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Some results on super root square mean labeling

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Abstract

Let G be a (p,q) graph and $f: V(G) \to \{1,2,3,\dots,p+q\}$ be an injective function. For each edge $= uv$, let $f^*(e=uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called a super root square mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,\dots,p+q\}$. A graph that admits a super root square mean labeling is called as super root square mean graph. In this paper we prove that Double triangular snake, Alternate double triangular snake, Double quadrilateral snake and Alternate Double quadrilateral snake graphs are super root square mean graphs.

Keywords: Root Square mean graph, Super Root Square mean graph, Triangular snake, Double triangular snake, Quadrilateral snake, Double quadrilateral snake.

AMS Subject Classification (2010): 05C78.

1 Introduction

All graphs in this paper are finite, simple and undirected graph $G = (V, E)$ with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. The concept of Root Square mean labeling was introduced by Sandhya, Somasundaram and Anusa in [3] and proved many results in [4, 5, 6, 7, 8]. In this paper we define super root square mean labeling and proved that $D(T_n), A(D(T_n), D(Q_n), A(D(Q_n)$ are Super root square mean graphs. The following definitions are useful for the present study.

Definition 1.1. Let G be a (p,q) graph and $f: V(G) \to \{1,2,3,\dots,p+q\}$ be an injective function. For each edge $= uv$, let $f^*(e=uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called a super root square mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,\dots,p+q\}$.

Definition 1.2. A Triangular snake T_n is obtained from a path $u_1, u_{2,} u_3, \cdots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le n-1$.

Definition 1.3. A Double Triangular Snake $D(T_n)$ consists of two Triangular snakes that have a common path.

Definition 1.4. An Alternate Double Triangular Snake $A(D(T_n))$ consists of two Alternate Triangular snakes that have a common path.

Definition 1.5. A Quadrilateral snake Q_n is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to new vertices v_i and w_i , $1 \le i \le n-1$ respectively and then joining v_i and w_i.

Definition 1.6. A Double Quadrilateral snake $D(Q_n)$ consists of two Quadrilateral snakes that have a common path.

Definition 1.7. An Alternate Double Quadrilateral snake $A(D(Q_n))$ consists of two Alternate Quadrilateral snakes that have a common path.

The following theorems are useful for the present study.

Theorem 1.8. Any Path P_n is a Super root square mean graph.

Theorem 1.9. Triangular snake T_n is a super root square mean graph.

Theorem 1.10. Quadrilateral snake Q_n is a super root square mean graph.

2 Main Results

Theorem 2.1. Double triangular snake $D(T_n)$ is a super root square mean graph.

Proof: Consider a path $u_1, u_2, u_3, \cdots, u_n$ .Join u_i and u_{i+1} ,$1 \le i \le n-1$ to two new vertices v_i , w_i, $1 \le i \le n-1$.Define a function $f: V(D(T_n)) \to \{1,2,\dots,p+q\}$ by

$f(u_i) = 8i-7, 1 \le i \le n,$

$f(v_i) = 8i-5, 1 \le i \le n-1,$

$f(w_i) = 8i-2, 1 \le i \le n-1.$

The edges are labeled as

$f(u_i u_{i+1}) = 8i-3, 1 \le i \le n-1,$

$f(u_i v_i) = 8i-6, \ 1 \le i \le n-1,$

$f(u_{i+1} v_i) = 8i-1, 1 \le i \le n-1,$

$f(u_i w_i) = 8i-4, 1 \le i \le n-1,$

$f(u_{i+1} w_i) = 8i, 1 \le i \le n-1.$

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,\dots,p+q\}$. Hence double triangular snake graph is a super root square mean graph.

Example 2.2. Super root square mean labeling of $D(T_5)$ is shown below.

