex of $\widetilde{K_n}$. Hence, every dominating set of $\widetilde{K_n}$ must contain the isolated vertex v_1. Consequently, every independent dominating set of $\widetilde{K_n}$ must contain v_1. Now, $V(\widetilde{K_n}) = V(K_{n-1}) \cup \{v_1\}$. Since there exists a vertex $u \in V(K_{n-1})$ such that $N[u] = V(K_{n-1})$, it follows that $i(K_{n-1}) = 1$. Thus, $i(\widetilde{K_n}) = i(K_{n-1}) + 1 = 1 + 1 = 2$. ∎

Definition 2.7. A shell S_n $(n > 3)$ is the graph obtained by taking $n-3$ concurrent chords in a cycle C_n. The vertex at which all the chords are concurrent is called the apex vertex. The shell S_n is also called fan f_{n-1}. That is, $S_n = f_{n-1} = P_{n-1} + K_1$.

Theorem 2.8. If $\widetilde{S_n}$ is the graph obtained by switching of an arbitrary vertex v of shell S_n then

$$i(\widetilde{S_n}) = \begin{cases} \left\lceil \frac{n-1}{3} \right\rceil + 1 & \text{if } v \text{ is the apex vertex ,} \\ \\ 2 & \text{if } v \text{ is not the apex vertex of } S_n\, (n > 5). \end{cases}$$

Proof: Let $\widetilde{S_n}$ denote the graph obtained by switching of a vertex v of S_n. Then, $\mid V(\widetilde{S_n}) \mid = n$.

Case (i): Let the switched vertex be the apex vertex c of S_n.

In this case, the apex vertex c of S_n becomes the isolated vertex of $\widetilde{S_n}$. Hence, every dominating set of $\widetilde{S_n}$ must contain c. Now, $V(\widetilde{S_n}) = V(P_{n-1}) \cup \{c\}$ and by Proposition 2.2, $i(P_{n-1}) = \left\lceil \frac{n-1}{3} \right\rceil$. Therefore, at least $\left\lceil \frac{n-1}{3} \right\rceil + 1$ pairwise non-adjacent vertices are required to dominate all the vertices of $\widetilde{S_n}$. Hence, for any independent dominating set S of $\widetilde{S_n}$, $\mid S \mid \geq \left\lceil \frac{n-1}{3} \right\rceil + 1$ implies that $i(\widetilde{S_n}) = \left\lceil \frac{n-1}{3} \right\rceil + 1$.

Case (ii): Let the switched vertex v be not the apex vertex of S_n $(n > 5)$.

Since there exists no vertex $u \in V(\widetilde{S_n})$ such that $N[u] = V(\widetilde{S_n})$, it follows that $i(\widetilde{S_n}) > 1$. Moreover, $deg(c) = \triangle(\widetilde{S_n})$ and the apex vertex c is adjacent to all the vertices of $\widetilde{S_n}$ except the switched vertex v. Hence, two non-adjacent vertices, namely, c and v dominate all the vertices of $\widetilde{S_n}$. Therefore, any independent dominating set of $\widetilde{S_n}$ must have at least two vertices which implies that $i(\widetilde{S_n}) = 2$.

Thus, we prove that

$$i(\widetilde{S_n}) = \begin{cases} \left\lceil \frac{n-1}{3} \right\rceil + 1 & \text{if } v \text{ is the apex vertex ,} \\ \\ 2 & \text{if } v \text{ is not the apex vertex of } S_n\, (n > 5). \end{cases}$$

∎

Definition 2.9. The helm H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each of its rim vertices.

Theorem 2.10. If $\widetilde{H_n}$ is the graph obtained by switching of an arbitrary vertex v of helm H_n then

$$i(\widetilde{H_n}) = \begin{cases} \left\lceil \frac{n-1}{3} \right\rceil + 1 & \text{if } v \text{ is the apex vertex ,} \\ 2 & \text{if } v \text{ is a pendant vertex ,} \\ 3 & \text{otherwise.} \end{cases}$$

Proof: Let $\widetilde{H_n}$ denote the graph obtained by switching of a vertex v of helm H_n. Then, $\mid V(\widetilde{H_n}) \mid = 2n-1$.