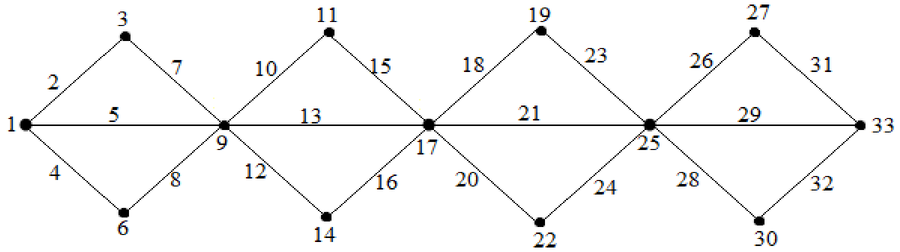

Figure 1: Super root square mean labeling of $D(T_5)$.

Theorem 2.3. Alternate double triangular snake $A(D(T_n))$ is a super root square mean graph.

Proof: Let G be the graph $A(D(T_n))$ .Consider a path $u_1, u_2, u_3, \cdots, u_n$ .To construct G ,join u_i and u_{i+1} (Alternatively) with two new vertices v_i and w_i, $1 \leq i \leq n-1$.There are two different cases to be considered.

Case 1: If the Double Triangle starts from u_1, then we consider two sub cases.

Sub Case 1(a): If n is even, then

Define a function $f: V(G) \to \{1,2, \dots p+q\}$ by

$f(u_{2i-1}) = 10i - 9, 1 \leq i \leq \frac{n}{2}$,

$f(u_{2i}) = 10i - 1, 1 \leq i \leq \frac{n}{2}$,

$f(v_i) = 10i - 7, 1 \leq i \leq \frac{n}{2}$,

$f(w_i) = 10i - 4,\ 1 \leq i \leq \frac{n}{2}$.

The edges are labeled as

$f(u_i u_{i+1}) = 5i\ ,\ 1 \leq i \leq n-1$,

$f(u_{2i-1} v_i) = 10i - 8,\ 1 \leq i \leq \frac{n}{2}$,

$f(u_{2i} v_i) = 10i - 3, 1 \leq i \leq \frac{n}{2}$,

$f(u_{2i-1} w_i) = 10i - 6, 1 \leq i \leq \frac{n}{2}$,

$f(u_{2i} w_i) = 10i - 2, 1 \leq i \leq \frac{n}{2}$.

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2, \dots, p+q\}$. Hence double triangular snake graph is a Super root square mean graph.

Sub Case 1(b): If n is odd then

Define a function $f: V(G) \to \{1,2, \dots, p+q\}$ by

$f(u_{2i-1}) = 10i - 9, 1 \leq i \leq \frac{n+1}{2}$,

$f(u_{2i}) = 10i - 1, 1 \leq i \leq \frac{n-1}{2}$,

$f(v_i) = 10i - 7, 1 \leq i \leq \frac{n-1}{2}$,

$f(w_i) = 10i - 4,\ 1 \leq i \leq \frac{n-1}{2}$.

The edges are labeled as

$f(u_i u_{i+1}) = 5i\ ,\ 1 \leq i \leq n - 1$,

$f(u_{2i-1} v_i) = 10i - 8,\ 1 \leq i \leq \frac{n-1}{2}$,

$f(u_{2i} v_i) = 10i - 3, 1 \leq i \leq \frac{n-1}{2}$,

$f(u_{2i-1} w_i) = 10i - 6, 1 \leq i \leq \frac{n-1}{2}$,

$f(u_{2i} w_i) = 10i - 2, 1 \leq i \leq \frac{n-1}{2}$.

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2, \dots, p+q\}$. Hence alternate double triangular snake graph is a super root square mean graph.

The labeling pattern is shown below.

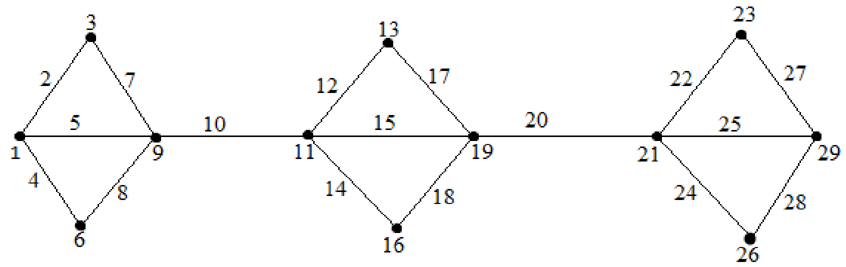

Figure 2

Case 2: If the triangle starts from u_2 , then we have to consider two sub cases.

Sub Case 1(a): If n is even, then

Define a function $f: V(G) \rightarrow \{1,2, \dots p+q\}$ by

$f(u_{2i-1}) = 10i - 9, 1 \leq i \leq \frac{n}{2}$,

$f(u_{2i}) = 10i - 7, 1 \leq i \leq \frac{n}{2}$,

$f(v_i) = 10i - 3, 1 \leq i \leq \frac{n-2}{2}$,

$f(w_i) = 10i - 4,\ 1 \leq i \leq \frac{n-2}{2}$.

The edges are labeled as

$f(u_{2i-1} u_{2i}) = 10i - 8\ ,\ 1 \leq i \leq \frac{n}{2}$,

$f(u_{2i} u_{2i+1}) = 10i - 2\ ,\ 1 \leq i \leq \frac{n-2}{2}$,

$f(u_{2i} v_i) = 10i - 5,\ 1 \leq i \leq \frac{n-2}{2}$,

$f(u_{2i+1} v_i) = 10i, 1 \leq i \leq \frac{n-2}{2}$,

$f(u_{2i+1} w_i) = 10i - 1, 1 \leq i \leq \frac{n-2}{2}$,

$f(u_{2i}w_i) = 10i - 6, 1 \le i \le \frac{n-2}{2}$.

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,\dots,p+q\}$. Hence alternate double triangular snake graph is a super root square mean graph.

Sub Case 1(b): If n is odd then

Define a function $f: V(G) \to \{1,2,\dots p+q\}$ by

$f(u_{2i-1}) = 10i - 9, 1 \le i \le \frac{n+1}{2}$,

$f(u_{2i}) = 10i - 7, 1 \le i \le \frac{n-1}{2}$,

$f(v_i) = 10i - 3, 1 \le i \le \frac{n-1}{2}$,

$f(w_i) = 10i - 4,\ 1 \le i \le \frac{n-1}{2}$.

The edges are labeled as

$f(u_{2i-1}u_{2i}) = 10i - 8,\ 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i}u_{2i+1}) = 10i - 2,\ 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i}v_i) = 10i - 5,\ 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i+1}v_i) = 10i, 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i+1}w_i) = 10i - 1, 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i}w_i) = 10i - 6, 1 \le i \le \frac{n-1}{2}$.

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,\dots,p+q\}$. Hence Alternate double triangular snake graph is a Super root square mean graph.

The labeling pattern is shown below.

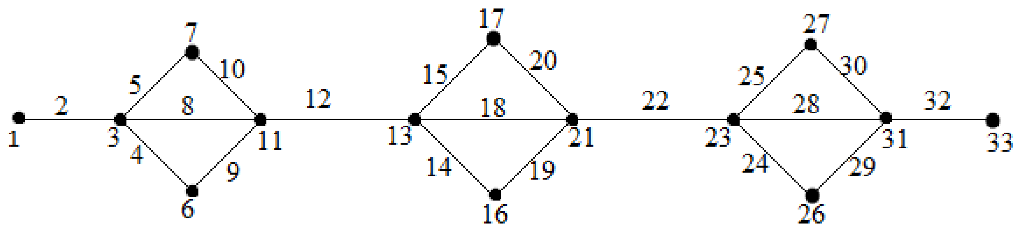

Figure 3

Theorem 2.4. Double quadrilateral snake graph $D(Q_n)$ are super root square mean graphs.