Case (i): Let the switched vertex be the apex vertex c of H_n.

In this case, the apex vertex c is adjacent to all the pendant vertices of H_n in $\widetilde{H_n}$ and hence, c dominates all the pendant vertices of H_n in $\widetilde{H_n}$. Now, the vertices other than the pendant vertices of H_n in $\widetilde{H_n}$ form a cycle C_{n-1}. Since $deg(c) = n-1 = \triangle(\widetilde{H_n})$ and by Proposition 2.2, $i(C_{n-1}) = \left\lceil \frac{n-1}{3} \right\rceil$, it follows that at least $\left\lceil \frac{n-1}{3} \right\rceil + 1$ pairwise non-adjacent vertices are essential to dominate all the vertices of $\widetilde{H_n}$. Therefore, for any independent dominating set S of $\widetilde{H_n}$, $\mid S \mid \geq \left\lceil \frac{n-1}{3} \right\rceil + 1$ which implies that $i(\widetilde{H_n}) = \left\lceil \frac{n-1}{3} \right\rceil + 1$.

Case (ii): Let the switched vertex v be a pendant vertex of H_n.

Here, by the definition of switching of a vertex, v being a pendant vertex in H_n, is adjacent to all the vertices of $\widetilde{H_n}$ except the vertex which is adjacent to v in H_n. Since $deg(v) = \triangle(\widetilde{H_n}) = \mid V(\widetilde{H_n}) \mid -1$, it follows that two vertices are enough to dominate all the vertices of $\widetilde{H_n}$. Moreover, these two vertices are not adjacent. Hence, any independent dominating set of $\widetilde{H_n}$ must have at least two vertices implies that $i(\widetilde{H_n}) = 2$.

Case (iii): Let the switched vertex v be neither the apex vertex nor a pendant vertex of H_n.

Since $\widetilde{H_n}$ has an isolated vertex u and there exists no vertex $x \in V(\widetilde{H_n})$ such that $N[x] = V(\widetilde{H_n}) - \{u\}$, it follows that every dominating set of $\widetilde{H_n}$ must contain more than two vertices of $\widetilde{H_n}$. Now, the switched vertex v is adjacent to all the pendant vertices of H_n in $\widetilde{H_n}$ and the apex vertex c is adjacent to the remaining vertices of $\widetilde{H_n}$ other than the isolated vertex. Hence, the three vertices namely, u, c and v dominate all the vertices of $\widetilde{H_n}$. Therefore, every dominating set of $\widetilde{H_n}$ must have at least three vertices of $\widetilde{H_n}$. Moreover, these three vertices are pairwise non-adjacent vertices. Hence, for any independent dominating set S of $\widetilde{H_n}$, $\mid S \mid \geq 3$ which implies that $i(\widetilde{H_n}) = 3$.

Thus, we prove that

$$i(\widetilde{H_n}) = \begin{cases} \left\lceil \frac{n-1}{3} \right\rceil + 1 & \text{if } v \text{ is the apex vertex ,} \\ 2 & \text{if } v \text{ is a pendant vertex ,} \\ 3 & \text{otherwise.} \end{cases}$$ ∎

Definition 2.11. A web graph is the graph obtained by joining the pendant vertices of a helm to form a cycle and then adding a single pendant edge to each vertex of this outer cycle.

The generalized web graph is the web graph with $t-$ cycles each of order $n-1$ and it is denoted by $W(t, n-1)$.