Proof: Let P_n be the path $u_1, u_2, u_3, \cdots, u_n$ .To construct $D(Q_n)$, join u_i and u_{i+1} to four new vertices $v_i, w_i, t_i$and s_i by the edges $u_iv_i,\ u_{i+1}w_i,\ v_iw_i,\ u_it_i,\ u_{i+1}s_i$ and s_it_i, for $1 \le i \le n-1$.

Define a function $f: V(D(Q_n)) \to \{1,2,\dots p+q\}$ by

$f(u_i) = 12i - 11, \quad 1 \le i \le n$,

$f(v_i) = 12i - 9, \quad 1 \le i \le n-1,$
$f(w_i) = 12i - 5, \ 1 \le i \le n-1,$
$f(t_i) = 12i - 6, \ 1 \le i \le n-1,$
$f(s_i) = 12i - 1, \ 1 \le i \le n-1.$

The edges are labeled as

$f(u_1u_2) = 9, \quad f(u_iu_{i+1}) = 12i - 4, 2 \le i \le n-1$
$f(u_iv_i) = 12i - 10, \ 1 \le i \le n-1$
$f(u_{i+1}w_i) = 12i - 2, 1 \le i \le n-1,$
$f(v_iw_i) = 12i - 7, 1 \le i \le n-1,$
$f(u_it_i) = 12i - 8, 1 \le i \le n-1,$
$f(u_{i+1}s_i) = 12i, 1 \le i \le n-1,$
$f(t_1s_1) = 8, \quad f(t_is_i) = 12i - 3, 2 \le i \le n-1.$

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2,\ldots,p+q\}$. Hence double quadrilateral snake graph is a super root square mean graph.

Example 2.5. The labeling pattern of $D(Q_4)$ is shown below.

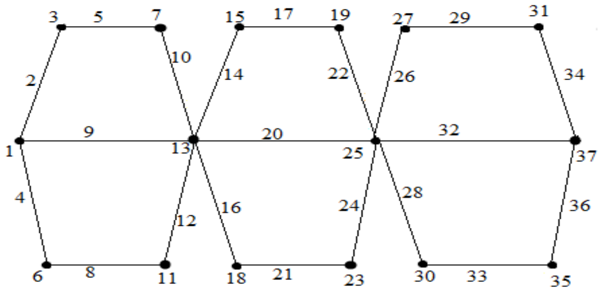

Figure 4

Theorem 2.6. Alternate Double Quadrilateral snake graphs $A(D(Q_n))$ is a super root square mean graph.

Proof: Let G be the alternate double quadrilateral snake $A(D(Q_n))$ .Consider a path $u_1, u_2, u_3, \cdots, u_n$ .Join u_i and u_{i+1} (Alternatively) with to four new vertices v_i, w_i, t_i and s_i.Here we consider two different cases.

Case 1: If the double quadrilateral starts from u_1 , then we consider two sub cases.

Sub Case 1(a): If n is even then

Define a function $f: V(G) \to \{1,2,\ldots p+q\}$ by

$f(u_{2i-1}) = 14i - 13, 1 \le i \le \frac{n}{2},$

$f(u_{2i}) = 14i - 1, 1 \le i \le \frac{n}{2},$

$f(v_i) = 14i - 11, 1 \le i \le \frac{n}{2},$

$f(w_i) = 14i - 7, \ 1 \le i \le \frac{n}{2},$

$f(t_i) = 14i - 8,\ 1 \le i \le \frac{n}{2}$,

$f(s_i) = 14i - 3,\ 1 \le i \le \frac{n}{2}$.

The edges are labeled as

$f(u_1u_2) = 9,\ f(u_{2i-1}u_{2i}) = 14i - 6\ ,\ 2 \le i \le \frac{n}{2}$,

$f(u_{2i}u_{2i+1}) = 14i\ ,\ 1 \le i \le \frac{n-2}{2}$,

$f(u_{2i-1}v_i) = 14i - 12,\ \ 1 \le i \le \frac{n}{2}$,

$f(v_iw_i) = 14i - 9,\ \ 1 \le i \le \frac{n}{2}$,

$f(w_iu_{2i}) = 14i - 4, 1 \le i \le \frac{n}{2}$,

$f(u_{2i-1}t_i) = 14i - 10, 1 \le i \le \frac{n}{2}$,

$f(t_1s_1) = 8,\ \ f(t_is_i) = 14i - 5,\ \ 2 \le i \le \frac{n}{2}$,

$f(s_iu_{2i}) = 14i - 2, 1 \le i \le \frac{n}{2}$.

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2, \dots , p + q\}$. Hence Alternate double quadrilateral snake graph is a super root square mean graph.

Sub Case 1(b): If n is odd then

Define a function $f: V(G) \to \{1,2, \dots p + q\}$ by

$f(u_{2i-1}) = 14i - 13, 1 \le i \le \frac{n+1}{2}$,

$f(u_{2i}) = 14i - 1, 1 \le i \le \frac{n-1}{2}$,

$f(v_i) = 14i - 11, 1 \le i \le \dfrac{n-1}{2}$,

$f(w_i) = 14i - 7,\ 1 \le i \le \frac{n-1}{2}$,

$f(t_i) = 14i - 8,\ 1 \le i \le \frac{n-1}{2}$,

$f(s_i) = 14i - 3,\ 1 \le i \le \frac{n-1}{2}$.

The edges are labeled as

$f(u_1u_2) = 9,\ f(u_{2i-1}u_{2i}) = 14i - 6\ ,\ 2 \le i \le \frac{n-1}{2}$,

$f(u_{2i}u_{2i+1}) = 14i\ ,\ 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i-1}v_i) = 14i - 12,\ \ 1 \le i \le \frac{n-1}{2}$,

$f(v_iw_i) = 14i - 9,\ \ 1 \le i \le \frac{n-1}{2}$,

$f(w_iu_{2i}) = 14i - 4, 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i-1}t_i) = 14i - 10, 1 \le i \le \frac{n-1}{2}$,

$f(t_1s_1) = 8,\ \ f(t_is_i) = 14i - 5,\ \ 2 \le i \le \frac{n-1}{2}$,

$f(s_i u_{2i}) = 14i - 2, 1 \leq i \leq \frac{n-1}{2}$.

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2, \dots, p+q\}$. Hence alternate double quadrilateral snake graph is a super root square mean graph.

The labeling pattern is shown below.

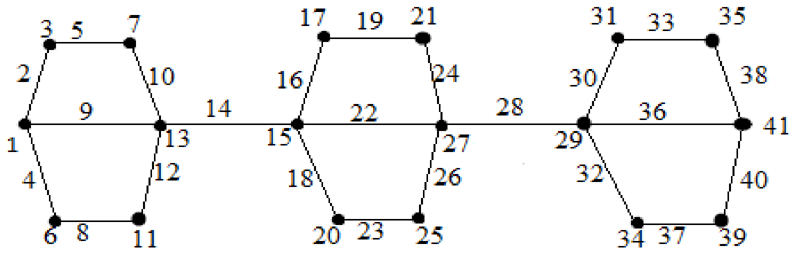

Figure 5

Case 2: If the double quadrilateral starts from u_2 , then we consider two sub cases.

Sub Case 2(a): If n is even then

Define a function $f: V(G) \to \{1,2, \dots p+q\}$ by

$f(u_{2i-1}) = 14i - 13, 1 \leq i \leq \frac{n}{2}$,

$f(u_{2i}) = 14i - 11, 1 \leq i \leq \frac{n}{2}$,

$f(v_i) = 14i - 7, 1 \leq i \leq \frac{n-2}{2}$,

$f(w_i) = 14i - 1, \ 1 \leq i \leq \frac{n-2}{2}$,

$f(t_i) = 14i - 5, \ 1 \leq i \leq \frac{n-2}{2}$,

$f(s_i) = 14i - 8, \ 1 \leq i \leq \frac{n-2}{2}$.