Theorem 2.12. If $\widetilde{W}(t, n-1)$ is the graph obtained by switching of an arbitrary vertex v of generalized web graph $W(t, n-1)$ then

$$i(\widetilde{W}(t, n-1)) = \begin{cases} 2 & \text{if } v \text{ is a pendant vertex ,} \\ 3 & \text{if } v \text{ is a vertex of the innermost cycle ,} \\ \left\lceil \frac{n-1}{3} \right\rceil + 1 & \text{if } v \text{ is the apex vertex ,} \\ 5 & \text{otherwise} (n \neq 4). \end{cases}$$

Proof: Let $\widetilde{W}(t, n-1)$ denote the graph obtained by switching of a vertex v of $W(t, n-1)$. Then, $| V(\widetilde{W}(t, n-1)) | = t(n-1) + n$.
Label the vertices of the generalized web graph $W(t, n-1)$ as follows:

Denote the vertices of the innermost cycle of $W(t, n-1)$ successively as $v_{1,1}, v_{1,2}, \ldots, v_{1,n-1}$. Then denote the vertices adjacent to $v_{1,1}, v_{1,2}, \ldots, v_{1,n-1}$ on the second cycle as $v_{2,1}, v_{2,2}, v_{2,3} \ldots$, $v_{2,n-1}$ respectively and the vertices adjacent to $v_{2,1}, v_{2,2}, \ldots, v_{2,n-1}$ on the third cycle as $v_{3,1}, v_{3,2}$, $\ldots, v_{3,n-1}$ respectively and the vertices on the t^{th} cycle (outermost cycle) as $v_{t,1}, v_{t,2}, \ldots, v_{t,n-1}$. Next denote the pendant vertices of $W(t, n-1)$ as $u_1, u_2, \ldots, u_{n-1}$ and the apex vertex of $W(t, n-1)$ as c.

Case (i): Let the switched vertex v be a pendant vertex of $W(t, n-1)$.

Since there exists no vertex $u \in V(\widetilde{W}(t, n-1))$ such that $N[u] = V(\widetilde{W}(t, n-1))$, it follows that $i(\widetilde{W}(t, n-1)) > 1$. Moreover, $N[v] = V(\widetilde{W}(t, n-1)) - \{v_1\}$ where v_1 is not adjacent to v in $\widetilde{W}(t, n-1)$. Therefore, two non-adjacent vertices of $\widetilde{W}(t, n-1)$ namely, v and v_1, dominate all the vertices of $\widetilde{W}(t, n-1)$. Hence, $i(\widetilde{W}(t, n-1)) = 2$.

Case (ii): Let the switched vertex be a vertex of the innermost cycle in $W(t, n-1)$.

Without loss of generality, let the switched vertex be $v_{1,1}$. By arguing as in above Case (i), $i(\widetilde{W}(t, n-1)) > 1$. Since $N[v_1] \cup N[v_2] \neq V(\widetilde{W}(t, n-1))$ for any two non-adjacent vertices

v_1 and v_2 in $\widetilde{W}(t,n-1)$), it follows that $i(\widetilde{W}(t,n-1)) > 2$. Now, the three pairwise non-adjacent vertices, namely, $v_{1,1}, v_{2,1}$ and c dominate all the vertices of $\widetilde{W}(t,n-1)$). Hence, for any independent dominating set S of $\widetilde{W}(t,n-1)$, $\mid S \mid \geq 3$ implies that $i(\widetilde{W}(t,n-1)) = 3$.

Case (iii): Let the switched vertex be the apex vertex of $W(t,n-1)$.

In this case, the apex vertex dominates all the vertices of $W(t,n-1)$ except the vertices of the innermost cycle C_{n-1} of $W(t,n-1)$. Now, by Proposition 2.2, $i(C_{n-1}) = \left\lceil \frac{n-1}{3} \right\rceil$. Therefore, at least $\left\lceil \frac{n-1}{3} \right\rceil + 1$ pairwise non-adjacent vertices are essential to dominate all the vertices of $i(\widetilde{W}(t,n-1))$. Hence, $i(\widetilde{W}(t,n-1)) = \left\lceil \frac{n-1}{3} \right\rceil + 1$.

Case (iv): Let the switched vertex be a vertex of m^{th} cycle C_{n-1} where $1 < m \leq t$.

Subcase (i) Let the switched vertex be a vertex of the outermost cycle (t^{th} cycle) in $W(t,n-1)$.