The edges are labeled as

$f(u_{2i-1} u_{2i}) = 14i - 12 , \ 1 \leq i \leq \frac{n}{2}$,

$f(u_{2i} u_{2i+1}) = 14i - 4 , \ 1 \leq i \leq \frac{n-2}{2}$,

$f(u_{2i} v_i) = 14i - 9, \ 1 \leq i \leq \frac{n-2}{2}$,

$f(v_i w_i) = 14i - 3, \ 1 \leq i \leq \frac{n-2}{2}$,

$f(w_i u_{2i+1}) = 14i, 1 \leq i \leq \frac{n-2}{2}$,

$f(u_{2i} t_i) = 14i - 10, 1 \leq i \leq \frac{n-2}{2}$,

$f(t_i s_i) = 14i - 6, \ 1 \leq i \leq \frac{n-2}{2}$,

$f(s_i u_{2i+1}) = 14i - 2, 1 \leq i \leq \frac{n-2}{2}$.

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2, \dots, p+q\}$. Hence alternate double quadrilateral snake graph is a super root square mean graph.

Sub Case 2(b): If n is odd then

Define a function $f: V(G) \to \{1,2, \dots p + q\}$ by

$f(u_{2i-1}) = 14i - 13, 1 \le i \le \frac{n+1}{2}$,

$f(u_{2i}) = 14i - 11, 1 \le i \le \frac{n-1}{2}$,

$f(v_i) = 14i - 7, 1 \le i \le \dfrac{n-1}{2}$,

$f(w_i) = 14i - 1,\ 1 \le i \le \frac{n-1}{2}$,

$f(t_i) = 14i - 5,\ 1 \le i \le \frac{n-1}{2}$,

$f(s_i) = 14i - 8,\ 1 \le i \le \frac{n-1}{2}$.

The edges are labeled as

$f(u_{2i-1}u_{2i}) = 14i - 12\,,\ 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i}u_{2i+1}) = 14i - 4\,,\ 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i}v_i) = 14i - 9,\ \ 1 \le i \le \frac{n-1}{2}$,

$f(v_iw_i) = 14i - 3,\ \ 1 \le i \le \frac{n-1}{2}$,

$f(w_iu_{2i+1}) = 14i, 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i}t_i) = 14i - 10, 1 \le i \le \frac{n-1}{2}$,

$f(t_is_i) = 14i - 6,\ \ 1 \le i \le \frac{n-1}{2}$,

$f(s_iu_{2i+1}) = 14i - 2, 1 \le i \le \frac{n-1}{2}$.

Then $f(V) \cup \{f^*(e): e \in E(G)\} = \{1,2, \dots, p + q\}$. Hence alternate double quadrilateral snake graph is a super root square mean graph.

The labeling pattern is shown below.

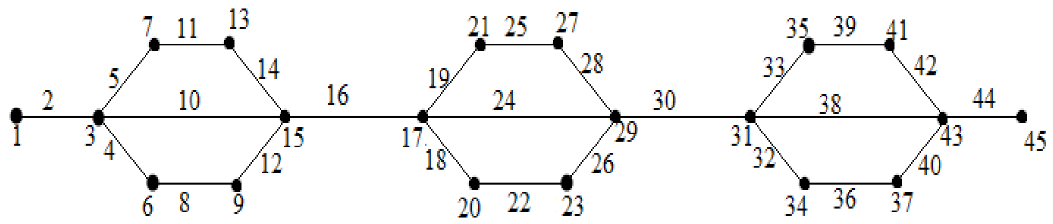

Figure 6

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