Without loss of generality, let the switched vertex be $v_{t,1}$. Here, the pendant vertex u_1 becomes the isolated vertex of $\widetilde{W}(t,n-1)$. Hence, every dominating set of $\widetilde{W}(t,n-1)$ must contain this isolated vertex. Since any independent set of graph G of order n containing a vertex of maximum degree $\triangle(G)$ contains at most $n - \triangle(G)$ vertices, $i(G) \leq n - \triangle(G)$. Hence, $i(\widetilde{W}(t,n-1)) \leq t(n-1) + n - [t(n-1) + n - 5] = 5$. Since $deg(u) \leq 5$ for all $u \in V(\widetilde{W}(t,n-1)) - \{v_{t,1}, c\}$ and from the structure of graph, one can observe that four pairwise non-adjacent vertices of $\widetilde{W}(t,n-1)$ are not enough to dominate all the vertices of $\widetilde{W}(t,n-1)$. But the five pairwise non-adjacent vertices namely, $u_1, v_{t,1}, v_{t,2}, v_{t,n-1}$ and $v_{(t-1),1}$ dominate all the vertices of $\widetilde{W}(t,n-1)$ and $i(\widetilde{W}(t,n-1)) \leq 5$. Therefore, $i(\widetilde{W}(t,n-1)) = 5$.

Subcase (ii): Let the switched vertex be a vertex of r^{th} cycle in $W(t,n-1)$ where $1 < r < t$.

Without loss of generality, let the switched vertex be $v_{2,1}$ of second cycle in $W(t,n-1)$. By arguing similar to Subcase (i), $i(\widetilde{W}(t,n-1)) > 4$ and $i(\widetilde{W}(t,n-1)) \leq 5$. Moreover, the five pairwise non-adjacent vertices, namely, $v_{2,1}, v_{2,2}, v_{2,n-1}, v_{3,1}$ and $v_{1,1}$ dominate all the vertices of $\widetilde{W}(t,n-1)$. Hence, $i(\widetilde{W}(t,n-1)) = 5$.
Thus, we prove that

$$i(\widetilde{W}(t,n-1)) = \begin{cases} 2 & \text{if } v \text{ is a pendant vertex ,} \\ 3 & \text{if } v \text{ is a vertex of the innermost cycle ,} \\ \left\lceil \frac{n-1}{3} \right\rceil + 1 & \text{if } v \text{ is the apex vertex ,} \\ 5 & \text{otherwise} (n \neq 4). \end{cases} \qquad \blacksquare$$

Remark 2.13. For $n = 4$ in Theorem 2.12., if we switch a vertex of m^{th} cycle where $1 < m \leq t$ then one can observe that at least four pairwise non-adjacent vertices are essential to dominate all the vertices of $\widetilde{W}(t,3)$. Therefore, for any independent dominating set S of $\widetilde{W}(t,3)$, $|S| \geq 4$ implies that $i(\widetilde{W}(t,3)) = 4$.

Illustration 2.14. In Figure 2, the graph $\widetilde{W}(2,4)$ obtained by switching of the apex vertex c is shown in which the set of solid vertices is its independent dominating set with minimum cardinality.

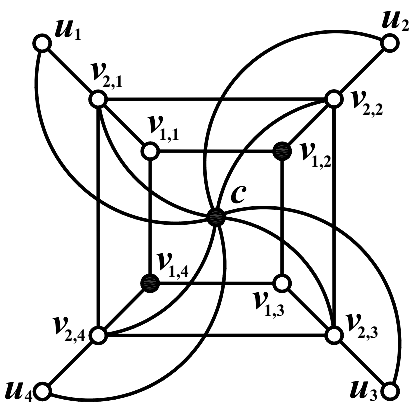

Figure 2

3 Concluding Remarks

Dominating sets of small cardinality are frequently used for backbone structures in any communication network. Many applications of domination in graphs can be extended to the theory of independent dominating sets. Independent dominating sets in wireless networks are used in a variety of applications, especially at the lower layers directly involved with communication strategies and the topology of the communication network. In any network when the existing link(s) between the nodes is/are failed, then the concept of switching of a vertex comes to rescue. This concept is widely used for fault detection and fault tolerance. The concept of independent domination in the context of switching of a vertex in various graphs is earlier studied by Vaidya and Pandit [14]. Here, we further explore the concept of independent domination in the context of switching of a vertex in graphs.

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